Numerical study of the vibrational behaviour of a motorcycle at different castor angle

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Abstract. The vibration behaviour of the vehicles undoubtedly affects both the smoothness of the movement and the health of the driver and passengers. In this study an in-plane dynamic model with six degrees of freedom is presented to understanding and doing evaluation of the vibration environment of the motorcycle rider and different motorcycle component bodies. The equations of motion for the model are formulated. The matrix form of the equations was written and simulations were performed in MATLAB software. The displacements, velocities and accelerations of the sprung masses of the motorcycle, seat and driver were found, taking into account the influence of the unsprung masses and the kinematics of the suspension. The vibration behaviour of the motorcycle in case of sinusoidal road disturbance at different castor angles has been studied numerically. The purpose of this paper is to determine the influence of the castor angle on the vibration behaviour of the motorcycle. The results of the work can be used for optimization in kinematics, damping and stiffness characteristics of the suspension.

1. Introduction

The behaviour of the suspension can be evaluated in terms of three parameters – discomfort, suspension work space and dynamic tire load [1]. Linear oscillations along the vertical axis and angular oscillations about the y axis have the strongest influence on the smoothness of motion [2, 3]. These oscillations exert a complex biological effect on humans, leading to reduced performance, functional changes, traumas, discussed in detail in [4-6] and others. The type of disturbance that most affects the comfort of movement is the random vibration caused by the irregularities of the road surface. In [7], a whole body dynamic vibration test was performed comparing the comfort of driving a car and a motorcycle. The risk to motorcycle riders is significantly greater as the back does not have a support, resulting in a greater absorption of vertical vibrations. In [8] a model with 6 degrees of freedom is examined, studying the vibrational behaviour of the motorcycle when changing the distance of the seat from the center of gravity. Significant studies to determine the comfort of motorcycle driving models with different degrees of freedom have been carried out in [9-12] and many others. Methods for assessing the smoothness of a car with are presented in [13-15], but they are applicable in the study of the dynamics of a motorcycle. The aim of the publication is to study the vibrational behaviour of the motorcycle and the driver in case of sinusoidal disturbance of the road with different castor angle.

2. Reduced stiffness and damping

In its plane of symmetry, the motorcycle can be represented as five rigid bodies whose vibration behaviour is described by six independent coordinates: vertical displacement of the seat and driver, vertical displacement of the centre of mass of the sprung parts, vertical displacement of the unsprung masses of the front and rear suspension (equivalent masses relative to the centre of the wheels) and rotation of the sprung masses. The half-car model (figure 1) is suitable for use. It takes into account the influence of the unsprung masses, and the sprung parts are regarded as a non-deformable beam with two degrees of freedom – the displacement of the mass centre along the vertical axis bounce mode, and the rotation motion (pitch).

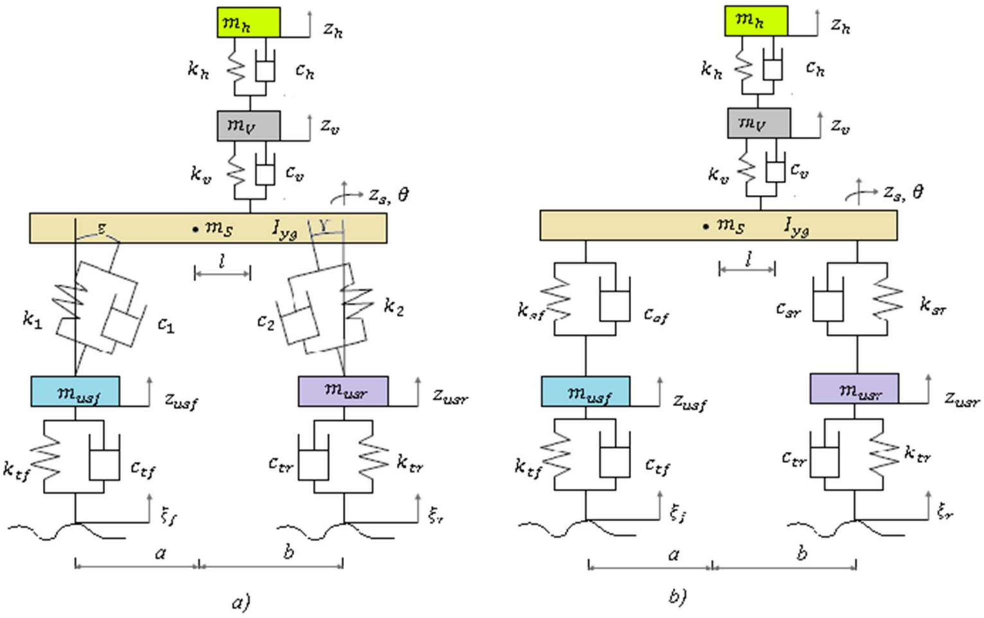

Figure 1. Schematic diagram of 6 DOF motorcycle model: a) real; b) equivalent.

In order to obtain more accurate results of the investigated model, it is necessary to use reduced values of the coefficients of stiffness and damping to obtain the equivalent of the examined suspension. The equivalent front suspension is expressed [12]:

$$k_{sf} = \frac{k_1}{cos^2\varepsilon}; \; c_{sf} = \frac{c_1}{cos^2\varepsilon}, \tag{1}$$

where k_1 is the coefficient of stiffness, ε is the caster angle, c_1 is the damping constant. With a telescopic fork in front, the equivalent stiffness and damping are equal to the sum of the stiffness and damping of the two springs and dampers. Since there are two groups of spring dampers arranged in parallel in the fork, the stiffness k_1 is equal to the sum of the stiffness of the two springs, and the damping c_1 is equal to the sum of the damping of the two dampers.

Due to the complexity of the rear suspension system, its equivalent coefficients in the reduced model have to be obtained by an energy conservation criterion [16]:

$$F_{sr}\frac{dz_{usr}}{dt} = F_{sr0}\frac{dq_{usr}}{dt}, \tag{2}$$

where F_{sr} is the force in the equivalent model, z_{usr} is the displacement in the equivalent suspension. The force in the actual suspension is F_{sr0} can be found by Hooke's law:

$$F_{sr0} = -k_2 q_{usr}, \tag{3}$$

A more accurate description of the force of the rear suspension spring can be found by polynomial regression of the equivalent force with the vertical travel of the rear wheel. To linearize the suspension in the equivalent model, the equivalent stiffness (k_{sr}) is taken as the absolute value of the first-order coefficient of the polynomial. Spring force of the rear suspension spring (F_{sr}) is described as a linear function of the vertical displacement of the rear wheel (z_{usr}):

$$F_{sr} = -k_{sr} z_{usr}. \tag{4}$$

A similar energy conservation criteria is followed to compute the equivalent rear suspension damping coefficient for the reduced model. To simplify the model, the rear suspension is represent in the same way as the front:

$$k_{sr} = \frac{k_2}{cos^2\gamma};\ c_{sr} = \frac{c_2}{cos^2\gamma}, \tag{5}$$

where γ is inclination angle of the spring – shock absorber group.

3. Differential equations of motion

The compilation of the equations of motion is based on the second-order Lagrange method [17]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \left(\frac{\partial T}{\partial q_i}\right) + \left(\frac{\partial \Pi}{\partial q_i}\right) + \left(\frac{\partial R}{\partial \dot{q}_i}\right) = Q, \tag{6}$$

where *T* is the kinetic energy; *Π* – potential energy; *R* – Relay dissipative function; q_i – generalized coordinate; *Q* – external force; *t* – time.

The kinetic energy of the system is:

$$T = \frac{1}{2}m_s \dot{z_s}^2 + \frac{1}{2}m_{usr}\dot{z}_{usr}^{\ 2} + \frac{1}{2}m_{usf}\dot{z}_{usf}^{\ 2} + \frac{1}{2}I_{yg}\dot{\theta}^2 + \frac{1}{2}m_v\dot{z}_v^{\ 2} + \frac{1}{2}m_h\dot{z}_h^{\ 2} \tag{7}$$

The potential energy is equal to:

$$\Pi = \frac{1}{2}k_{sf}[z_s - z_{usf} - a\theta]^2 + \frac{1}{2}k_{sr}[z_s - z_{usr} + b\theta]^2 + \frac{1}{2}k_{tf}(z_{usf} - \xi_f)^2 + \frac{1}{2}k_{tr}(z_{usr} - \xi_r)^2 + \frac{1}{2}k_v(z_v - z_s + l\theta)^2 + \frac{1}{2}k_h(z_h - z_v)^2 \tag{8}$$

Relay function:

$$R = \frac{1}{2}c_{sf}[\dot{z}_s - \dot{z}_{usf} - a\dot{\theta}]^2 + \frac{1}{2}c_{sr}[\dot{z}_s - \dot{z}_{usr} + b\dot{\theta}]^2 + \frac{1}{2}c_{tf}[(\dot{z}_{usf} - \dot{\xi}_f]^2 + \frac{1}{2}c_{tr}(\dot{z}_{usr} - \dot{\xi}_r)^2 + \frac{1}{2}c_v[(\dot{z}_v - \dot{z}_s + l\dot{\theta}]^2 + \frac{1}{2}c_h(\dot{z}_h - \dot{z}_v)^2 \tag{9}$$

The equations of motion:

$$m_{usf}\ddot{z}_{usf} + c_{sf}(\dot{z}_{usf} - \dot{z}_s - a\dot{\theta}) + k_{sf}(z_{usf} - z_s - a\theta) + k_{tf}(z_{usf} - \xi_f) + c_{tf}(\dot{z}_{usf} - \dot{\xi}_f) = 0 \tag{10}$$

$$m_{usr}\ddot{z}_{usr} + c_{sr}(\dot{z}_{usr} - \dot{z}_s + b\dot{\theta}) + k_{sr}(z_{usr} - z_s + b\theta) + k_{tr}(z_{usr} - \xi_r) + c_{tr}(\dot{z}_{usr} - \dot{\xi}_r) = 0 \tag{11}$$

$$m_v\ddot{z}_v + c_v(\dot{z}_v - \dot{z}_s + l\dot{\theta}) + k_v(z_v - z_s + l\theta) + c_h(\dot{z}_v - \dot{z}_h) + k_h(z_v - z_h) = 0 \tag{12}$$

$$m_s\ddot{z}_s + c_{sf}(\dot{z}_s - \dot{z}_{usf} + a\dot{\theta}) + c_{sr}(\dot{z}_s - \dot{z}_{usr} - b\dot{\theta}) + k_{sf}(z_s - z_{usf} + a\theta) + k_{sr}(z_s - z_{usf} - b\theta) + k_v(z_s - z_v - l\theta) + c_v(\dot{z}_s - \dot{z}_v - l\dot{\theta}) = 0 \tag{13}$$

$$I_{yg}\ddot{\theta} + ac_{sf}(\dot{z}_s - \dot{z}_{usf} + a\dot{\theta}) - bc_{sr}(\dot{z}_s - \dot{z}_{usr} - b\dot{\theta}) + lc_v(\dot{z}_v - \dot{z}_s + l\dot{\theta}) + ak_{sf}(z_s - z_{usf} + a\theta) - bk_{sr}(z_s - z_{usr} - b\theta) + lk_v(z_s - z_v + l\theta) = 0 \tag{14}$$

$$m_h\ddot{z}_h + c_h(\dot{z}_h - \dot{z}_v) + k_h(z_h - z_v) = 0 \tag{15}$$

The equations of motion can be combined, rearranged and expressed in matrix form:

$$[\mathrm{M}]\ddot{q} + [C]\dot{q} + [K]q = [Q] \tag{16}$$

where [M] – mass matrix; $[C]$ – damping matrix; $[K]$ – stiffness matrix; $[Q]$ – force matrix; q – state vector.

$$[M]=\begin{bmatrix} m_{usf} & 0 & 0 & 0 & 0 & 0\\ 0 & m_{usr} & 0 & 0 & 0 & 0\\ 0 & 0 & m_v & 0 & 0 & 0\\ 0 & 0 & 0 & m_S & 0 & 0\\ 0 & 0 & 0 & 0 & I_{yg} & 0\\ 0 & 0 & 0 & 0 & 0 & m_h \end{bmatrix}, q=\begin{bmatrix} z_{usf}\\ z_{usr}\\ z_v\\ z_s\\ \theta\\ z_h \end{bmatrix} \dot{q}=\begin{bmatrix} \dot{z}_{usf}\\ \dot{z}_{usr}\\ \dot{z}_v\\ \dot{z}_s\\ \dot{\theta}\\ \dot{z}_h \end{bmatrix} \ddot{q}=\begin{bmatrix} \ddot{z}_{usf}\\ \ddot{z}_{usr}\\ \ddot{z}_v\\ \ddot{z}_s\\ \ddot{\theta}\\ \ddot{z}_h \end{bmatrix}, [Q]=\begin{bmatrix} k_{tf}\xi_f+c_{tf}\dot{\xi}_f\\ k_{tr}\xi_r+c_{tr}\dot{\xi}_r\\ 0\\ 0\\ 0\\ 0 \end{bmatrix},$$

$$[C]=\begin{bmatrix} c_{sf}+c_{tf} & 0 & 0 & -c_{sf} & -ac_{sf} & 0\\ 0 & c_{sr}+c_{tr} & 0 & -c_{sr} & bc_{sr} & 0\\ 0 & 0 & c_v+c_h & -c_v & lc_v & -c_h\\ -c_{sf} & -\ c_{sr} & -c_v & c_{sr}+c_{sf}+c_v & -c_{sr}b+c_{sf}a-c_vl & 0\\ -ac_{sf} & bc_{sr} & lc_v & -c_{sr}b+c_{sf}a-c_vl & a^2c_{sf}+b^2c_{sr}+l^2c_v & 0\\ 0 & 0 & -c_h & 0 & 0 & c_h \end{bmatrix},$$

$$[K]=\begin{bmatrix} k_{sf}+k_{tf} & 0 & 0 & -k_{sf} & -\ ak_{sf} & 0\\ 0 & k_{sr}+k_{tr} & 0 & -k_{sr} & bk_{sr} & 0\\ 0 & 0 & k_v+k_h & -k_v & lk_v & -k_h\\ -k_{sf} & -\ k_{sr} & -k_v & k_{sr}+k_{sf}+k_v & -\ k_{sr}b+k_{sf}a-k_vl & 0\\ -ak_{sf} & bk_{sr} & lk_v & -k_{sr}b+k_{sf}a-k_vl & a^2k_{sf}+b^2k_{sr}+l^2k_v & 0\\ 0 & 0 & -k_h & 0 & 0 & k_h \end{bmatrix},$$

The equations are solved in MATLAB. Since numerical integration functions are capable of solving systems of first order differential equations, it is necessary to canonize them by the method described in [18]. The state-space formulation and output formulation are as follows:

$$\dot{S}=AS+BU;\ Y=DS+EU, \tag{17}$$

where S is the state vector; A – state matrix; B – input impact matrix; D and E are output variable matrices; U – input vector; Y – output vector.

4. Model parameters and conditions

A sinusoidal road profile is used to perform the simulation. The disturbance is described by the following equations:

$$\xi_f=\xi_o sin(2\pi vt);\ \xi_r=\xi_o \sin(2\pi vt+t_1), \tag{18}$$

where v is the frequency of motion imposed on the system by the irregularities of the road and represents the ratio between the velocity of movement and the wave length. The rear axle disturbance follows the same profile with a delay of t_1 equal to the motorcycle wheelbase divided by the speed of travel. The following assumptions were adopted when designing the model:

- the values of the vector of the initial conditions are assumed to be zero;
- during the considered period the speed is constant;
- the characteristics of the stiffness and damping elements are linear and no pre-load is taken into account;
- the tire damping coefficients are assumed to be 0, since in low-frequency mode their influence is not significant;
- tires are represented as a simple spring without mass;
- the driver is presented as a body with one degree of freedom. The position of the rider on the motorcycle does not change during the study period. Approximate values of the stiffness and damping properties of the rider are taken from [19].

The other parameters of the model are described in table 1.

Table 1. Model parameters.

Parameter	Definition	Value
Front unsprung mass, kg	m_{usf}	18
Rear unsprung mass, kg	m_{usr}	20
Sprung mass, kg	m_s	192
Mass of seat, kg	m_v	5
Inertial moment above y, kgm^2	I_{yg}	63.125
Front suspension stiffness, N/m	k_1	18000
Front suspension damping, Ns/m	c_1	800
Rear suspension stiffness, N/m	k_{sr}	30000
Rear suspension stiffness, Ns/m	c_{sr}	1490
Castor angle, °	ε	Variable
Distance between CG and front axle, m	a	0.755
Distance between CG and rear axle, m	b	0.755
Inclination of rear shock absorber group, °	γ	145
Front tire stiffness, N/m	k_{tf}	130000
Rear tire stiffness, N/m	k_{tr}	141000
Length of the wave, m	L	10
Velocity, m/s	V	13.88
Seat stiffness, N/m	k_v	15000
Seat damping, Ns/m	c_v	200
Distance between CG and seat, m	l	0.25
Time, s	t	10
Weight of irregularities, m	ξ_0	0.025
Rider's mass, kg	m_h	70
Rider's stiffness, N/m	k_h	390000
Rider's damping, Ns/m	c_h	4224

5. Results

The results of the experiment are presented graphically at castor angle of 27°. In figure 2 is represented the acceleration obtained in the front unsprung masses. It is noticed that the initial acceleration has very large values, which quickly stabilize to at an amplitude of 2.2 m/s^2 after few seconds. Maximum displacement in front unsprung mass is 0.030 m – a little bit above the weight of the irregularities. This could be described as a vibrational state of the wheel caused by road profile as the model of the motorcycle travels at. This is a result of inertia and elastic properties of the wheel while hitting a hump profile at some speed, which leads to pitch motion of the body of the motorcycle about a small angle θ. Similar results are obtained for the response of the rear unsprung masses. The vibration acceleration has larger maximum (by about 8 m/s^2) shown in figure 3. This this behaviour is described by the difference in stiffness and damping properties and difference in weight. Maximum displacement in rear unsprung mass is 0.029 m.

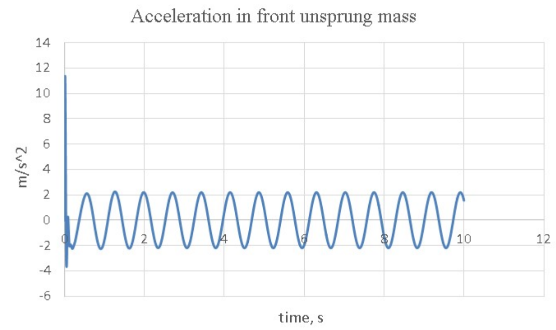

Figure 2. The resulting acceleration in the front unsprung as a function of time.

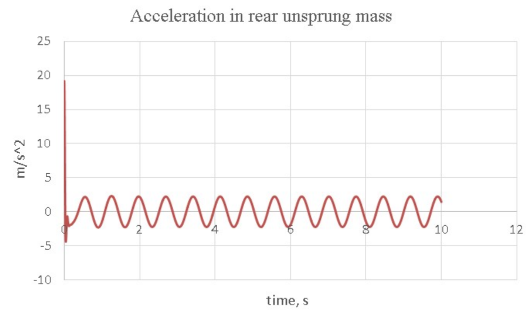

Figure 3. The resulting acceleration in the rear unsprung as a function of time.

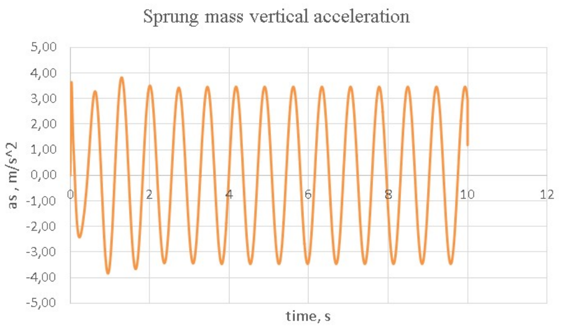

Figure 4. The resulting vertical acceleration in the sprung mass as a function of time.

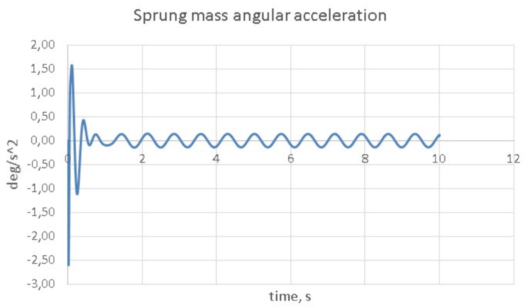

Figure 5. The resulting angular acceleration in sprung as a function of time.

In figures 4 and 5 is represented the behaviour of the motorcycle body, described by the vertical and angular accelerations. A part of the vibrations from the tyres have been absorbed by the suspension and hence resulting in reduced vibration magnitude values. Maximum vertical acceleration is 3.84 m/s^2 and maximum angular acceleration is 2.601.

Perhaps the most important are the results obtained for the displacements and accelerations in the seat and the driver. The vibrational behavior of model seat and driver in terms of time domain were as indicated respectively in figures 6 and 7. Although they almost same magnitudes, the responses of the model seat and rider had similar behavior. Accelerations reach their maximum in 1.65 seconds – 6.910 m/s^2 in seat and 6.992 at human.

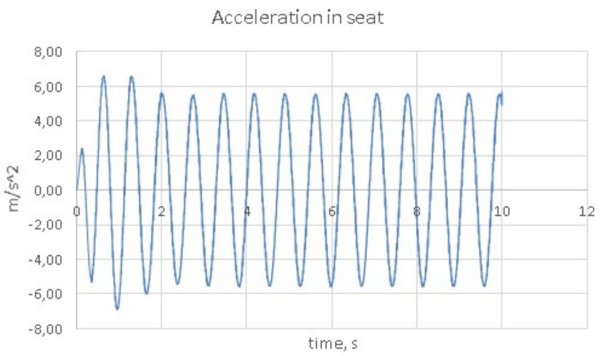

Figure 6. Vertical acceleration in motorcycle seat.

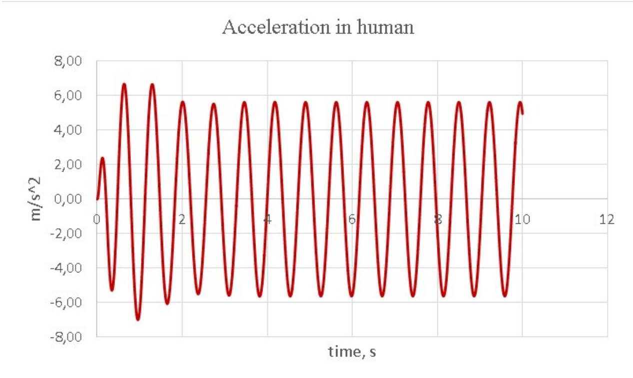

Figure 7. Vertical acceleration in human.

Numerical experiments were performed while maintaining the actual stiffness and damping coefficients of the front suspension when changing the castor angle from 27 to 35 degrees with a step of 1 degree. The other parameters of the model remain unchanged. The variation in vertical and angular accelerations are not shown graphically (due to insufficient clarity), but are presented in table 2.

Table 2. Maximum accelerations at different castor angles.

Castor angle, °	$\ddot{z}_s$, m/s^2	$\ddot{z}_v$, m/s^2	$\ddot{z}_h$, m/s^2	$\ddot{\theta}$, °/s^2
27	3.835	6.910	6.992	2.601
28	3.821	6.909	6.989	2.580
29	3.807	6.908	6.986	2.558
30	3.791	6.902	6.983	2.535
31	3.775	6.904	6.979	2.511
32	3.759	6.893	6.975	2.487
33	3.742	6.890	6.970	2.463
34	3.725	6.878	6.966	2.438
35	3.707	6.873	6.961	2.412

According to the international standard ISO 2631-1: 1997, which was introduced in 2004 under the name BDS ISO 2631-1: 2004, as a Bulgarian standard, the main method for estimating the intensity of oscillations uses the weighted root mean square (r.m.s.) of acceleration [20]:

$$\ddot{z}_{rms} = \sqrt{\frac{1}{T}\int_0^T \ddot{z}_h{}^2(t)dt}, \tag{19}$$

where T is the duration of the measurement in seconds, $\ddot{z}_h$ is the weighted acceleration as a function of time. The results are represented in table 3.

Table 3. Weighted root mean square (r.m.s.) of acceleration.

Castor angle, °	$\ddot{z}_{rms}$, m/s^2
27	4.036
28	4.023
29	4.009
30	3.996
31	3.982
32	3.968
33	3.953
34	3.938
35	3.924

Compared to the values given in ISO 2631-1, the calculated values of r.m.s show that the sensation in humans is very uncomfortable. This is a result of the high speed with which the road unevenness is passed, the relatively short base of the motorcycle (compared to other type of transport) and the suspension parameters. It is noticed that with the increase of the castor angle, the driving comfort increases insignificant and is not enough to achieve driving comfort. In order to increase the comfort, it is necessary to reduce the speed of passing the irregularities and optimize the characteristics of the suspension.

6. Conclusions

The results show that with increasing castor angle while keeping the other parameters unchanged, the vertical accelerations decrease.

In order to achieve reducing vibration magnitudes of the motorcycle and rider caused by the road condition it is necessary to use suspension system which may have variable characteristics (active suspension). The model can be used to study the importance of suspension characteristics and their optimization.

The model allows the study of driving comfort.

The model can be used to optimize suspension parameters in order to reduce the pitch and bounce and reducing the tendency to break (exhaustion of the stroke) in suspension when passing road irregularities. As the influence of the castor angle is insignificant, an adjustment in the suspension is needed to improve the vibrational behaviour.

The model can be used on motorcycles with different types of suspension if linearized coefficients are used in the stiffness and damping matrices.

As a disadvantage it should be noted that the assumptions made in rider's position introduce some error in the results.

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