Investigation of the Method of RMS Measurement Based on Moving Averaging

Anna A. Kostina[1], Plamen M. Tzvetkov[2] and Andrey N. Serov[3]

Abstract – **Currently, the measurement method of root mean square value (RMS), based on samples accumulation, is the most popular. The advantages of this method include ease of implementation, high accuracy and additional opportunities to reduce the methodological component of the total measurement error. Studies show that increasing measurement time also contributes to a reduction in error. However, in a number of measurement and automatic control problems, one of the main requirements is to reduce the measurement time. This problem can be solved by a method based on moving averaging, that includes the advantages of the "classical" method, but is largely devoid of its shortcomings. An algorithm of RMS measurement for implementation on the basis of recursive and non-recursive moving average filters is proposed. The influence of the sinusoidal signal parameters on the RMS measurement error is analyzed. An analytical expression is obtained that allows to estimate the RMS measurement error from the signal parameters and the number of samples. Approaches are proposed for reducing the RMS measurement error when applying the measurement method under consideration. It is shown that the application of post-filtration of RMS measurement results can reduce the final RMS measurement error. The veracity of the analytical expressions obtained in the paper is confirmed by the results of simulation at check points.**

Keywords – **Root mean square, Simulation, Digital filtration, Measurement accuracy, Frequency deviation.**

I. INTRODUCTION

The root mean square (RMS) of a periodic signal is the parameter, which is determined by the expression [1]:

$$X_{id} = \sqrt{\frac{1}{T_A}\int_0^{T_A} x^2(t)dt}\ , \quad (1)$$

where T_A – total observation (measurement) time, which is selected proportional (multiple) of the real value of the input signal period; $x(t)$ – input periodical signal.

For a polyharmonic input signal,

$$x(t) = \sum_{i=1}^{M} X_i \sqrt{2}\sin(\omega_i t + \alpha_i), \quad (2)$$

expression (1) can be represented as follows [1]:

$$X_{id} = \sqrt{\sum_{i=1}^{M} X_i^2}\ , \quad (3)$$

where X_i – RMS of the i-th spectral component of the input signal; ω_i – angular frequency of the i-th spectral component; α_i – initial phase of the i-th spectral component; M – number of considered spectral components.

For digital measuring devices, the instantaneous values of the input signal come in the form of discrete samples by using an analog-to-digital converter (ADC). For this reason, the direct implementation of the definition of RMS (see formula (1)) is impossible. There are a large number of digital methods for RMS measurement, which are based on the analysis of the signal in the time [2]-[8] or frequency domain [9]-[10]. Some digital RMS measurement methods are designed to measure sinusoidal input signals. These methods include the three-sample method [8] and the method based on the application of the second derivative of the signal [2]. The application of these methods in conditions of nonsinusoidal signals results to significant measurement errors. Currently, the operation algorithms of most digital measuring instruments for RMS measurement are based on the application of a samples accumulation method (ASS) of the input signal [7].

The investigated method has several advantages. First of all, this method is applicable for measurement sinusoidal and polyharmonic signals. The small number of required arithmetic operations ensures the simplicity of its practical implementation. Due to the additional measurement of the signal frequency [11]-[16] and the adjustment of the number of accumulated samples (or the initial phase of the signal), it is possible to significantly reduce the methodological error. One of the parameters that influence the measurement error is the measurement time. With increasing measurement time, the methodological error tends to decrease. However, in a number of problems of measurement and control theory, measurement time is a critical parameter, which imposes serious limitations on this measurement method.

Thus, the problem arises of finding high accuracy RMS measurement methods, that requires a short measurement time for the case of the real electrical networks. In the paper, a samples accumulation method is analyzed. This method satisfies the properties, which are describe above.

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II. THE RMS SAMPLES ACCUMULATION MEASUREMENT METHOD

A. General Description

This method is based on the application of a zero-order polynomial as a restoring function and the subsequent implementation of the RMS determination for the reconstructed signal (see expression (1)). Moreover, RMS can be obtained by using the following formula [5]-[6]:

$$X = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1} x^2[i]}\,, \qquad (4)$$

where N – number of averaged samples; $x[i]$ – signal samples.

The number of averaged samples is related to the measurement time by the following relation:

$$T_A = NT_S = \frac{N}{f_S} = mT\,, \qquad (5)$$

where m – number of averaged input signal periods; T – input period value; T_S – sampling time; f_S – sampling frequency.

B. Measurement Error

In [5]-[6], a relation was obtained for calculation the RMS measurement error for the case of a sinusoidal input signal:

$$\delta_X = -\frac{\cos((N-1)\omega T_S + 2\alpha)\sin(\omega N T_S)}{2N\sin(\omega T_S)}\,, \qquad (6)$$

where ω – input signal angular frequency (true value); α – initial phase of the input signal.

The dependence of the error on the input signal frequency is shown in Fig. 1. The figure shows that there are singular points where the dependence takes on a zero.

Special points where the error takes zero values are achieved when the following conditions are met:

$$\begin{cases} \sin(\omega N T_S) = 0, \\ \cos((N-1)\omega T_S + 2\alpha) = 0. \end{cases} \qquad (7)$$

The fulfillment of the upper condition of expression (7) can be ensured by adjusting the number of averaged samples:

$$N = \frac{k}{fT_S} = \frac{kT}{T_S} = \frac{T_A}{T_S}, \qquad (8)$$

where k – integer value (in expression (8) is accepted $k = m$).

Since the number of processed samples is a discrete, and the frequency of the input signal is unstable in time, relation (8) is not absolutely accurate. This results to the appearance of a methodological error. In practice, the following formula can be used to determine the number of measured samples:

$$N = round(T_A \cdot f_S) = round(m \cdot T_{nom} \cdot f_S), \qquad (9)$$

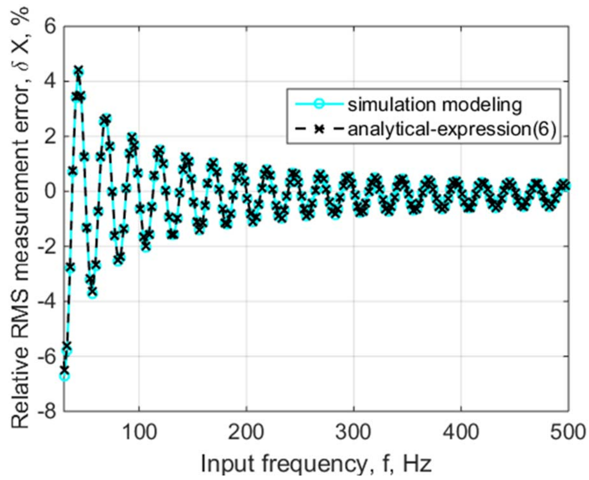

Fig. 1. The dependence of the RMS measurement error (moving averaging method) on the input signal frequency

where T_{nom} – nominal or measured value of the input signal period.

The second approach to reducing the error is to ensure that the lower equation of condition (7) is satisfied. If the initial phase of the signal satisfies the condition:

$$\alpha = 0.25\pi - 0.5(N-1)\omega T_S \pm 0.5\pi k\,, \qquad (10)$$

then the RMS measurement error for the case of a sinusoidal signal is equal to zero.

As in the previous case, since the frequency of the signal is unstable, and the initial phase of the signal is discrete (due to the discrete nature of the signal samples), condition (10) cannot be fulfilled absolutely precisely. This results to the RMS measurement error.

III. THE RMS MOVING AVERAGING MEASUREMENT METHOD

A. General Description

As already described above, one of the drawbacks of the samples accumulation method is a large measurement time. Since N samples are required to obtain a measurement result, the measurement time is equal to $NT_S = T_A$. That is, the sampling time of the RMS measurement result is T_A. The update rate of the RMS measurement result can be increased. It is achieved if for the calculating of the next measurement result, the signal samples, which are used for the previous RMS result. The sampling time of the RMS measurement results is equal to the sampling time of the input signal if the RMS measurement is performed for the N-1 previous samples of the input signal relative to the current n-th sample. In this case, the formula for RMS calculating takes the following form:

$$X[n] = \sqrt{\frac{1}{N}\sum_{i=0}^{N-1} x^2[n-i]}\,. \qquad (11)$$

As for the previous method, the number of averaged samples can be obtained by expression (5). Expression (11) can be implemented on the basis of a moving average non-recursive filter of the N-th order. The RMS measurement transducer can also be implemented by applying a recursive structure:

$$X[n] = \sqrt{\frac{x^2[n]}{N} + X[n-1] - \frac{x^2[n-N]}{N}} . \quad (12)$$

B. Measurement Error

The error of the n-th measured RMS sample of the considered method can be calculated by using expression (6) (provided that the initial phase of expression (6) is corrected, the input signal is sinusoidal):

$$\delta_X(n) = -\frac{\cos((N-1)\omega T_S + 2(\alpha + n\omega T_S))\sin(\omega N T_S)}{2N\sin(\omega T_S)}, \quad (13)$$

where α – initial phase of the input sinusoidal signal at the first sample for the total measurement time.

Fig. 2 shows the time dependence of the RMS measurement error with the application of expression (11). It can be seen from Fig. 2 and expression (13) that the measurement error in time domain characterized by a sinusoidal behavior with a frequency equal to the real value of the signal frequency ω.

To reduce the measurement error, as well as for the samples accumulation method, the following approaches can be applied:

– increase of the total measurement time;

– adjustment of the number of averaged samples according to the dependence (9);

– adjustment of the initial phase of the signal according to the relation (10);

– application of additional low-pass filtration.

As can be seen from dependence (13), an increase in the measurement time results to a decrease of the RMS relative measurement error. For large measurement time values, the maximum RMS error can be represented as:

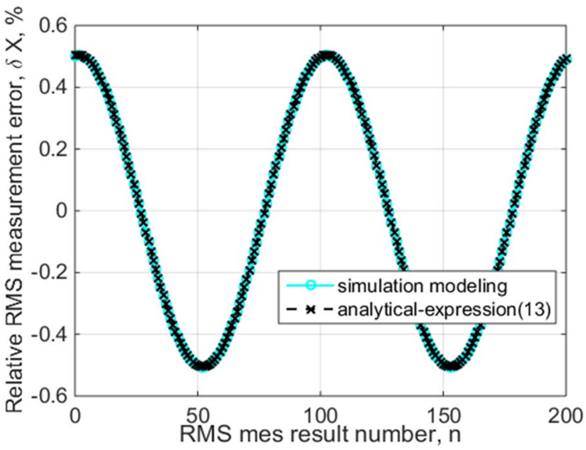

Fig. 2. The dependence of the RMS measurement error for the method of moving averaging on the number of measurement result

$$\delta_X(n) \le -\frac{1}{2N\sin(\omega T_S)} . \quad (14)$$

However, an increase in the measurement time results to a deterioration in the dynamic characteristics of the method. Since the result of the RMS measurement with the use of this method is the average value of the RMS for the considerate measurement interval, changes in the RMS of the input signal will more slowly manifest themselves on the measurement result. For this reason, the application of this approach in the considerate measurement method is ineffective, since it neutralizes the advantages of this method compared to the samples accumulation measurement method.

Using the adjustment of the number of samples allows to reduce the value of the factor $sin(\omega N T_S)$ in expression (13). The finite error of frequency measurement and the discrete nature of the number of averaged samples (parameter N) interfere with ensuring the equality of this factor to zero. In the case of using this approach, the relative RMS measurement error can be represented as:

$$\delta_X(n) = -\frac{\left(\delta_f + \frac{1}{2N}\right)\omega\cos((N-1)\omega T_S + 2(\alpha + n\omega T_S))}{2 f_S \sin(\omega T_S)} . \quad (15)$$

where δ_f – frequency relative measurement error.

The application of adjustment of the initial phase for this measurement method is not justified, since such adjustment violates the measurement algorithm represented by expressions (11) and (12).

The application of low-pass filtration of the RMS measurement results makes it possible to reduce the influence of a time-variable factor of expression (13) – $cos((N–1)\omega T_S+2(\alpha+n\omega T_S))$. In this case, the applied low-pass filter should be designed to suppress the non-dc spectral components with the frequency of the input signal (frequency of the variable component of expression (13)). In addition, the constant (DC) component of the RMS measurement result (useful signal) should be transmitted to the filter output unchanged. The RMS relative measurement error for this approach takes the form:

$$\delta_X(n) = -\frac{H(\omega)\cos((N-1)\omega T_S + 2(\alpha + n\omega T_S) + \varphi(\omega))}{2N\sin(\omega T_S)\sin^{-1}(\omega N T_S)}, \quad (16)$$

where $H(\omega)$ – magnitude response of the applied low-pass filter for the frequency (true value) of the input signal ω; $\varphi(\omega)$ – phase response of the applied low-pass filter for the frequency (true value) of the input signal ω.

IV. Simulation Modeling

Simulation mathematical modeling was performed in Simulink software package. By Simulink a simulation model of the RMS measurement transducer was developed based on the method of moving averaging of samples (MA) and based on the samples accumulation measurement method (ASS). At the output of the measurement converter, a digital IIR filter is

TABLE 1.
SIMULATION RESULTS FOR RMS MEASUREMENT TRANSDUCERS OF DIFFERENT TYPES (ERROR IS REPRESENTED IN PERCENT)

RMS MC type	Frequency relative deviation, δ_f, %				
	-5.0	-1.0	0.0	1.0	5.0
ASS	2.6	0.51	0.0	0.50	2.4
MA	2.6	0.51	0.0	0.50	2.4
MA+BF	$1.8\ 10^{-2}$	$9.2\ 10^{-4}$	$8.0\ 10^{-5}$	$8.8\ 10^{-4}$	$1.5\ 10^{-2}$
MA+CF	$8.7\ 10^{-2}$	0.10	0.10	0.10	$9.0\ 10^{-2}$
ASS+SA	0.11	$5.0\ 10^{-3}$	0.0	$5.0\ 10^{-3}$	0.13
MA+SA	0.11	$5.0\ 10^{-3}$	0.0	$5.0\ 10^{-3}$	0.13
MA+BF+SA	$1.6\ 10^{-4}$	$8.0\ 10^{-5}$	$8.0\ 10^{-5}$	$8.2\ 10^{-5}$	$1.6\ 10^{-4}$
MA+CF+SA	0.10	0.10	0.10	0.10	0.10

applied, constructed according to the analog prototype of 4-th order Butterworth filter (BF) or 3-rd order Chebyshev filter (CF). The developed model provides for the possibility of additional adjustment of the number of averaged samples (SA) according to the frequency measurement result. The simulation results are shown in Table 1. When performing the simulation, a sinusoidal signal with a unit amplitude and a zero initial phase was applied. The sampling frequency is equal to 10 kHz, the measurement time of one frame is equal to 0.02 seconds. The total simulation time is 2 seconds. When designing the output post-filters, the following characteristics was applied: passband limit – 10 Hz; stopband limit is 45 Hz; maximum passband deviation – 0.1 %; minimum stopband rejection is 0.01.

V. CONCLUSION

As a result of the research done, the following results were obtained:

1. The method of moving averaging of samples allows to get the RMS measurement result in less time compared to the "classical approach".

2. An analytical expressions (13)-(16) has been obtained to take into account the measurement error of the proposed method. The application of these expressions makes it possible to calculate the maximum value of the RMS measurement error for the given parameters of the input signal and RMS measurement transducer. In addition, the inverse problem can be solved: for a given maximum value of the RMS measurement error, the maximum deviation of the signal frequency can be estimated (which is equal to the maximum frequency measurement error in the case of adjusting the number of samples).

3. The approaches to reduce the RMS measurement error by the proposed method are developed. The approaches consist in adjusting the number of averaged input signal samples and post-filtration of the RMS measurement results. It was shown that adjusting the number of averaged samples and low-pass post-filtration can significantly (up to 10^3 times) reduce the RMS measurement error.

4. By application of Simulink, a simulation model of RMS measurement transducer is developed.

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