ОБЩЕСТВО НА ТРИБОЛОЗИТЕ В БЪЛГАРИЯ

SOCIETY OF BULGARIAN TRIBOLOGISTS

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The Technical University of Sofia is the largest educational and scientific complex in Bulgaria in the field of technical and applied sciences with an institutional accreditation grade of 9.5 (on the scale of 10) for the period 2012 – 2018.

The Faculty of Industrial Technology (FIT) prepares specialists in the field of engineering and technology. Ever since the first academic year in 1945/1946 the Faculty has trained thousands of engineers and has always been a leading national center in the field of scientific research and applied activities. One of the Laboratories in FIT is the **Scientific & Production Laboratory "Tribology".**

Scientific and Production Laboratory "Tribology" – 44 years National Tribology Centre

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The scientific and production **Laboratory "Tribology"** is headed by Prof. Dr Mara Kandeva. The **Laboratory "Tribology"** was founded at the **Technical University Sofia** in 1974 by **Prof. DSc Nyagol Manolov**, and acts as National Tribology Centre in Bulgaria. It is the starting place for feeding the National WEB in the tribospace, which is a contact network of researchers/educators, customers and producers, and their achievements in the field of tribology and tribotechnologies. In sight are problems related to the management of friction, wear, lubrication, hermeticity, serviceability and reliability of tribotechnical elements and systems in their operation and maintenance. Lubricants, additives and surface coatings are the thoroughly developed topics of the latest years.

The latest tribotechnologies developed in the Laboratory "Tribology" at the Technical University – Sofia are **tribotechnologies for application of wear-resistant gas-flame and ultrasonic powder coatings** in collaboration with the Belgian company "GMA-Technologies. Another tribotechnology actual for Bulgaria and the region is a **tribotechnology for qualification and regeneration of air filters in motorcar and truck transportation**. The method and technology are patent of the Laboratory "Tribology", Sofia.

The Laboratory for Tribology, with the support of the Society of Bulgarian Tribologists, organizes the International Conferences on Tribology BULTRIB.

CONTENTS

1. **OPENING SPEECH**: *Prof. Dr. Mara KANDEVA*, Chairman of the Society of Bulgarian Tribologists 10
2. Vyara POZHIDAEVA, Mara KANDEVA, Emilia ASSENOVA, **DIAGNOSTICS OF THE BEARINGS THROUGH THE CONTACT ELEMENTS ROUGHNESS** 12
3. Nina KOSTOVA, Marcela ACHIMOVIČOVÁ, Martin FABIÁN, **MECHANOCHEMICAL SYNTHESIS OF ZINC SELENIDE IN INDUSTRIAL MILL FOR PHOTOCATALYTIC DEGRADATION OF AZO DYE** 16
4. Evgenia KOZHOUKHAROVA, Sergei V. FEDOROV, Emilia ASSENOVA, **FRICTION AND WAVE EFFECTS ON THE GEOLOGICAL PROCESSES IN THE EARTH'S CRUST** 22
5. Christo ARGIROV, Yavor LUKARSKI, Sasho POPOV, **INVESTIGATION OF THE WEAR-RESISTANT OF HIGH NITROGEN CHROME-MANGANESE STEELS** 29
6. Juliana JAVOROVA, Svetla LEKOVA, Sandra SOVILJ-NIKIC, Virgil ILIUTA, **GRAPHENE - SOME BASIC ASPECTS** 35
7. Juliana JAVOROVA, **3D BIOPRINTING FOR TISSUE ENGINEERING** 44
8. Stefan GARABITOV, **EQUIVALENT VISCOUS DAMPING FOR FRICTION IN GEAR TRAIN** 53
9. Zdravka KARAGUIOZOVA, Grzegorz CIESLAK, Anna MAZUREK, **SPACE APPLICATION OF ELECTROLESS NICKEL COATING** 61
10. Zdravka KARAGUIOZOVA, **ELECTROLESS NICKEL COATINGS PLATED ON 2024 ALUMINIUM ALLOY** 70
11. Marko PANTIĆ, Miroslav BABIĆ, Slobodan MITROVIĆ, Danimir JEVREMOVIĆ, Dragan DŽUNIĆ, Tatjana KANJEVAC, Ivana LUKOVIĆ, **EFFECTS OF DIFFERENT FINISHING PROCEDURE ONTO NANOMECHANICAL CHARACTERISTICS OF VENEERING CERAMIC** 79
12. Dragan DžUNIĆ, Slobodan MITROVIĆ, Miroslav BABIĆ, Marko PANTIĆ, Ilija BOBIĆ, **TRIBOLOGICAL BEHAVIOR OF ZA-27 ALLOY BASED NANOCOMPOSITE REINFORCED WITH SIC NANOPARTICLES** 86
13. Anna PETROVA, Adelina MITEVA **THE ABRASIVE PROPERTIES OF DETONATION NANODIAMONDS** 93
14. Adelina MITEVA, Anna PETROVA, **DETONATION NANODIAMONDS - SOME BIOMEDICAL APPLICATIONS** 98
15. Bogdan SOVILJ, Sandra SOVILJ-NIKIC, Juliana JAVOROVA, **TRIBOLOGICAL RESEARCHES OF TRIBOELEMENTS TOPOGRAPHY OF HOB MILLING PROCESS OF CYLINDRICAL GEAR SERRATION** 104
16. P. SINAPOV, **MODELING A BAR ELEMENT TO STUDY IMPACT PROCESSES IN ELASTIC-PLASTIC MECHANICAL SYSTEMS** 106
17. P. SINAPOV, I. IGNATOV, **FRICTION INDUCED VIBRATIONS OF A CANTILEVER BEAM** ..107
18. I. LEVANOV, E. ZADOROZHNAYA, M. KANDEVA, **Zh.KALITCHIN**, N. DOLGUSHINA, E. POLYACKO, **INFLUENCE OF THE FRICTION GEO-MODIFIER ON THE ANTI-WEAR PROPERTIES OF PLASTIC LUBRICANTS** 108
19. J. GENOV, M. TODOROV, **DYNAMICAL STRESSES IN THE HIGH CLASS WIND TURBINE BLADES CAUSED BY THE VERTICAL WIND SPEED GRADIENT. PART 1 - AERODYNAMICAL LOADS** 109
20. J. GENOV, M. TODOROV, **DYNAMICAL STRESSES IN THE HIGH CLASS WIND TURBINE BLADES CAUSED BY THE VERTICAL WIND SPEED GRADIENT. PART 2 - STRESS ANALYSIS IN THE TURBINE BLADES** 110

21. Todor TODOROV, Nikolay NIKOLOV, Rumen NIKOLOV, Yavor SOFRONOV, Lachezar KOCHEV, **OPTIMIZATION OF MEMS PIEZOELECTRIC ENERGY HARVESTERS WITH INTERDIGITATED ELECTRODES**……………. 111
22. Krasimir NEDELCHEV, Ivan KRALOV**, ACOUSTIC METHOD FOR IDENTIFICATION OF RAILWAY WHEEL DISC STRUCTURAL VIBRATIONS USING COMSOL**……….. 112
23. J. KALEICHEVA, V. MISHEV, Z. KARAGUIOZOVA, **TRIBOLOGICAL CHARACTERISTICS OF AUSTEMPERED DUCTILE IRONS WITH NANOSIZED PARTICLES** 113
24. Stelian ALACI, Luminta IRIMESCU, Florina Carmen CIORNEI, Mara KANDEVA, **DEVICE AND METHOD FOR SIMULTANEOUS DETERMINATION OF ROLLING AND SPINNING FRICTION IN A CONCENTRATED CONTACT** 114
25. M. KANDEVA, Zh. KALITCHIN, P. SVOBODA, S. SOVILJ-NIKIC, **GENERAL METHODOLOGY FOR STUDYING THE TRIBOLOGICAL PROCESSES ON THE BASIS OF THE COMMUNICATIVE POTENTIAL**.. 115
26. M. KANDEVA, P. SVOBODA, Zh. KALITCHIN, T. PENYASHKI, G. KOSTADINOV, **WEAR OF GAS-FLAME COMPOSITE COATINGS WITH TUNGSTEN AND NICKEL MATRIX. PART I. ABRASIVE WEAR** 116
27. M. KANDEVA, P. SVOBODA, Zh. KALITCHIN, T. PENYASHKI, G. KOSTADINOV, **WEAR OF GAS-FLAME COMPOSITE COATINGS WITH TUNGSTEN AND NICKEL MATRIX. PART II. EROSIVE WEAR** ……………………………………………………..…… 117
28. A. PETROVA, **INFLUENCE OF ADDITIVES ON STRUCTURAL AND PHYSICO-MECHANICAL PROPERTIES OF NANO-MICROCRYSTAL ALUMINIUM ALLOYS** 118
29. G. TODOROV, Y. SOFRONOV, M. ZAGORSKI, **DIRECT METAL DEPOSITION FOR HYBRID MANUFACTURING** 119
30. G. TODOROV, **VIRTUAL PROTOTYPE OF FAMILY BASED GRAPPLE DESIGN** 120
31. N.M. VAXEVANIDIS, A. VENCL, E. ASSENOVA, M. KANDEVA, P. PSYLLAKI, **SCIENTIFIC LITERATURE ON THERMAL SPRAY COATINGS FROM SOUTHEASTERN EUROPE: A TEN YEARS BIBLIOMETRIC ANALYSIS** 121
32. Valentin KAMBUROV, Rayna DIMITROVA, Kiril NIKOLOV, **ELECTROLESS NICKEL COATING OF CARBON MICRO FIBERS AND NANOTUBES INTENDED FOR A REINFORCING PHASE IN MMC AND PMC**.. 122
33. Maria ORMANOVA, Dimitar DECHEV, Nikolai IVANOV, Peter PETROV, **CHARACTERIZATION ON ZRN COATINGS DEPOSITED BY DC MAGNETRON SPUTTERING**............................... 123
34. M. KANDEVA, V. KAMBUROV, K. NIKOLOV, L. DIMITROV, **ABRASIVE WEAR OF ULTRA-HIGH-MOLECULAR-WEIGHT POLYETHYLENE, MODIFIED WITH CARBON NANOTUBES** 124
35. T. PENYASHKI, G. KOSTADINOV, E. DIMITROVA, I. MORTEV, **IMPROVING THE TRIBOLOGICAL CHARACTERISTICS OF HELICAL DRILLS FROM HIGH SPEED STEEL BY MEANS OF A CONTACTLESS LOCAL ELECTRICAL DEPOSITION WITH COMPOSITE CARBIDE ELECTRODES BASED ON TiC AND TiN** 125
36. T. PENYASHKI, G. KOSTADINOV, D. RADEV, M. KANDEVA, **COMPARATIVE STUDIES OF TRIBOLOGICAL CHARACTERISTICS OF CARBON STEELS WITH GAS FLAME COATINGS FROM NEW MULTI-COMPONENT CARBIDE COMPOSITE MATERIALS**.............................. 126
37. K. GAVRILOV, A. DOIKIN, Y. GORITSKIY, Zh. KALICHIN, **MATHEMATICAL MODEL FOR THE ANALYSIS OF OPERATING CONDITIONS OF HYDRODYNAMIC TRIBO-SYSTEMS AND FRICTION PROCESSES AND WEAR OF THEIR CONTACT SURFACES** 127
38. .J. GENOV, I. KRALOV, I. ANGELOV, **BEM THEORY ADAPTATION TAKING INTO ACCOUNT NON-UNIFORM, NON-STATIONARY WIND FIELD** 128
39. I. KRALOV, J. GENOV, I. ANGELOV, **MODIFIED BEM THEORY APPLICATION FOR DETERMINING THE AERODYNAMIC FORCES ACTING ON THE BLADE OF WIND TURBINE** 129

40. J. GENOV, I. KRALOV, I. ANGELOV, **DYNAMIC STRESS ANALYSIS OF A BLADE OF WIND TURBINE GENERATOR TAKING INTO ACCOUNT VERTICAL WIND SPEED GRADIENT** ... 130

41. Aleksandar VENCL, Veljko ŠLJIVIĆ, Ilija BOBIĆ, **METAL-METAL COMPOSITES WITH ZN-AL ALLOY BASE AND ADDITION OF TI MICROPARTICLES REINFORCED WITH CERAMIC NANOPARTICLES** 131

42. Victor BALABANOV, **SELF-ORGANIZATION OF FRICTION CONDITIONS WHEN USING REPAIR AND RECOVERY ADDITIVES TO LUBRICANTS** 132

43. Lucian CÃPITANU, Liliana - Laura BÃDIȚÃ, Constantin TIGÃNŞTEANU, Virgil FLORECSCU, Liviu Florin ISVORANU**, A GREAT TECHNICAL TAMPING WHICH WAS CONSIDERED A PROMOTING DISCOVERY. MICRO-TEXTURING OF THE SURFACE OF THE PROSTHETIC FEMORAL HEAD TO REDUCE FRICTION AND WEAR**.. 133

44. Lucian CÃPITANU, Liliana - Laura BÃDIȚÃ, Constantin TIGÃNŞTEANU, Virgil FLORECSCU, Liviu Florin ISVORANU**, THE TRUNION FRETTING AND WEAR OF THE STEM OF TOTAL HIP PROSTHESIS AND ITS INFLUENCE ON THE PROSTHESIS STABILITY**.............................. 134

45. Sergey V. FEDOROV, **ABOUT ENERGY BALANCE OF FRICTION AND MACHINE AS TRIBOSUPERSYSTEM**... 135

46. I. MUKCHORTOV, E. ZADOROZHNAYA, M. KANDEVA, I. LEVANOV, K. YAKUNINA, O. DOROKHOVA**, NON-TOXIC ANTIWEAR ADDITIVE FOR FOOD AND BIODEGRADABLE LUBRICANTS**... 136

47. Igor LEVANOV, Elena ZADOROZHNAYA, Vladimir LASHMANOV, Michail ESCHIGANOV, **PREDICTING LIFETIME OF INTERNAL COMBASTION ENGINE CRANKSHAFT JOURNAL BEARINGS AT THE DESIGN STAGE**... 137

37. Elena ZADOROZHNAYA, M. KANDEVA, Vlad HUDYAKOV, Igor DOLGUSHIN, **INVESTIGATION OF THE EFFECT OF NON-ISOTHERMAL FLOW OF NON-NEWTONIAN FLUID IN A THIN LAYER AND THERMAL STATE OF THE TURBOCHARGER RADIAL BEARINGS ON THE ROTOR DYNAMICS**.. 138

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EQUIVALENT VISCOUS DAMPING FOR FRICTION IN GEAR TRAIN

Stefan GARABITOV

Abstract: *An energy method was proposed to calculate the mesh damping using friction. The calculation result showed how to model vibrations of gears. The mesh damping was presented as a function. As a pattern to which the results were derived, the mathematical model of the gear tooting was used, in which the characteristics of the gearing were modeled with a non-continuous function describing friction zone. The numerical results revealed that the gear system primarily performs a non-harmonic-single-periodic motion.*
Keywords: *gear mesh; gear tooting; mesh damping; friction; nonlinear dynamics*

1. INTRODUCTION

For a better understanding of dynamic gear behavior and gear damage analysis, dynamic modeling of the vibration of the gear is widely used. The main source of vibration in a geared transmission system is usually the meshing action of the gears. The vibration models of the gear-pair in the mesh have been developed taking into account the most important dynamic factors such as the effects of the friction forces at the meshing interface, gear-backlash, the time-varying mesh stiffness, and the excitation from gear transmission errors [2]. Mostly used a dynamic model of a pair of gears is shown (see Fig. 1) [3]. The gear mesh is modeled as a pair of rigid disks connected by a spring-damper set along the line of contact.

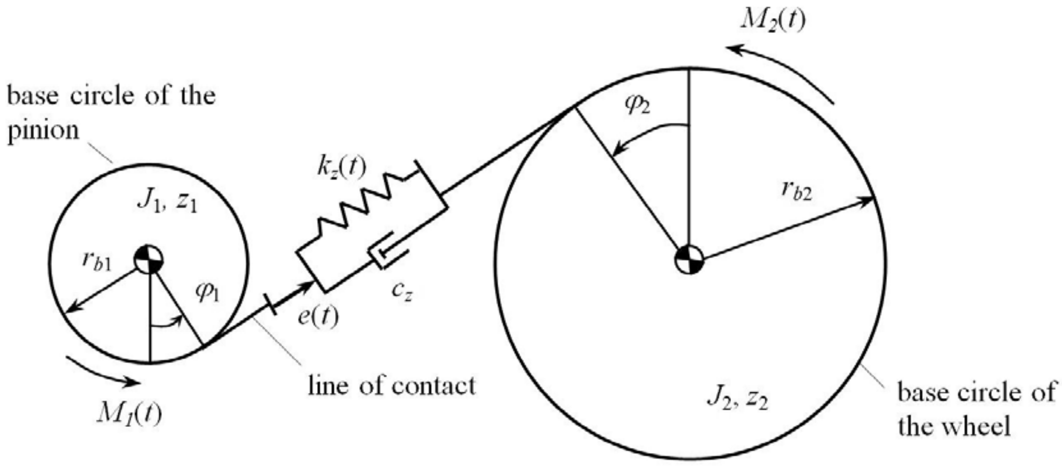

Fig. 1. Dynamic model of a pair of gears

Determination of the coefficient k_z is easy using methods of finite elements. More difficulty offers calculation damper coefficient c_z . Damping in the gear system usually is an unknown quantity, but it has an important effect in resonance vibration. In the gear train, the main energy losses are friction in the teeth meshing. damping dissipates energy constantly because of sliding friction. The magnitude of sliding friction is a constant value, independent of surface areas, displacement or position, and velocity. The system undergoing damping is periodic or oscillating and restrained by the sliding friction

2. BASICS OF THE EQUIVALENT VISCOUS DAMPING

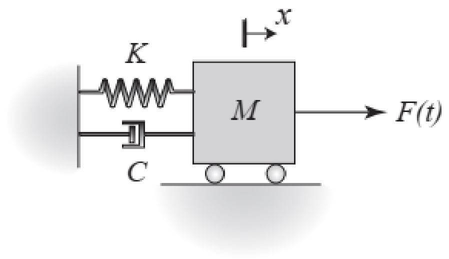

Fig. 2. Forced mass-spring-damper system (3)

The energy lost per cycle in a damper in a harmonically forced system may be expressed as:

$$W_d = \oint F_d dx, \tag{1}$$

where F_d. represents the damping force. The simplest case mathematically is that of viscous damping where $F_d = c\dot{x}$. Letting the steady-state solution be expressed as

$$x = X \sin(\omega t) \tag{2}$$

$$\dot{x} = \omega X \cos(\omega t) \tag{3}$$

$$W_d = \oint c\dot{x}\, dx = \oint c\dot{x}^2\, dt \tag{4}$$

$$W_d = C\omega^2 (kX)^2 \int_0^{2\pi/\omega} \cos^2(\omega t)\, dt = \pi C\omega (kX)^2 \tag{5}$$

3. PROBLEM

The problem is that the friction is not in the direction of movement. The friction occurs on the teeth surfaces in the area of engagement and is in the direction of the common tangent which is different from the teeth engagement. The relationship between the two displacements must be determined to determine the equivalent friction power loss in the model. Defining the ratio occurs in contact of working surfaces of the gears.

$$h = \frac{v_{slid}}{\dot{x}} = \frac{y}{x} \tag{6}$$

$$y = hX \sin(\omega t) \tag{7}$$

$$\dot{y} = h\omega X \cos(\omega t) \tag{8}$$

$$W_d = \oint C\dot{y} dx = \oint C (h\dot{x})^2\, dt \tag{9}$$

$$W_d = C\omega^2 (hX)^2 \int_0^{2\pi/\omega} \cos^2(\omega t)\, dt = \pi C\omega (hX)^2 \tag{10}$$

$$C = \frac{W_d}{\pi\omega (hX)^2} \tag{11}$$

4. EQUIVALENT VISCOUS DAMPING FOR FRICTION

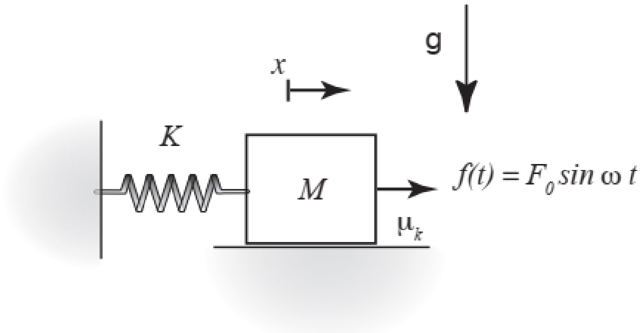

Fig. 3. Simple friction model

The resistance force, F_c, in the case of friction dissipates W_c=4FcX in energy over each quarter cycle as shown in Figure 4, hence, equating the total dissipative work per cycle to that done by a viscous damper, we have

$$W_c = 4F_c hX = \pi c\omega\left(hX\right)^2 \tag{12}$$

$$c = \frac{4F_c}{\pi\omega hX} \tag{13}$$

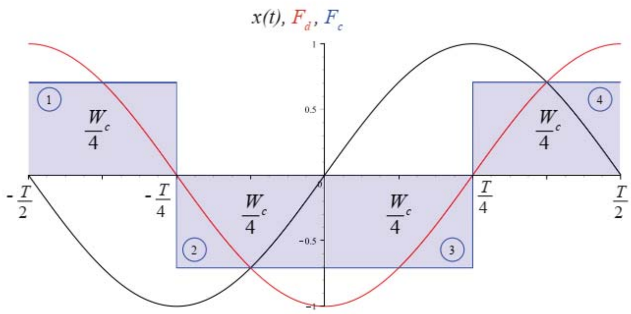

Fig. 4. Normalized viscous and friction resistance force, and displacement over one period. (3)

5. DEFINING THE RATIO

$$h = \frac{|v_{sl}|}{|\dot{x}|} = \frac{|\dot{y}|}{|\dot{x}|} = \frac{y}{x} \tag{14}$$

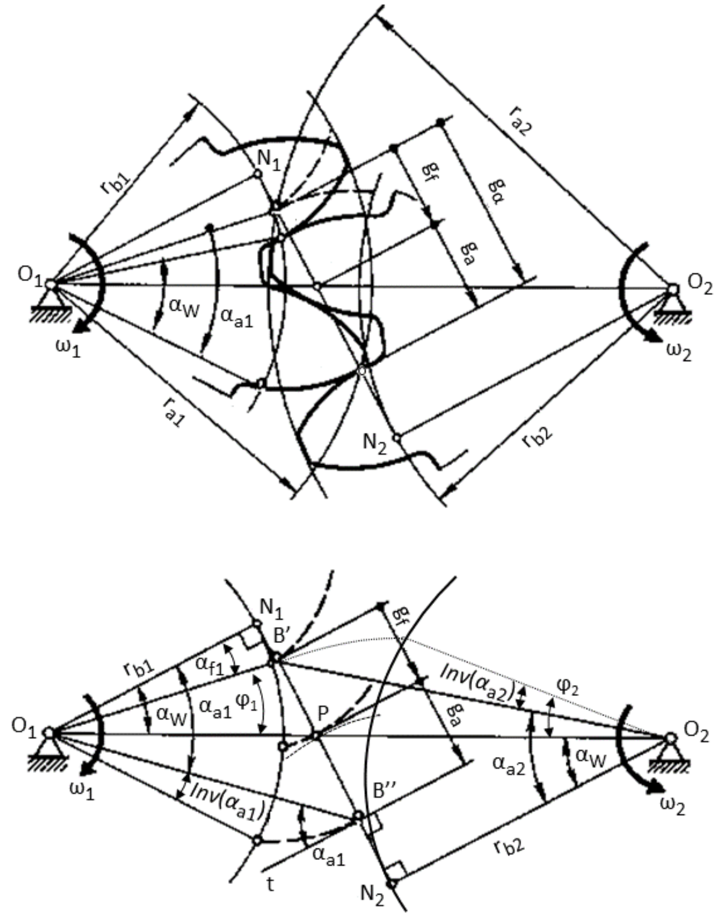

Fig. 5. Geometry of gearing

All characters in the formulas 14 to 20 for the theory of gear tooting correspond to [1]:

$$\varphi_1 = \alpha_w - \alpha_{f_1} - \nu_{\alpha_{f_1}} \tag{15}$$

$$\alpha_1 = \alpha_w - \varphi_1 - \nu_{\alpha_{f_1}} - \omega_1 t \tag{16}$$

$$\begin{gathered}\varphi_1 \in \left[\alpha_w - \alpha_{f_1} - inv\left(\alpha_{f_1}\right); -\left(\alpha_{a_1} - \alpha_w + inv\left(\alpha_{a_1}\right)\right)\right] \Rightarrow \\ t \in \left[0, \frac{\left(-\alpha_{a_1} - inv\left(\alpha_{a_1}\right) + \alpha_{f_1} + inv\left(\alpha_{f_1}\right)\right)}{\omega_1}\right]\end{gathered} \tag{17}$$

$$\varphi_2 = -\alpha_w + \alpha_{a_2} + \nu_{\alpha_{a_2}} \tag{18}$$

$$\alpha_2 = \alpha_w + \varphi_2 - \nu_{\alpha_{a_2}} - \omega_2 t \tag{19}$$

$$t \left[0, \frac{\left(-\alpha_{a_1} - inv\left(\alpha_{a_1}\right) + \alpha_{f_1} + inv\left(\alpha_{f_1}\right)\right)}{\omega_1}\right] \tag{20}$$

$$\cos\left(\alpha_y\right) = \frac{r_b}{r_y} \Rightarrow \alpha_y = \text{Arccos}\left(\frac{r_b}{r_y}\right) \tag{21}$$

The radii of curvature and the slip velocity are respectively (1):

$$\rho_1 = r_{b_1} tg\left(\alpha_1\right) \tag{22}$$

$$\rho_2 = r_{b_2} tg\left(\alpha_2\right) \tag{23}$$

Relative sliding speed is:

$$\vec{v}_{sl} = \rho_1\vec{\omega}_1 - \rho_2\vec{\omega}_2 = \rho_1\vec{\omega}_1 - \rho_2 i_{21}\vec{\omega}_1 \tag{24}$$

$$k = \frac{|v_{sl}|}{|\dot{x}|} \tag{25}$$

$$\dot{x} = r_{b_1}\omega_1 \tag{26}$$

$$\vec{F}_n = \frac{\vec{T}_1}{r_{b_1}} \tag{27}$$

Friction force is:

$$F_c = \mu F_n \tag{28}$$

$$F_c = \mu\vec{F}_n\vec{v}_{sl} = \mu\vec{F}_n\left(\rho_1\vec{\omega}_1 - \rho_2 i_{21}\vec{\omega}_1\right) \tag{29}$$

The total dissipative work per cycle to that done by a damper is:

$$W_c = \oint F_c dx \tag{30}$$

Vibrating of the model mass is:

$$M\ddot{x} + c\dot{x} + Kx = F_0 \sin\left(\omega t\right) \tag{31}$$

The steady-state magnitude may be written:

$$|X| = \frac{F_0}{\sqrt{\left(K - M\omega^2\right)^2 + c^2\omega^2}} \tag{32}$$

$$|X| = \frac{\sqrt{F_o^2\pi^2h^2 - 16F_c^2}}{\pi h\sqrt{\left(K^2 - M\omega^2\right)^2}} \tag{33}$$

$$|X| = \frac{F_0\sqrt{1 - \frac{16F_c^2}{F_o^2\pi^2h^2}}}{K\left(1 - \frac{\omega^2}{\left(K/M\right)}\right)} = \frac{F_0\sqrt{1 - \left(\frac{4F_c}{F_o\pi\ h}\right)^2}}{K\left(1 - \left(\frac{\omega}{\omega_r}\right)^2\right)} \tag{34}$$

Friction resistance force:

$$F_c \le \pi h F_0 \tag{35}$$

Coefficient of friction is:

$$\mu \le \frac{\pi h F_0}{F_n} \tag{36}$$

Note that the amplitude grows unbounded as $\omega \to \omega_n$ In addition, for real (physically meaningful) solutions:

$$\frac{4F_c}{F_o \pi\ h} \le 1 \tag{37}$$

6. NUMERIC SAMPLE

Using

Table 1. Example

Number of teeth Pinion / Gear	z	17	44	
Normal module	m_n	10	10	(mm)
Center distance (working)	a_w	305	305	(mm)
Pressure angle	α	0.348	0.348	(rad)
Tip diameter	d_a	190	460	(mm)
Reference diameter	d	170	440	(mm)
Base diameter	d_b	159,74	413,46	(mm)
Root diameter	d_f	145	415	(mm)
Operating pitch diameter	d_w	170	440	(mm)
Addendum	h_a	10	10	(mm)
Dedendum	h_f	12,5	12,5	(mm)
Tip profile angle	α_a	0,572079	0,453689	(rad)
Start profile angle	α_f	0	0,086041	(rad)

$$\varphi_1 = \alpha_w - \alpha_{f_1} - \nu_{\alpha_{f_1}} = 0.3490 \tag{38}$$

$$\alpha_1 = \alpha_w - \varphi_1 - \nu_{\alpha_{f_1}} - \omega_1 t = t \tag{39}$$

$$\varphi_2 = -\alpha_w + \alpha_{a_2} + \nu_{\alpha_{a_2}} = 0.1212 \tag{40}$$

$$\alpha_2 = \alpha_w + \varphi_2 - \nu_{\alpha_{a_2}} - \omega_2 t = 0.4363 - \frac{17}{44} t \tag{41}$$

$$t_{\max} = \frac{\left(-\alpha_{a_1} - inv\left(\alpha_{a_1}\right) + \alpha_{f_1} + inv\left(\alpha_{f_1}\right)\right)}{\omega_1} = \frac{8\pi}{45} = 0.5585 \tag{42}$$

$$\rho_1 = r_{b_1} tg(\alpha_1) = \frac{159}{2} tg(t) \tag{43}$$

$$\rho_2 = r_{b_2} tg(\alpha_2) = \frac{413}{2} tg\left(0.4363 - \frac{17}{44} t\right) \tag{44}$$

$$|\vec{v}_{sl}| = |\rho_1 \vec{\omega}_1 - \rho_2 \vec{\omega}_2| = \left|\rho_1 - \rho_2 \frac{17}{44}\right| = \left|-\frac{3485}{44} tg\left(0.4363 - \frac{17}{44} t\right) + 79 tg(t)\right| \tag{45}$$

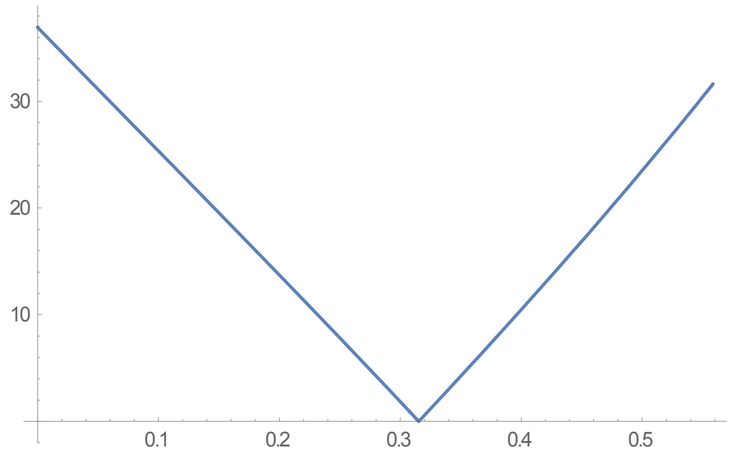

Fig. 4. Glide speed during engagement

$$\int_0^{t_{\max}} v_{sl} dt = 9.6 \tag{46}$$

$$\overline{v}_{sl} = \frac{\int_0^{t_{\max}} v_{sl} dt}{t_{\max}} = 17.1985 \, {}^{mm}\!/_{sek} \tag{47}$$

$$h = \frac{\overline{v}_{sl}}{r_{b_1} \omega_1} = 0.22 \tag{48}$$

$$T_1 = 25882 Nmm \Rightarrow F_n = 323N \tag{49}$$

$$\mu = 0.1 \tag{50}$$

$F_c = 1.625 \mu F_n = 52.49N$ overlap factor 1.2

The final form of the formula for determining dissipation is obtained from formulas (1) to (50) and has the following form:

$$c = \frac{4F_c}{\pi \omega h X} \tag{51}$$

7. CONCLUSION

Formula (51) enables the calculation of dissipation with acceptable accuracy with further use to calculate the amplitudes of the natural and forced frequencies

As a pattern to which the results were derived, the mathematical model of the gear tooting was used, in which the characteristics of the gearing were modeled with a non-continuous function describing friction zone.

The analysis was carried out for the zero initial conditions (correction of teeth is zero) which allowed more easy calculations for all considered values of the system parameters.

The source of the excited mechanical vibrations of meshing gear teeth is the so-called performance and location errors. They are mainly caused by radial beating and geometric deviations of the tooth profile. This parameter depends on the accuracy of the production and assembly of the cooperating wheels. The entire mentioned factors mean that the phenomenon of energy dissipation in cooperating toothed wheels is a complex issue. However, due to the complexity of the energy dissipation phenomenon in meshing, the energy losses are usually modeled with a viscous damper. Taking into account these factors leads to a nonlinear mathematical model of a gear transmission in which chaotic phenomena may occur.

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