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Modeling and Simulation of Thermal-Hydraulic Processes in General PWR Core in MATLAB Environment

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Abstract. This paper present direct numerical simulation (DNS) of the generic PWR core and its thermal-hydraulic characteristics. For this simulations SIMPLE method is used and applied in MATLAB environment. The model will be used in subsequent simplified simulator development.

INTRODUCTION

It is known that there are a number of computer codes which are carrying out calculations of thermal-hydraulic processes occurring in nuclear reactors. One of these codes is RELAP5 which is developed for best estimate simulation of light water reactor cooling system during postulated accidents [1]. The programs' main issue is that they are with closed source codes and the user can't make any changes in models and computation procedures. Therefore, sometimes it is useful to develop your own code.

That report considers the development of such code in MATLAB environment. It presents one dimensional flow of water coolant at generic PWR's core. As a basis for creating the model Navier-Stokes differential equation system is used, namely – conservation of mass equation, momentum and energy conservation equations with two additional thermodynamic algebraic equations. Following the approach which is used in RELAP5 [1] some terms are neglected: in equation of momentum – Reynolds term and in equation of energy – diffusion term.

PWR CORE SHORT DESCRIPTION

There are two major systems utilized to convert the heat generated in the nuclear fuel into electrical power for industrial and commercial use. The primary system which is frequently referred to Reactor Coolant System transfers the heat generated in the fuel pellets to the steam generator (SG), which is the link between primary and secondary

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systems. The steam produced in the SG is transferred by the secondary system to the main turbine generator, where it is converted into electricity.

The primary system consists of several main components, namely the reactor vessel, the steam generators, the reactor coolant pumps, a pressurizer, and the connecting piping.

The reactor core, and all associated support and alignment devices, are housed within the reactor vessel. The major components are the reactor vessel, the core barrel, the reactor core itself, and the upper internals package [2]. The primary coolant is maintained at a pressure of ~ 15.5 MPa, enters the core at about 288 °C and leaves it at about 324 °C depending upon the core configuration [3].

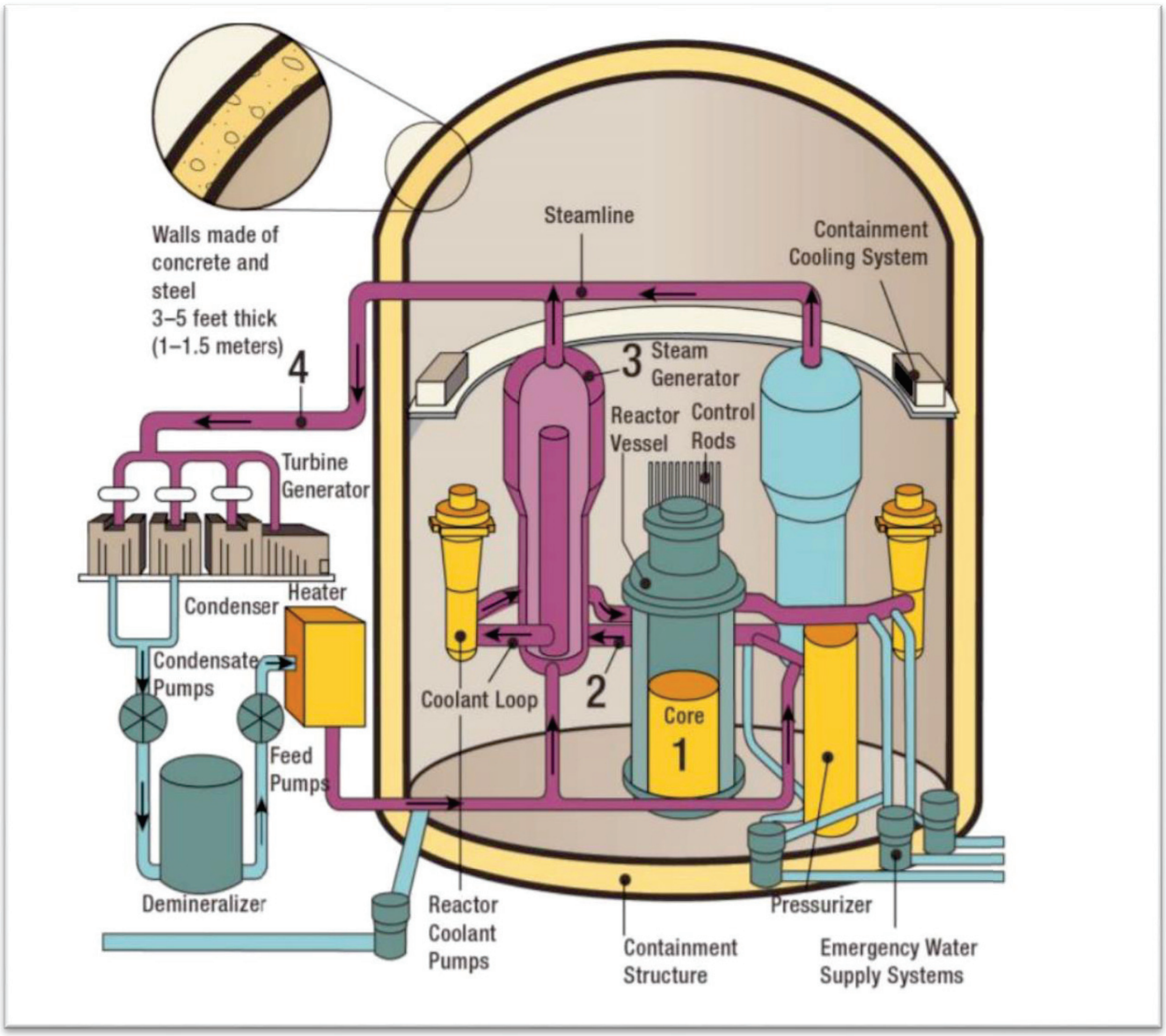

FIGURE 1. Schematic picture of nuclear power plant with pressurizer water reactor [4]

ASSUMPTIONS AND POSSIBILITIESFOR PWR CORE MODELING

Thermal-hydraulic performance in any reactor core is an essential factor in the nuclear power plant design [5]. Hence, for a thermal-hydraulic core modeling, analysis, and simulations as a part of the main equipment of the nuclear power plant it is necessary to reproduce its inlet and outlet parameters in a proper way.

There are several possibilities for the thermal-hydraulic modeling and analyzes of PWR core. Mainly this is done by using sub-channel analysis codes. In this codes the system pressure, coolant inlet temperature, as well as coolant flow rate and thermal power and its distributions are considered as the key parameters for sub-channel analysis. Also in these codes, the governing equations of mass, momentum and energy are solved in control volumes which are connected in both radial and axial directions. The flow distributions in the rod bundle geometry are estimated by considering lateral momentum balance and the inter channel mixing models to account for the cross flow between the adjacent sub-channels. [6]

In this paper, we describe one-dimensional (1D) thermal-hydraulic model of reactor core and it parameters based on SIMPLE method and implemented in MATLAB environment which is described below.

MATHEMATICAL MODEL

The system that governs the process is

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)=0 \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u.u)}{\partial x}=-\frac{\partial P}{\partial x} \tag{2}$$

$$\frac{\partial(\rho i)}{\partial t}+\frac{\partial(\rho i.u)}{\partial x}=-P\frac{\partial u}{\partial x}+Q \tag{3}$$

$$P=P(\rho,T) \qquad and \qquad i=c_v T \tag{4}$$

The second state equation from (4) can be used to rewrite the energy equation (3) with respect to the temperature T.

$$\frac{\partial}{\partial t}(\rho c_v T)+\frac{\partial}{\partial x}(\rho c_v T.u)=-P\frac{\partial u}{\partial x}+Q \tag{5}$$

where c_v is the thermal conductivity at constant volume.

The system of equations should be solved with predefined initial and boundary conditions corresponding to the problem that is considered.

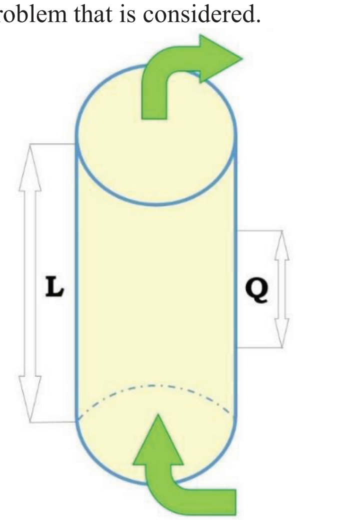

FIGURE 2. Model representation of the PWR core

Here we consider thermal-hydraulic processes in the PWR core. The problem is considered in 1D and the reactor core is presented as a vertical tube with height L and on the walls of the duct heat source Q is given with a temperature T_Q. The coolant is light pressurized water with inlet temperature T_b and going into the reactor core with inlet velocity u_b, pressure p_b and density ρ_b. Heated water leaves the core at the top outlet with a lack of heat flux, $\left.\frac{\partial T}{\partial n}\right|_L=0$. Beside the boundary conditions some initial conditions have to be given depending on the stage of the process at the beginning of simulation. Usually, the initial conditions of the parameters of the problem are taken corresponding to their inlet values.

NUMERICAL PROCEDURES

The system of equations (1), (2) and (5) is nonlinear. So, it has to be solved numerically. For this purpose, the finite volume method [7] is used. The domain under consideration which is the tube that represent of the PWR's core is divided into N finite volumes, presented, for convenience horizontally on Fig.3.

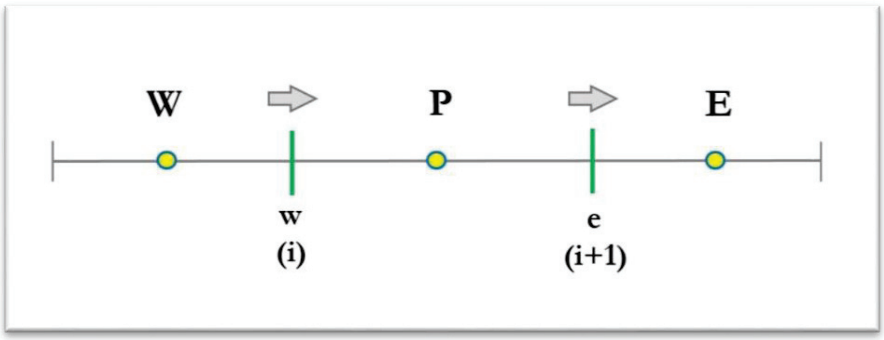

FIGURE 3. A given control volume with its neighbor volumes

Let's P is a middle point in one such control volume. The point in the neighbor volumes are denoted with W and E, correspondingly. The faces surrounding the point P are denoted with w and e correspondingly. It is co called staggered grid where scalars like T, p, ρ are presented in the middle point P of the volume and the vector n on the faces.

The solving process of the equations (1), (2), and (5) is split into two parts. First part concerns equation (5) with respect to the temperature T solved at one time step with predefined velocities u, pressure p, and density ρ. Then at the second part at the same time step are solved the equations (1) and (2) with respect to u, p, ρ. These equations are two and unknown values are three. So, a form of the equation of state (4) should be used. It should connect the temperature of equation (5) with the rest of unknown values. As there is non-theoretical form of first equation of (5) except for case of ideal gas this requires an empirical formula $\rho=\rho(T)$ to be derived. In this study several empirical models have been tested. The simplest one – linear model have been found in the form

$$\rho = 1385.8 - 2.2 * T \tag{6}$$

The data for this model have been taken from [8] for pressure $P = 15.5 MPa$. Fig. 4 shows the agreement between data and the model.

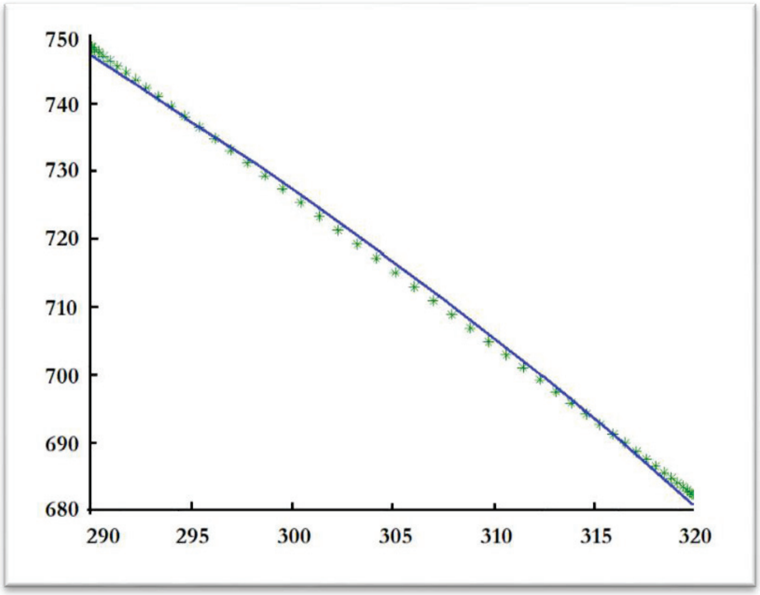

FIGURE 4. Empirical model for the density as a function of temperature

After obtaining the temperature distribution in one time step then transfer to momentum and continuity equations with density distribution following equation (6) again for the same time step. And following these procedures the algorithm proceeds in time.

Here, in the study we applied a slightly different procedure. In [9] the energy equation (5) is solved with reasonably predefined velocity, pressure, and density distribution again using the finite volume method. The solution of the energy equation – the temperature distribution was waited for until it reached its steady-state.

Here, this temperature distribution was taken as an input value to solve steady-state for equations (1) and (2) with respect to velocities and pressure, again using (6) for densities.

Applying the finite volume method, the momentum equation is integrated over a control volume

$$\int_{c_v} \frac{\partial}{\partial x}(\rho u.u)dx = F_e u_e - F_w u_w \tag{7}$$

Here, with F_e and F_w the flow fluxes through volume faces $(\rho u)_e$ and $(\rho u)_w$ are denoted. And using upwind scheme following Fig. 3

$$u_e = \frac{u_{i+1} + u_i}{2} \quad , \quad u_w = \frac{u_i + u_{i-1}}{2} \tag{8}$$

If we denote with F_i the quantities $F_i = \frac{\rho_i + \rho_{i-1}}{2} u_i$ then the fluxes F_e and F_w can be calculated by

$$F_e = (\rho u)_e = \frac{F_{i+1} + F_i}{2} \ , \quad F_w = \frac{F_i + F_{i-1}}{2}$$

Equations (7) and (8) leads to

$$\frac{F_e}{2}u_{i+1} + \frac{F_e - F_w}{2}u_i - \frac{F_w}{2}u_{i-1} \tag{9}$$

Integrating the right hand side of equation (2) gives

$$-\int_{c_v} \frac{\partial p}{\partial x} dx = -p_p + p_w = p_{i-1} - p_i \tag{10}$$

Equations (9) and (10) give the equation $\frac{F_e}{2}u_{i+1} + \frac{F_e - F_w}{2}u_i - \frac{F_w}{2}u_{i-1} = p_{i-1} - p_i$.

This equation can be presented in the form

$$a_i u_i + \sum a_{nb} u_{nb} = p_{i-1} - p_i \tag{11}$$

where u_{nb} present the velocities in the neighbor points of (i).

We use the SIMPLE (Semi-Implicit Method for Pressure Linked Equations) algorithm to solve this equation. It is an iterative method [7].

Let's assume that we know the velocities in an approximation, firstly initial velocities. That means that the coefficients in equation (11) can be computed and the pressures can be taken as known, as well. Then we are able to solve equation (11) for a provisional velocities u^*.

$$a_i u_i^* + \sum a_{nb} u_{nb}^* = p_{i-1}^* - p_i^* \tag{12}$$

Let's define corrections $u_i^{'}$ and $p_i^{'}$ of velocities and pressures to the true values u_i and p_i. Then we will have $u_i = u_i^* + u_i^{'}$ and $p_i = p_i^* + p_i^{'}$. Let's subtract equation (12) from equation (11) which will lead to equation for the corrections

$$a_i u^{'} + \sum a_{nb} u_{nb}^{'} = p_{i-1}^{'} - p_i^{'} \tag{13}$$

And here we make the main approximation of SIMPLE procedure – neglect the term $\sum a_{nb} u_{nb}^{'}$. Thus there is a direct relation between velocity and pressure corrections $a_i u^{'} = p_{i-1}^{'} - p_i^{'}$ or

$$u_i^{'} = \frac{p_{i-1}^{'} - p_i^{'}}{a_i} = d_i\left(p_{i-1}^{'} - p_i^{'}\right) \tag{14}$$

with $d_i = \frac{1}{a_i}$.

Now from the integration of continuity equation over the control volume we have $(\rho u)_e - (\rho u)_w = 0$ or $\rho_{i+1}\left(u_{i+1}^* + d_{i+1}\left(p_i^{'} - p_{i+1}^{'}\right)\right) - \rho_i\left(u_i^* + d_i\left(p_{i-1}^{'} - p_i^{'}\right)\right) = 0$.

Then we will have

$$-\rho_i d_i p_{i-1}^{'} + \left(\rho_{i+1} d_{i+1} + \rho_i d_i\right) p_i^{'} - \rho_{i+1} d_{i+1} p_{i+1}^{'} = -\rho_{i+1} u_{i+1}^* + \rho_i u_i^* \tag{15}$$

which is an equation for pressure corrections.

We can write this equation for any internal point of discretization shown on Fig. 3. For boundary points, the boundary conditions are $u\big|_0 = u_1$ and $\left.\dfrac{\partial u}{\partial n}\right|_L = 0$.

Solving equation (15) for pressure corrections and using equation (14) for velocity corrections we have $u = u^* + u'$ and $p = p^* + p'$ i.e. we have a new approximations for velocity and pressure. Then follows repetition of the procedure – solve equation (12) for u^* and then equation (15) for p'. Thus it proceeds until u' and p' are kept in a tolerance.

Fig. 5 presents the successive iteration of the velocities to the target distribution given whit (*) while Fig. 6 presents iteration of the flow rate (ρu) to its constant value, depending on the boundary velocity u_1.

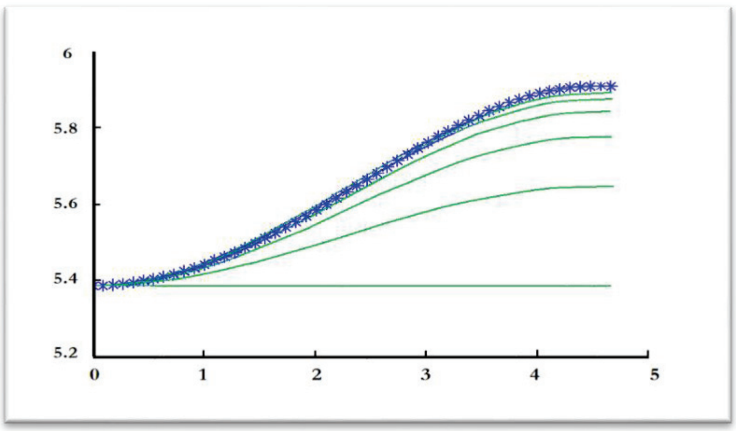

FIGURE 5. Successive iterations of velocities converging to a target values in (*)

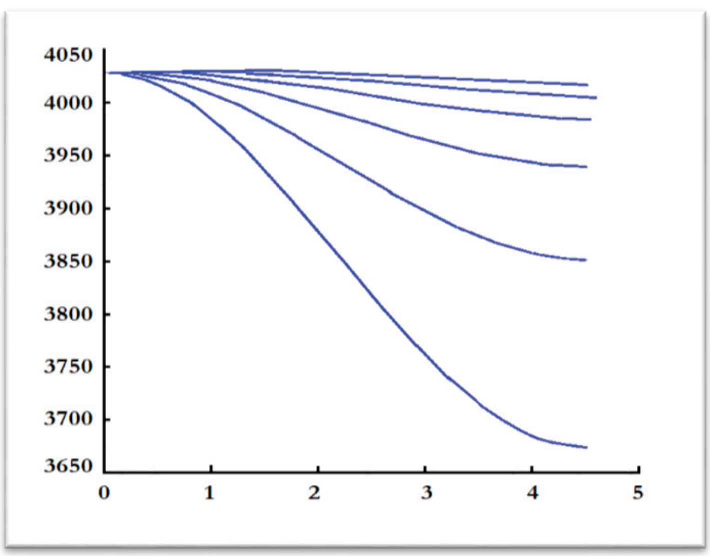

FIGURE 6. Flow rate converging to a constant

CONCLUSIONS AND OUTLOOK

The method presented above was implemented in MATLAB software. The results show that SIMPLE algorithm works for continuity and momentum equations. It allows combining energy equation solution with the continuity and momentum equation solutions' for each time step. It gives the possibility to integrate the computations of processes that take place in the steam generator presented in [10]and the current study. These two models we be connected in a simplified simulator of primary circuit of PWR.

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