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SUMMARY: *The paper suggests a simple procedure for the design of a low order robust controller for an anti-vibration system. Anti-vibration systems are used to reduce the influence of vibrations generated in the surroundings and are essential when precise operations or measurements are required. Our design approach is based on the frequency-domain internal-model controller technique, suitably modified to account for the sensor and actuator dynamics. It uses a simplified nominal plant model with multiplicative description of both structured and unstructured plant model uncertainties. The robust performance design criterion is adopted. Simulation investigations, carried out with MATLAB and SIMULINK, are used to estimate the robustness achieved.*

Key Words: Internal model control; robust performance; frequency domain; anti-vibration system; simulation.

1 INTRODUCTION

Precise measurements and operations require a special environment with reduced perturbing vibrations. Anti-vibration control is essential in calibration systems,[1] mobile equipment and suspension systems,[2] warehouse stackercrane machines,[3] etc. The effectiveness of such control depends on the ability of the designed system to reduce the impact of the plant model uncertainties. The plant uncertainty is considered in the control system design by using either the fuzzy logic[3] or the robust approach.[1, 4, 5]

The fuzzy logic controller is based on a model of an experienced operator actions when a simple and accurate enough plant description cannot be derived. In the case when a nominal plant model and a plant uncertainty description are available the robust approach can be used.

A robust anti-vibration control system, based on H_∞ technique where a nominal model of a physical plant and of the plant uncertainty is derived was used to describe various systems subjected to vibrations.[2] The application of the H_∞ technique,[4] even when assisted by the corresponding software,[6] makes the design procedure rather complex. It requires the controller order reduction to ease implementation, special representation of the performance specifications, solution of an optimization problem to account for possible local minima and iterations in case robustness is not achieved in the first attempt.

On the other hand, the internal model controller technique[5] leads directly to a low order robust controller thanks to the simplified nominal plant model used. The advantages of the method are: easy implementation, simple design procedure and direct relationship between model uncertainty and robustness achieved. This technique has been already successfully applied in the design of the robust control for anaerobic wastewater treatment processes[7] and for measurement systems.[8, 9, 10]

The aim of the present investigation is to develop a simple procedure for the design of a low order robust anti-vibration controller that accounts for the dynamics of the sensor in the feedback and the actuator. Simulation results will be used to estimate the achieved robustness.

2 DESIGN OF A MODIFIED ROBUST INTERNAL MODEL CONTROLLER

The plant consists of two platforms A and B with masses m_1 and m_2 correspondingly. The vibrations of the ground are transmitted to platform B with attenuation due to the passive damper-spring systems at each corner of platforms A and B. In order for platform B to become an inertial body, active electromagnetic force generators controlled by a controller via actuators at each corner of platform A are introduced. The simplified plant model[1] used considers the following assumptions.

1. The ground and the two platforms are rigid bodies.
2. The ground moves only vertically.
3. The platform structure has perfect symmetry at the four corners.
4. Nonlinearities can be neglected.

These assumptions allow the four identical elements at each corner of the platforms to be equivalently represented by only one as shown in Figure 1. The plant input is the vertical component of the ground acceleration $\ddot{x}_g$, which is the main disturbance d. The plant output y to be controlled is the vertical component of the platform B acceleration $\ddot{x}_2$. The control is the electric current i that drives the electromagnetic force generator to produce the required force: $f = k_F i$.

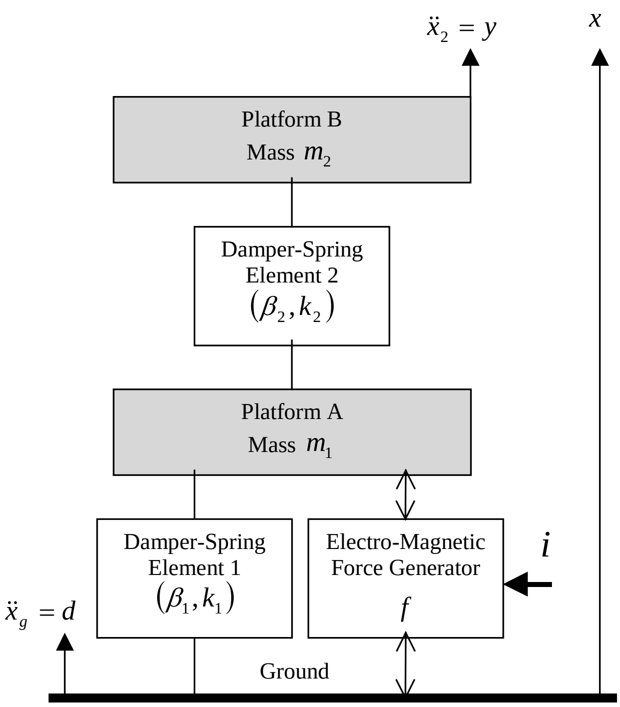

Figure 1: A Simplified Plant Model

The internal model controller $C(s)$, depicted in Figure 2, includes[5] a controller $Q(s)$ with a positive feedback. The feedback transfer function is determined by the nominal plant model $P^0(s,q^0)$, where $\mathbf{q^0} = [\mathbf{q_i^0}]$ is the vector of the nominal plant parameters in case assumptions 1-4 hold. In Figure 2 H and l denote the transfer function of the plant $H(s)$ with respect to the disturbance d and the transfer function of the multiplicative plant uncertainty $l(s)$ respectively. The reference input r is set to zero.

The controller transfer function for minimum phase stable plants is $Q(s) = F(s)\left[P^0(s,q^0)\right]^{-1}$. The filter for settling input signals or trends that ensures zero steady state error is in the form $F(s) = (\lambda s + 1)^{-n}$. The order n is selected to make $Q(s)$ proper, while λ is tuned to ensure an acceptable tradeoff between performance robustness and fast system response.[8]

A modification of the internal model controller with the block diagram represented in Figure 3 is suggested to account for the sensor $M(s)$ in the feedback and the actuator $A(s)$, denoted with M and A respectively. Thus the transfer function of the robust controller becomes:

$$R(s) = \frac{F(s)\left[P^0(s,q^0)\right]^{-1}}{1 - F(s)} A^{-1}(s) M^{-1}(s). \tag{1}$$

To ensure the practical realisation of the controller $R(s)$ the order of the filter $F(s)$ in this case is selected to make $R(s)$ proper.

The system sensitivity $S(s)$ and the complementary sensitivity $T(s)$ that represents the transfer function of the closed-loop system shown in Figure 3, are respectively:

$$S(s) = \left[1 + G_{ol}(s)\right]^{-1} \tag{2}$$

$$T(s) = G_{ol}(s)\left[1 + G_{ol}(s)\right]^{-1}, \tag{3}$$

$$T(s) + S(s) = 1,$$

where the open-loop transfer function is:

$$G_{ol}(s) = A(s) P^0(s,q^0)\left[1 + l(s)\right] M(s) R(s)$$

The overall multiplicative uncertainty $l(s)$ is assumed to consist of a series connection of two components:

$$\left[1 + l_1(s)\right].\left[1 + l_2(s)\right] = 1 + l(s)$$

$$\therefore l(s) = l_1(s) + l_2(s) + l_1(s) l_2(s), \quad (4)$$

where :

$l_1(s)$ is a multiplicative unstructured uncertainty, related to the unmodelled dynamics, that reflects the ground deformability and nonperfect structure symmetry, which gives rise to rotating motion components of the platforms;

$l_2(s)$ is a multiplicative structured uncertainty resulting from the plant parameter variations Δq_i, that accounts for the stiffness of the springs sustaining platform A, the ground elasticity and the asymmetry of the structure, neglected in the nominal model.

If all possible characteristics of the plant for $q \in D_q$ are enveloped by the characteristics of the plants $P(s, q^1)$ and $P^0(s, q^0)$ then

$$l_2(s) = \frac{P(s, q^1) - P^0(s, q^0)}{P^0(s, q^0)}.$$

The tuning of the controller $R(s)$ consists of the determination of the smallest possible value of the filter time constant $\lambda\ (\lambda > 0)$ that satisfies the robust performance criterion:

$$|W(j\omega)|_{\max} |S(j\omega)| + |l(j\omega) T(j\omega)| < 1 \quad (5)$$

for $\omega \in D_\omega$,

where $|W(j\omega)|$ is the amplitude of the input signal with $|W(j\omega)|_{\max} = 0.5$ for the frequency range of interest D_ω.[5]

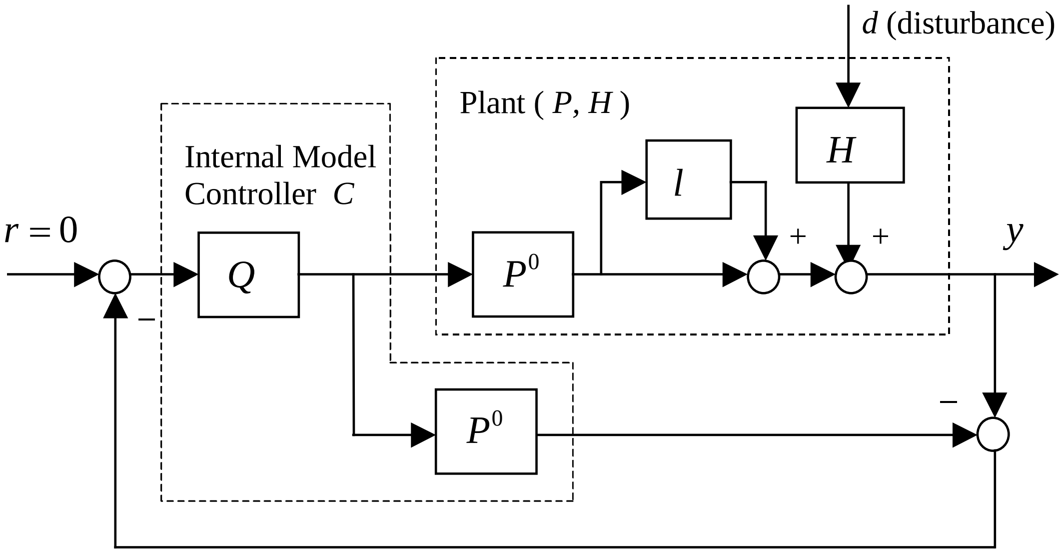

Figure 2: A Block Diagram of a Robust Control System with Internal Model Controller

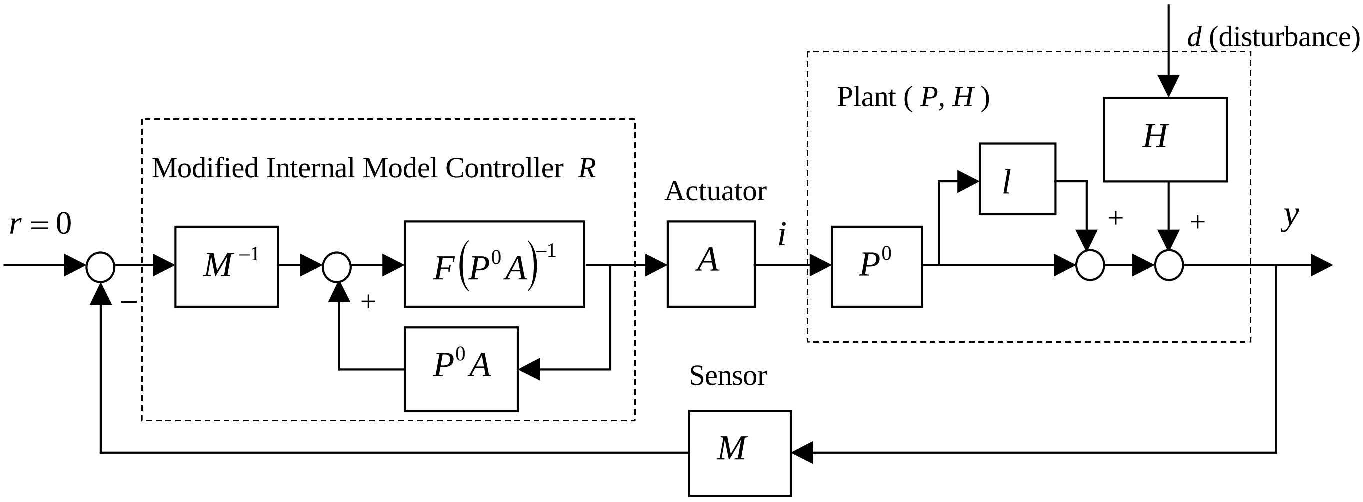

Figure 3: A Block Diagram of a Robust Control System with Modified Internal Model Controller

3 SIMULATION INVESTIGATIONS

The suggested modified internal model controller technique is applied to design a robust anti-vibration system for the control of a plant. The plant transfer functions with respect to control $P(s,q)$ and disturbance $H(s,q)$ as well as the transfer functions of the actuator $A(s)$ and the sensor $M(s)$ are defined as: [1]

$$P(s,q) = \frac{k_F k_1^{-1} s^2 (\beta_2 k_2^{-1} s + 1)}{D(s,q)} \quad (6)$$

$$H(s,q) = \frac{(\beta_1 k_1^{-1} s + 1)(\beta_2 k_2^{-1} s + 1)}{D(s,q)} \quad (7)$$

$$A(s) = k_a \quad (8)$$

$$M(s) = k_m (T_m s + 1)^{-1} \quad (9)$$

where:

$$\begin{aligned} D(s,q) &= m_1 m_2 k_1^{-1} k_2^{-1} s^4 + \\ &+ \left[m_1 \beta_2 + m_2 \left(\beta_1 + \beta_2\right)\right] k_1^{-1} k_2^{-1} s^3 + \\ &+ \left[m_1 k_1^{-1} + m_2 \left(k_1^{-1} + k_2^{-1}\right) + \beta_1 \beta_2 k_1^{-1} k_2^{-1}\right] s^2 + \\ &+ \left(\beta_1 k_1^{-1} + \beta_2 k_2^{-1}\right) s + 1 \end{aligned}$$

m_1 and m_2 are the masses of the platforms,
k_1 and k_2 are the stiffness coefficients of the springs and
β_1 and β_2 are the damping coefficients of the dampers.

The parameters k_a, k_m, T_m and k_F take values from the data sheets of the corresponding components and are considered as constants with values:

$$k_a = 10.0 AV^{-1},$$
$$k_m = 10^5 Vs^3 m^{-1},$$
$$T_m = 2s,$$
$$k_F = 8.7 NA^{-1}.$$

The mass m_2 is a constant too ($m_2 = 440kg$), since it is directly measured. Thus the vector of the plant parameters can be represented by:

$$q^T = [k_1 \; k_2 \;\; \beta_1 \; \beta_2 \; m_1] = [q_i],$$

where:

$q_i = q_i^0 \pm \Delta q_i$ and $q_i \in \left[q_i^-, q_i^+\right]$,

where:

$$q_i^- = q_i^0 - \Delta q_i,$$
$$q_i^+ = q_i^0 + \Delta q_i,$$

The following parameters are estimated from the experimental data and their values are: [1]

$$k_1 = (1.4 \pm 0.3) 10^6 \, Nm^{-1},$$
$$k_2 = (1 \pm 0.3) 10^6 \, Nm^{-1},$$
$$\beta_1 = (4.8 \pm 1) 10^4 \, Nsm^{-1},$$
$$\beta_2 = (1.7 \pm 0.3) 10^4 \, Nsm^{-1},$$
$$m_1 = (4.2 \pm 0.7) 10^3 \, kg.$$

The parameter q_i is known to belong to the parameter uncertainty set, defined as:

$$D_q = \left\{ q \in R_i^5 \left\| q - q^0 \right\|_\infty^{\Delta q} \le 1 \right\} .$$

The transfer function $l_1(s)$ is experimentally derived from the magnitude frequency characteristic, which envelops the peaks in the plant magnitude frequency characteristic at high frequencies $\omega > 1000 rad.s^{-1}$ and is represented by: [1]

$$l_1(s) = \frac{1.22s}{1000 + s} . \quad (10)$$

The simulation shows that characteristics of the plant with various parameters $q \in D_q$ are located between the characteristics of the nominal plant $P^0(s,q^0)$ and of the minimum parameter plant $P^-(s,q^-)$, hence:

$$l_2(s) = \frac{\left[P^-(s,q^-) - P^0(s,q^0)\right]}{P^0(s,q^0)} . \quad (11)$$

The control objective is to achieve robust performance in the closed-loop system with the uncertainty defined by (10) and (11) and parameters $q \in D_q$ and disturbance attenuation at a restriction for the r.m.s. value for the current of the actuator $i \le 10A$. The frequency range where the spectral density of the disturbance has more then 95% of its power is experimentally obtained as: [1]

$$D_\omega = \left[0.63 \div 57\right] rad.s^{-1} .$$

The modified robust internal model controller for the plant defined by (6) and (7) according to (1), has the transfer function:

$$R(s) = \frac{T_m s + 1}{\lambda(\lambda s + 1) k_m k_a} \frac{[P^0(s, q^0)]^{-1}}{s} . \qquad (12)$$

The tradeoff from the model accuracy requirements with the simplification in the nominal plant model with respect to order reduction is compensated in the robustness design criterion by defining the corresponding multiplicative uncertainty.

The order of the filter $F(s)$ that makes $R(s)$ proper is $n = 2$. The parameter λ is tuned to satisfy the robust criterion (5) and the constraint $i \leq 10A$. It is obtained to be $\lambda = 0.004$. Figure 4 shows the plot of $|0.5S(j\omega)| + |l(j\omega)T(j\omega)|$ for $\lambda = 0.004$. It can be seen that the robustness criterion

$$|0.5S(j\omega)| + |l(j\omega)T(j\omega)| < 1$$

holds over the frequency range of interest.

Without any approximations or order reduction, the order of the controller $R(s)$ given by (12) is five and it depends on the transfer functions of the simplified nominal plant model, the actuator, the sensor and the required filter.

The application of the H_∞ approach[1] results initially in a robust controller with the order that is much higher than the order of controller $R(s)$ given by (12). After order reduction the H_∞ controller is reduced to a 4th order controller with the following transfer function:

$$R_H(s) = 1.4 \frac{(2s+1)(0.02s+1)^2(0.001s+1)}{0.005s^4 + s^3} . \qquad (13)$$

We would like to point out that the approach presented in this paper is simpler and gives a lower order controller than the H_∞ technique[1] before the order reduction.

The performance of the robust controller is investigated by comparing the robust properties of the three systems:

- system without the controller
- system with the controller based on the H_∞ approach[1] and
- system with the robust modified internal model controller presented in this paper.

The three systems are simulated using MATLAB and SIMULINK with sinusoidal disturbances d of magnitude 0.001 and frequencies $\omega_1 = 50 rad.s^{-1}$ and $\omega_2 = 5 rad.s^{-1}$ from the frequency range of interest D_ω.

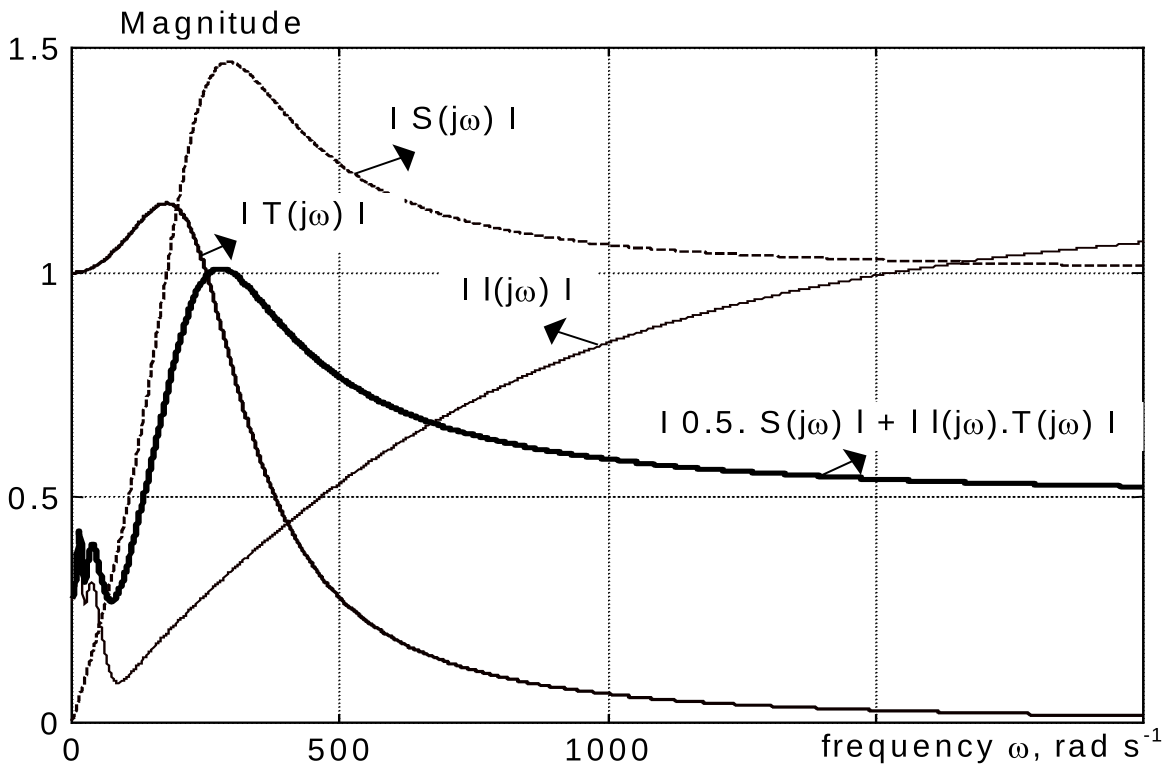

Figure 4: Robust Performance of the Control System

Responses of the three systems are comparatively shown in Figure 5 and Figure 6 and are labeled as:

- y_{ol} - output of the system without a controller
- y_H - output of the system with a controller based on H_∞ approach[1] and
- y_r - output of the system with the robust modified internal model controller.

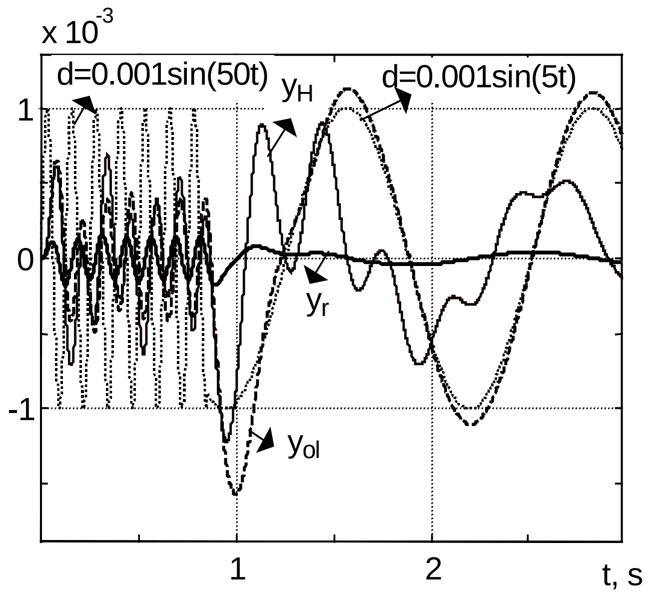

Figure 5: Control System Responses for the Nominal, Unperturbed Plant (P^0, H^0).

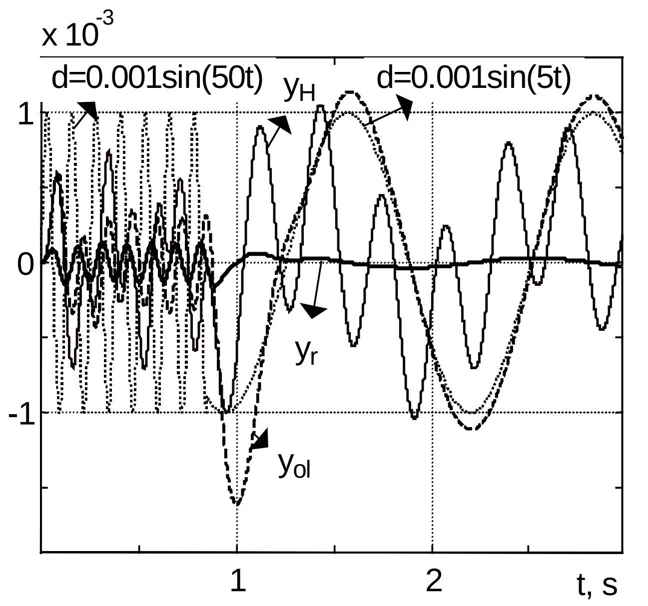

Figure 6: Control System Responses for the Perturbed Plant (P, H)

Figure 5 shows simulation results for those three systems for the nominal plant model without perturbations. Figure 6 shows responses of the three systems for the perturbed plant (minimum parameter plant with unmodelled dynamics). These results also demonstrate another advantage of the suggested approach. At the higher frequency disturbance the output y_r of the system with the modified internal model controller has the amplitude approximately two times smaller than both the output y_{ol} of the system without a controller and the output y_H of the system with H_∞ based controller. At the lower frequency disturbance the amplitude of y_r becomes insignificantly small unlike the amplitudes of y_{ol} and y_H, which have nearly doubled. Besides, only the response y_r is not influenced by the plant uncertainties, which is the aim of achieving a good robustness.

4 CONCLUSION

A simple procedure for the design of a low order robust anti-vibration controller is developed, based on a modification of the internal model controller technique. The procedure is applied for the control of the plant, described in[1] under plant uncertainties. The modified controller accounts for the presence of a sensor in the feedback, and an actuator. The plant structured uncertainties are represented in a multiplicative form and are united with the unstructured multiplicative uncertainties. Then the required filter time constant is tuned to satisfy the robust performance criterion with respect to both structured and unstructured plant uncertainties and the expected disturbances.

The procedure leads to a low order controller and is simpler than H_∞ technique[1]. The responses of the three systems: the system without a controller, the system based on the new approach presented in this paper and the system with the controller based on H_∞ technique[1] are simulated using MATLAB and SIMULINK. The responses are then compared with respect to the rejection of low and high frequency disturbances when the plant is nominal and when perturbed. The system designed using the modified internal model controller technique displays nearly two times better robust properties, thus serving its purpose in compensating vibrations to a higher degree. In addition, when compared with the H_∞ technique,[1] the proposed method gives the lower order controller when implemented directly, ie. without introducing simplifications and order reduction.

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