

Intelligent Approaches for Control of Nonlinear Plants with Significant Time Delay

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Abstract—The nonlinear plants with significant time delays are difficult to be controlled by classical means. In the present paper intelligent approaches are applied for the design of a nonlinear Smith predictor for compensation of the plant time delay based on a Takagi-Sugeno-Kang plant model and a fuzzy logic parallel distributed compensation (PDC). The design and the advantages of the PDC-Smith are illustrated in temperature control. The good compensation of both the plant time delay and nonlinearity results in faster system step responses in regard to a linear PI, a Smith and a PDC control systems.

Keywords— *genetic algorithms; nonlinear plant; parallel distributed compensation; Takagi-Sugeno-Kang model; temperature control; time delay compensation*

I. INTRODUCTION AND STATE-OF-THE-ARTS

Most industrial plants respond with time delay to applied inputs. The pure time delay τ is a mathematical generalization for transient, transport and approximation time delay due to plant parameters distribution in space, multi-capacity (high order), inertia, transportation operations and approximation error in modelling [1-5].

Plants with time delay are difficult to be controlled since the time delay decreases the system stability margins. In order to preserve the control system stability the controller's gain is restricted which results in long settling time and increased impact of disturbances. For plants with high relative time delays with respect to the dominating plant time constant $T - \tau/T > 0.5$, no effective tuning approaches for linear controllers exist that can ensure system stability and high performance (reduced settling time and overshoot, increased dynamic accuracy) unless special measures for compensation of the time delay are first applied.

A closed loop system with Smith predictor $R(s)$ for compensation of the plant time delay of a linear plant [3-5], is shown in Fig. 1, where y is the system output and y_r its reference. The plant is represented by a Ziegler-Nichols (ZN) plant model with transfer function:

$$P(s) = P_o(s)e^{-\tau s}, P_o(s) = K(Ts + 1)^{-1}. \quad (1)$$

The transfer function of the local feedback $C_{fb}(s)$ to a standard linear controller $C(s)$ is specified from the requirement to eliminate the time delay term $e^{-\tau s}$ in the system characteristic polynomial (the denominator of the

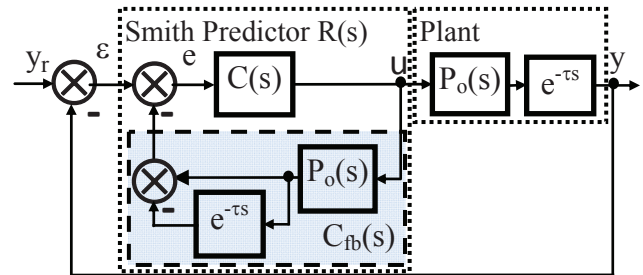


Figure 1. Smith predictor control system

closed loop system transfer function) which determines the system stability:

$$C_{fb}(s) = P_o(s)[1 - e^{-\tau s}] \quad (2)$$

The closed loop system transfer function with $C_{fb}(s)$ becomes:

$$\Phi_{\text{Smith}}(s) = C(s)P_o(s)e^{-\tau s} [1 + C(s)P_o(s)]^{-1}. \quad (3)$$

Thanks to $C_{fb}(s)$ the tuning of the controller $C(s)$ is based on the plant with no time delay $P_o(s)$ in a first or second order system. The controller can have a high gain without violating the system Nyquist stability which ensures good disturbance rejection and dynamic accuracy. The value of the gain should, however, comply with the amplitudes of the expected error ϵ in order to keep the operation of the controller in the linear range.

Despite of the advantages of the Smith predictor its industrial application is difficult because of the demand for a precise plant model in $C_{fb}(s)$. The system performance may be greatly deteriorated in case of deflection of the plant from its model caused by a shift of the operation point along nonlinear characteristics, aging with time, etc.

Most of the contemporary plants with significant time delay are nonlinear, operate in different ranges and have model uncertainties. In order to make the Smith predictor more robust and to extend its application to nonlinear plants a fuzzy Smith predictor (FSP) based on the principle of parallel distributed compensation (PDC) is developed [6, 7]. The FSP is built on the basis of a Takagi-Sugeno-Kang (TSK) plant model of local ZN linear plants with significant time delays. It consists of two PDC fuzzy logic controllers (FLCs) in series, derived after an approximation of the local time delays

$e^{-\tau_i s} \approx (\tau_i s + 1)^{-1}$. The FSP system Lyapunov stability is also proven based on its TSK-PDC model.

Another approach for compensation of the time delay of nonlinear plants is suggested in [8] on the basis of Sugeno neuro-fuzzy structures, trained to predict the plant behavior and introduced in the system feedback to supply the PI controller with advance information about the plant output thus improving the overall performance of the control system. The proper selection of the prediction horizon, processing of the training data, generalization test and real time validation are discussed.

The aim of the present research is to develop an approach for the design of a simple PDC-based Smith predictor for a nonlinear plant with significant relative time delay using intelligent techniques – fuzzy logic and genetic algorithms (GAs). For assessment of its advantages the PDC-Smith system performance is compared to designed systems with linear PI control, linear Smith predictor (compensating the plant average time delay) and PDC with local linear PI controllers (compensating the plant nonlinearity). The novelty is the similarity to the linear Smith predictor structure but with PDC main controller and a feedback to it based on a TSK plant model with no approximation of the time delay. The approach is developed using the example of air temperature control in a laboratory-scale furnace and MATLABTM [9, 10]. The GAs are selected as an efficient gradient-free optimization approach for random parallel search of global extremum of complex nonlinear multimodal cost functions of many parameters, computed in simulations [10].

II. PLANT EXPERIMENTAL STUDY AND TSK MODELLING

The plant is a laboratory-scale electrical furnace [7], shown in Fig. 2. Its output is the inside air temperature $T, ^\circ\text{C}$ - $Y(t)=T(t)$, $Y \in [0-150]$, $^\circ\text{C}$, controlled by the output control action $U(t)$, $U \in [0-12]$ of a controller completed in a MATLABTM-Simulink model. A software Pulse-Width-Modulation turns $U(t)$ into a corresponding duty ratio which via a Solid State Relay powers an electrical heater during the pulse thus ensuring a proper average heat, emitted to the furnace air.

The experimentally obtained plant responses Y_{plant} to plant input of successive step increases with equal magnitudes $\Delta U=2$, shown in Fig. 3, are approximated by ZN models with parameters (K_i, T_i, τ_i) . The models have significant relative time delays - $\tau_i/T_i > 1$, and different

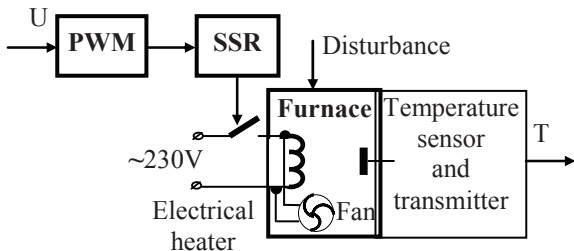


Figure 2. Laboratory-scale electrical furnace

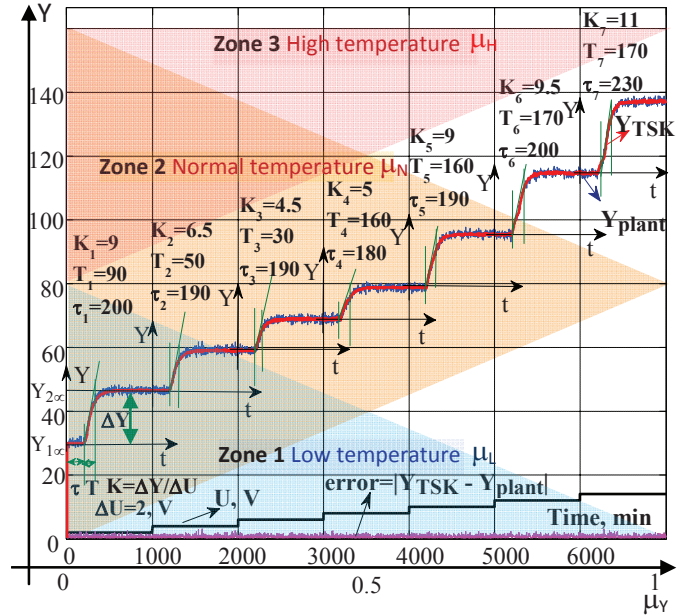


Figure 3. Plant step responses in different operation points

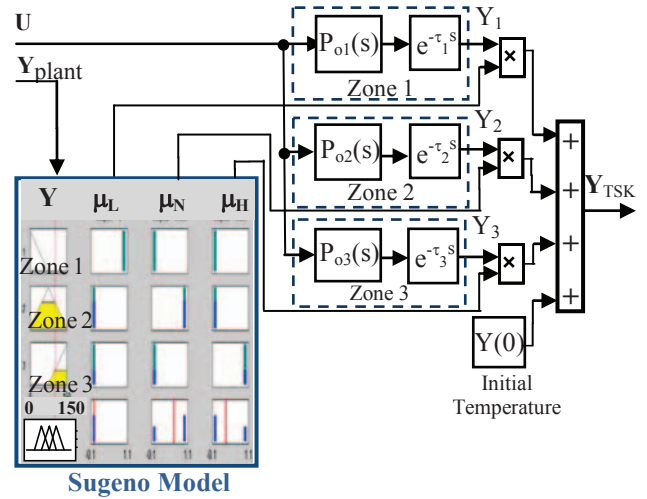


Figure 4. Structure of TSK plant model

parameters in each operation point, defined by the settled values of the plant input and output $(U_{i\alpha}, Y_{i\alpha})$, $i=1-7$ – a proof for plant nonlinearity.

The plant nonlinearity motivates to derive a nonlinear plant model using intelligent techniques based on fuzzy logic (FL) and GAs. In Fig. 3 three overlapping zones along Y can be distinguished, denoted by triangles of different colours, in which the adjacent step responses are similar and their ZN models have close parameters. So, in each zone the plant can be considered linear. This allows the TSK plant model structure in Fig. 4 to be suggested that consists of three linearization zones with dynamic properties in each k -th zone described by a local linear ZN model where $P_{ok}(s) = K_k (T_k s + 1)^{-1}$. A Sugeno Model with orthogonal triangle input membership functions (MFs) defines the three temperature zones in Fig. 3 where

the MFs μ_Y are in colours. Input to the Sugeno Model is the measured temperature Y_{plant} or the TSK model output Y_{TSK} when an accurate model is derived. It is used to recognize the operation point in which the current Y_{plant} falls. The Sugeno Model has three outputs each with MFs – the singletons “1” and ”0”. The fuzzy rules are:

- R1:** If Y_{plant} is L then Output 1 is 1, Output 2 is 0, Output 3 is 0
- R2:** If Y_{plant} is N then Output 1 is 0, Output 2 is 1, Output 3 is 0
- R3:** If Y_{plant} is H then Output 1 is 0, Output 2 is 0, Output 3 is 1.

Each measured value for the plant temperature Y_{plant} is turned into MFs values $\mu_1=\mu_L$, $\mu_2=\mu_N$, $\mu_3=\mu_H$ of belonging to each of the three temperature (linearization) zones - “Low” (L), “Normal” (N) and “High” (H) respectively. Since in the k -th rule only “Output R ” for $R=k$ ($k, R=1\div 3$) is nonzero, the defuzzification using weighted average for orthogonal MFs $\sum_{k=1}^3 \mu_k = 1$ yields:

$$\text{Output } R^0 = \sum_{k=1}^3 \mu_k (\text{Output } R)_k / \sum_{k=1}^3 \mu_k = \mu_{k=R}. \quad (4)$$

In this way the Sugeno Model maps at its three outputs the three MFs of the input. The current values of these MFs scale the outputs Y_k of the three parallel local ZN models that model the dynamics of the plant in each zone. Input to the local ZN models is the plant input U . The output of the nonlinear TSK plant model is computed as a sum of the scaled outputs of the local ZN models:

$$Y_{\text{TSK}}^0 = \sum_{k=1}^3 \mu_k Y_k / (\sum_{k=1}^3 \mu_k = 1). \quad (5)$$

The unknown parameters of the TSK plant model $\mathbf{q}_{\text{TSK}}=[K_1 K_2 K_3 T_1 T_2 T_3 \tau_1 \tau_2 \tau_3 Y(0)]$, shown in Fig. 4, are computed to ensure close step responses to the one and the same applied input to the TSK model $Y_{\text{TSK}}(t)$ from simulation and to the real world plant $Y_{\text{plant}}(t)$ from experimentation. The parameter optimization problem is solved using GAs for minimization of the following fitness function:

$$\mathbf{F}_{\text{TSK}} = \int [Y_{\text{TSK}}(t) - Y_{\text{plant}}(t) / H_{\text{plant}}(t)]^2 dt \rightarrow \min_{\mathbf{q}_{\text{TSK}}} \quad (6)$$

The optimal parameters $\mathbf{q}_{\text{TSK}}^0=[K_1=12, K_2=6, K_3=8; T_1=80, T_2=70, T_3=50; \tau_1=230, \tau_2=200, \tau_3=280; Y(0)=30]$ show that all local plants have significant relative time delays - $\tau_1/T_1=2.89; \tau_2/T_2=2.86; \tau_3/T_3=5.6$. The step responses of the TSK plant model Y_{TSK} and the absolute modelling error are depicted in Fig. 3. The modelling error is within 2% and remains small in model validation when applying different step inputs both to the plant and to its TSK model. So, the TSK plant model is accurate and the system simulations and the PDC design based on it can be trusted.

III. DESIGN OF PDC-BASED SMITH PREDICTOR

The plant nonlinearity is better compensated using a nonlinear control. In case of an available TSK plant model a PDC is a suitable nonlinear controller – easy to

design and tune based on the mastered and widely applied in the engineering practice linear systems concept.

The PDC follows the structure of the transfer function based TSK plant model. The same Sugeno Model is used for defining of the linearization zones and recognition of the degrees of belonging of the current measured temperature to each zone. The dynamic part consists of parallel operating linear controllers, each designed with respect to the corresponding local linear plant to ensure local linear system stability and desired performance [4, 11-15]. The PDC control is computed as a fuzzy blending of the local control actions using the weighted average defuzzification.

A PDC for the TSK plant model in Fig. 4 with three parallel local linear controllers, here selected to be PI with transfer functions $C_{\text{PI}k}(s)=K_{\text{pk}}(1+1/T_{\text{ik}}s)$, is shown in Fig. 5. The PDC tuning parameters are the local PI controllers gains K_{pk} and integral action times T_{ik} – $\mathbf{q}_{\text{PDC}}=[K_{\text{p1}} K_{\text{p2}} K_{\text{p3}} T_{\text{i1}} T_{\text{i2}} T_{\text{i3}}]$, computed from empirical engineering relationships that ensure minimal overshoot σ and settling time t_s of the local linear closed loop control systems [3-5]:

$$K_{\text{pk}}=\mathbf{A}.T_k/(K_k.\tau_k), T_{\text{ik}}=\mathbf{B}.T_k, \mathbf{A}=0.1\div 2, \mathbf{B}=0.1\div 3. \quad (7)$$

The classical tuning approaches, however, do not apply for local linear plants with significant time delays. A delay-compensation scheme is first applied and then - tuning methods for systems with no time delay. The linear Smith predictor is ineffective since the linear plant model in its structure differs from the real world nonlinear plant which parameters vary with the operation point. Therefore, a nonlinear Smith predictor similar in structure to the linear $R(s)$ in Fig. 1 is suggested in Fig. 6 where the linear controller $C(s)$ and the feedback $C_{\text{fb}}(s)$, based on the linear plant model $P(s)$, are substituted by a PDC and a feedback based on the nonlinear TSK plant model. Thus in each operation point the PDC produces different equivalent PI control and feedback signals which comply with the real world plant, i.e the PDC Smith predictor adapts to the changing parameters of the nonlinear plant it controls.

The parameters of the local PI controllers are

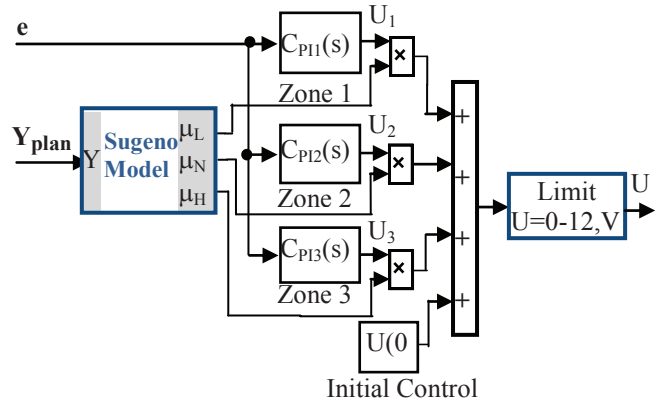


Figure 5. Structure of TSK-based PDC

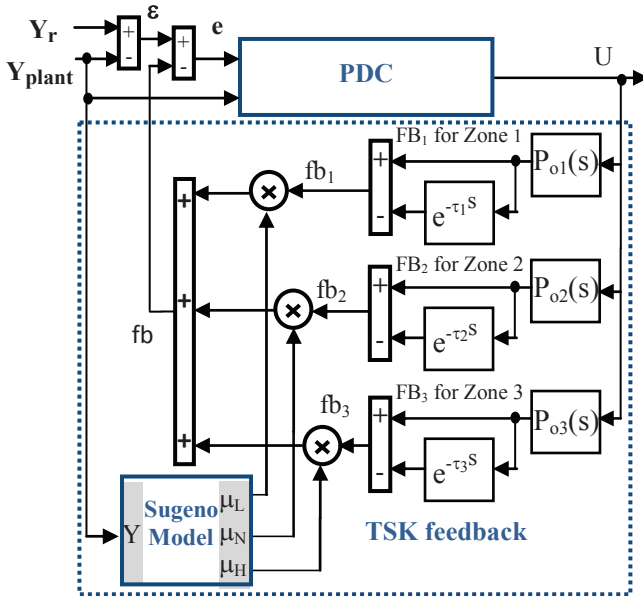


Figure 6. Nonlinear PDC Smith predictor

computed accounting for the corresponding local plant models $P_{ok}(s)$ with no time delay. Maximal possible gains are accepted - $K_{pk}=1/K_{avTSK}$, where K_{avTSK} is the average gain of the local plants. The integral action times T_{ik} are computed to ensure overdamped step responses of the local closed loop systems from [3-5]:

$$T_{ik} = 4m^2 T_k K_k K_{pk} [(1+m^2)(K_k K_{pk} + 1)^2]^{-1}.$$

The PDC parameters obtained are:

$$\mathbf{q}_{PDC} = [K_{p1}=0.11, K_{p2}=0.11, K_{p3}=0.11; T_{i1}=160, T_{i2}=140, T_{i3}=100].$$

The stability of the closed loop nonlinear system with the PDC-TSK based Smith predictor can be studied using a TSK plant model and a PDC FLC representation of the closed loop system and the derived in [4, 11-14] Lyapunov sufficient conditions or based on local Nyquist plot concept developed in [15]. Therefore, an approximation of each local feedback in Fig. 6 is searched for in order to simplify the TSK-based state space description of the PDC feedback. The role of the feedback in the linear Smith predictor and hence each local feedback in the PDC Smith predictor is to force the controller compute an adequate control action before the plant responds to reference changes with time delay, i.e. the Smith controller has to predict the plant response. Its effect after the time delay diminishes to zero. The study of the local feedback step response fb_1 for the feedback in zone 1 FB_1 , shown in Fig. 7(b), confirms that the feedback acts as a differentiator. So, a third order differentiator $FB_k(s) = K_{fk} T_{dk} s [(T_{fk1}s+1)(T_{fk2}s+1)(T_{dk}s+1)]^{-1}$ is assumed as a proper approximation model for each local feedback as shown in Fig. 7(a). The parameters of each $FB_k(s)$ are computed from the minimization of the mean absolute modelling error using GAs:

$$\mathbf{F}_{FBk} = \left[\int |fb_k(t) - fb_k^a(t)| dt \right] / N \rightarrow \min_{\mathbf{F}_{FBk}} \quad (8)$$

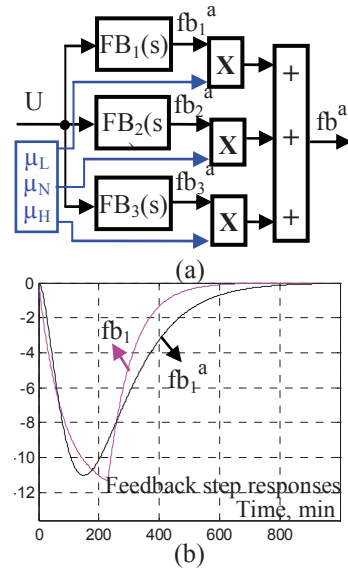


Figure 7. TSK-based feedback in the PDC Smith predictor: (a) Approximating local models; (b) Step responses

The computed optimal parameters of the approximated local feedbacks FB_k , $k=1-3$, are $\mathbf{q}_{FB^0} = [K_{f1}=65, T_{f11}=120, T_{f12}=70, T_{d1}=50; K_{f2}=25, T_{f21}=50, T_{f22}=80, T_{d2}=50; K_{f3}=25, T_{f31}=90, T_{f32}=40, T_{d3}=80]$.

The approximating fb_1^a and the original fb_1 step responses for FB_1 are close as seen in Fig. 7(b).

Then the step responses to different reference changes of the closed loop PDC-Smith control systems with the TSK-based feedback and its simple approximation are studied via simulations. The absolute errors for Y and U , introduced as measures for close step responses of the two systems, is about $\pm 2, ^\circ\text{C}$, and $\pm 0.52, \text{V}$, for actual ranges $110, ^\circ\text{C}$, and $12, \text{V}$, respectively which shows that the PDC feedback approximation is precise enough. Thus the time delay term in the TSK-based feedback is eliminated. This facilitates the classic description of the PDC-Smith predictor by state space models in the fuzzy rules conclusions enabling the system stability analysis by application of the approaches from [4, 11-15]. The PDC-Smith feedback approximation simplifies also the implementation of the PDC-Smith predictor in programmable logic controllers (PLC) for real time control of industrial processes.

IV. SIMULATION RESULTS AND DISCUSSION

For the purpose of assessment of the advantages of the designed PDC-Smith controller a comparison is provided with designed linear PI controller (uncompensated plant high time delay and nonlinearity), linear Smith controller (compensated linear plant high time delay) and PDC (compensated plant nonlinearity).

The standard linear PI controller is selected as the simplest widely used in the engineering practice. It is tuned with respect to a linear plant model, here the

average ZN plant model in Fig. 3 with parameters (K_{av} , T_{av} , τ_{av}). The same engineering approach from [3-5] for ensuring minimal closed loop system overshoot and settling time results in $K_{pPI} = \mathbf{A} \cdot T_{av} / (K_{av} \cdot \tau_{av}) = 0.031$, $T_{iPI} = \mathbf{B} \cdot T_{av} = 119$ for $\mathbf{A} = 0.4$, $\mathbf{B} = 1$.

The linear Smith predictor provides compensation of the significant time delay of linear plants. It is built on a standard linear PI controller and a feedback to it, based on the linear plant model, here the average ZN plant model. The PI controller is tuned for a gain, computed accounting for the highest plant gain of all ZN models in Fig. 3 $K_{max} = \max(K_i) = 11$ - $K_{pmax} = 1/K_{max}$ and $T_{iSmith} = T_{av} = 119$.

The PDC is a simple nonlinear controller for compensating the plant nonlinearity. It is based on the derived TSK plant model in Fig. 4 and has the structure, shown in Fig. 5, with local linear PI controllers. Their parameters are tuned with respect to the local plants and their high relative time delays using an engineering approach which for local systems stability reasons requires restrictions on the controllers' gains:

$$K_{pkPDC} = 0.25/K_k, T_{ikPDC} = 2T_{ik}$$

The computed parameters of the local PI controllers are [$K_{p1PDC} = 0.021$, $T_{i1PDC} = 160$; $K_{p2PDC} = 0.042$, $T_{i2PDC} = 140$; $K_{p3PDC} = 0.031$, $T_{i3PDC} = 100$].

Simulation investigations are carried out on the corresponding control systems with the four types of designed controllers for comparison of the system performance and energy efficiency when the reference changes in the whole operation range. The performance is assessed from the settling time t_s and the overshoot σ of the step responses. The smaller t_s and σ , the more accurate the control system is. The energy efficiency is estimated on the basis of the mean control action –

$EEF = \Sigma(U_n) / N$, where U_n is the control action in discrete time $t_n = n\Delta t$ with $n = 1 \div N$ and sample time Δt , and the settling time of the control t_{sU} . The greater EEF, the less efficient the system is since more energy is used for control. The step responses for the temperature Y of the four systems are presented in Fig. 8.

The assessed performance indices of systems with designed controllers are presented in Table 1. The PDC-Smith system step responses have negligible σ and are the fastest with the shortest settling time for the control t_{sU} - an evidence for good compensation of the significant plant time delay. The control, however, changes with a high peak at the beginning and therefore the EEF index is higher. In the different operation points the PDC-Smith system preserves similar step responses – a proof for a good compensation of the plant nonlinearity. The PI control system is the most economic with no overshoot but at the expense of slow step responses. The other systems are not only difficult to design but also have longer t_s and higher σ .

V. CONCLUSION

A novel approach for the control of nonlinear plants with significant plant delays combining the linear Smith predictor with the TSK-PDC principles is developed. The suggested nonlinear Smith predictor is of simple structure, easy to design and to tune and is effective in compensation both of the plant time delay and nonlinearity.

The future research will focus on the validation of the stability of the closed loop system with the designed PDC-Smith controller and the PLC implementation of the controller for the real time control of the air temperature in the laboratory-scale furnace.

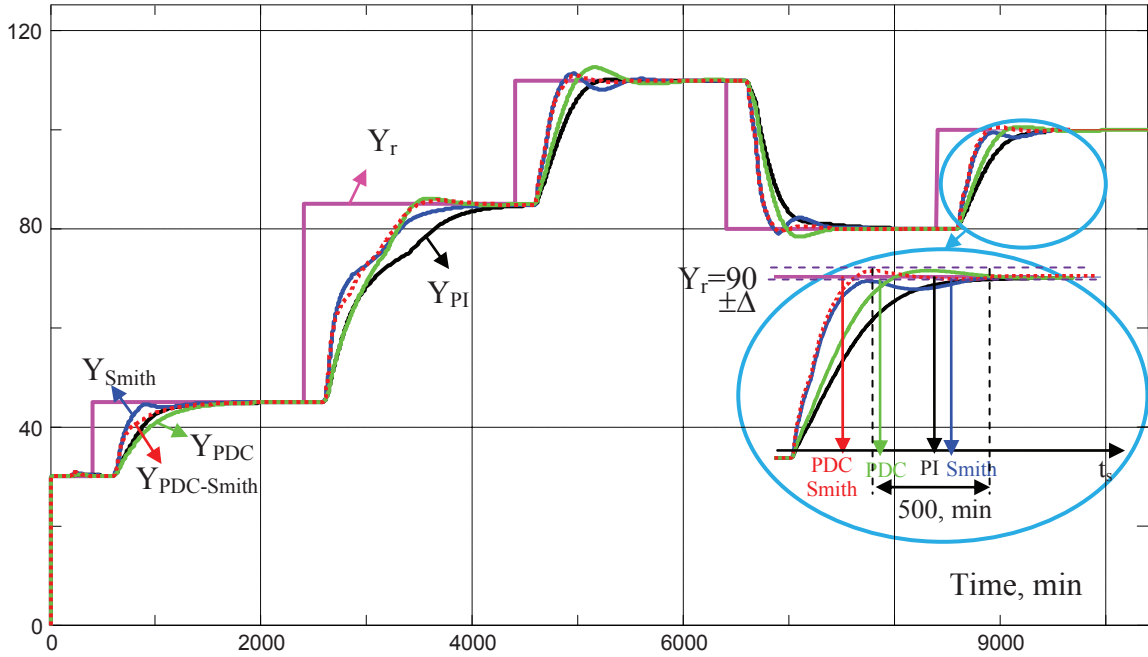


Figure 8. Temperature step responses to reference changes

Table 1 Assessed performance indices of systems with designed controllers

Reference changes	Performance indicators	PDC-Smith system	PDC System	Smith System	PI System
	EEF, V	7.7	7.61	7.7	7.55
$\Delta Y_r=30-45, ^\circ\text{C}$	$\sigma, \%$	0	0	0	0
	t_s, min	800	1100	900	900
	t_{sU}, min	200	350	200	200
$\Delta Y_r=45-85, ^\circ\text{C}$	$\sigma, \%$	2	5	0	0
	t_s, min	1000	1200	1800	1900
	t_{sU}, min	400	400	600	800
$\Delta Y_r=85-110, ^\circ\text{C}$	$\sigma, \%$	5	10	7	0
	t_s, min	300	1000	1000	400
	t_{sU}, min	200	900	800	250
$\Delta Y_r=110-80, ^\circ\text{C}$	$\sigma, \%$	2	4	7	0
	t_s, min	300	800	700	650
	t_{sU}, min	200	400	500	500
$\Delta Y_r=80-100, ^\circ\text{C}$	$\sigma, \%$	2	4	0	0
	t_s, min	300	400	600	500
	t_{sU}, min	250	350	400	350

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