Choice of optimal replacement of equipment in the warehouse of the company

Silvia Baeva and Kiril Tomov

Citation: **1789**, 020022 (2016); doi: 10.1063/1.4968443
View online: http://dx.doi.org/10.1063/1.4968443
View Table of Contents: http://aip.scitation.org/toc/apc/1789/1
Published by the American Institute of Physics

Choice of optimal replacement of equipment in the warehouse of the company

Silvia Baeva [1] and Kiril Tomov [2]

[1] *Technical University – Sofia, FAMI, e-mail: sbaeva@tu-sofia.bg*
[2] *Technical University – Sofia, FAMI, e-mail: tomov_kiril@abv.bg*

Abstract. The aim of this paper is to find an optimal replacement of equipment in the warehouse of the company. Real data for the machines are used and are processed with appropriate software. The mathematical model is made for this problem as problem for the shortest path from graph theory. The solution is made by Bellman's principle.

INTRODUCTION

Many technical, research and managerial tasks can be formulated and solved as tasks for analysis and optimization of networks. These include for example the design of highways of the oil and gas pipelines in such a way that the cost of construction of the object to be minimal, the creation of electrical equipment and installations with a minimum length of cable connections, or determining the shortest path between two villages at a set road network. Widespread use first in electrical engineering and construction, network methods and models play an important role in the planning of production, processing and transportation processes. They are a handy tool for analysis and synthesis of structures - technical, manufacturing, information, social and others.[1]÷[7]

The aim of this paper is to find an optimal replacement of equipment in the warehouse of the company. Real data for the machines are used and are processed with appropriate software. The mathematical model is made for this problem as problem for the shortest path from graph theory. The solution is made by Bellman's principle and software lingo.

DESCRIPTION OF THE PROBLEM

In the warehouse purchased new machines for the production. In the company will be introduced new production system after N years and these machines will no longer be needed. Nevertheless, the expenses of operation and maintenance increase over time - due to amortization. It may be more profitable a machine to be replaced with a new first or second or etc. $(N\text{-}1)^{th}$ years. Persons expert mechanical engineers have identified net expenses, related to the purchase of a new machine in particular i^{th} year, $i=1,2,...,N-1$ and replacement through j^{th} year, $j=2,3,...,N$. Let purchased machines are n number for each of these experts has identified net expenses in the tables:

Applications of Mathematics in Engineering and Economics (AMEE'16)
AIP Conf. Proc. 1789, 020022-1–020022-10; doi: 10.1063/1.4968443
Published by AIP Publishing. 978-0-7354-1453-2/$30.00

TABLE 1. Net expenses for each machine, $k=1,2,\ldots,n$

$i \backslash j$	2	3	$\ldots$	N
1	d_{k12}	d_{k13}	$\ldots$	d_{k1N}
2	$\backslash$	d_{k23}	$\ldots$	d_{k2N}
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$N-1$	$\backslash$	$\backslash$	$\ldots$	d_{kN-1N}

The aim of the problem is to determine when each of the machines to be replaced with a new one, so that the total expenses of the first to the N^{th} year for maintenance to be minimal. So the problem can be described through a network of graph theory. Each year compares corresponding vertex and arc length d_{kij} comparing the costs of the machine, $k=1,2,\ldots,n$, $i=1,2,\ldots,N-1$, $j=2,3\ldots,N$. It is necessary to determine the shortest path from 1 to N for each of the machines.

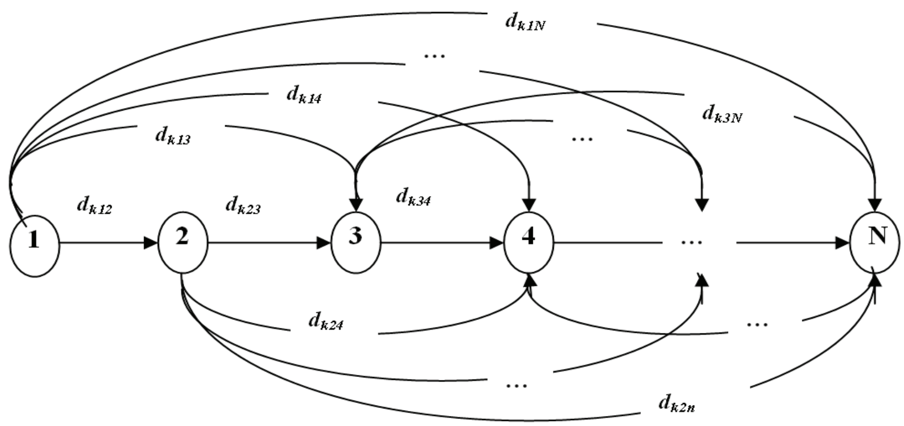


FIGURE 1. The graph describe the expenses yearly for machine k, $k=1,2,\ldots,n$

The principle of Bellman is attached to each machine to find optimal and replacement as criteria is used minimal expenses from the first to the N^{th} year for the maintenance. [8]÷[10]

The principle of Bellman states: each stage is optimal - at every stage of solving the problem of the shortest path leading to optimality. [8]÷[10]

A function of Bellman is introduced with recurrent formula:

$$f_{kj} = \min\{d_{kj} + f_{ki}\}, i < j, (i, j) \in D, k = 1 \div n \quad (1)$$

For each machine $f_{kN} = f_{k\min}, k = 1 \div n, f_{k\min}$ the minimum net expenses. [8]÷[10]

For each machine shortest route is: [8]÷[10]

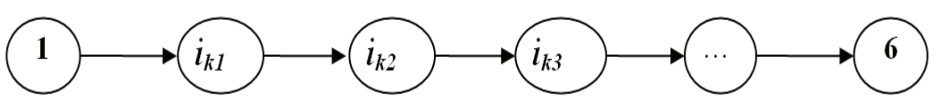


FIGURE 2. The years of replacement - $i_{k1}, i_{k2}, \ldots, k = 1 \div n$

FOR EXAMPLE

Many technical, research and managerial tasks can be formulated and solved as tasks for analysis and The mathematical model is applied in the warehouse of the company for processing of secondary raw materials. The production line consists of the company have *16* machines, for which are given net expenses (bgn) for maintenance from the *1*st to the *7*th year:

Machine 1

Year of replacement / *Year of buying*	*2*	*3*	*4*	*5*	*6*	*7*
1	*220*	*440*	*750*	*970*	*1190*	*3500*
2	-	*220*	*440*	*750*	*970*	*1190*
3	-	-	*220*	*440*	*750*	*970*
4	-	-	-	*220*	*440*	*750*
5	-	-	-	-	*220*	*440*
6	-	-	-	-	-	*220*

Machine 2, Machine 8

Year of replacement / *Year of buying*	*2*	*3*	*4*	*5*	*6*	*7*
1	*60438*	*120876*	*211514*	*273952*	*334390*	*425028*
2	-	*60438*	*120876*	*211514*	*273952*	*334390*
3	-	-	*60438*	*120876*	*211514*	*273952*
4	-	-	-	*60438*	*120876*	*211514*
5	-	-	-	-	*60438*	*120876*
6	-	-	-	-	-	*60438*

Machine 3

Year of replacement / *Year of buying*	*2*	*3*	*4*	*5*	*6*	*7*
1	*260*	*520*	*870*	*1130*	*1390*	*3740*
2	-	*260*	*520*	*870*	*1130*	*1390*
3	-	-	*260*	*520*	*870*	*1130*
4	-	-	-	*260*	*520*	*870*
5	-	-	-	-	*260*	*520*
6	-	-	-	-	-	*260*

Machine 4

Year of buying \ Year of replacement	2	3	4	5	6	7
1	580	1160	2740	3320	3900	5480
2	-	580	1160	2740	3320	3900
3	-	-	580	1160	2740	3320
4	-	-	-	580	1160	2740
5	-	-	-	-	580	1160
6	-	-	-	-	-	580

Machine 5

Year of buying \ Year of replacement	2	3	4	5	6	7
1	2010	4020	6030	8340	10350	12360
2	-	2010	4020	6030	8340	10350
3	-	-	2010	4020	6030	8340
4	-	-	-	2010	4020	6030
5	-	-	-	-	2010	4020
6	-	-	-	-	-	2010

Machine 6, Machine 9

Year of buying \ Year of replacement	2	3	4	5	6	7
1	875	1750	2625	3800	4675	5550
2	-	875	1750	2625	3800	4675
3	-	-	875	1750	2625	3800
4	-	-	-	875	1750	2625
5	-	-	-	-	875	1750
6	-	-	-	-	-	875

Machine 7, Machine 10, Machine 12

Year of buying \ Year of replacement	2	3	4	5	6	7
1	590	1180	1770	2360	2950	3540
2	-	590	1180	1770	2360	2950
3	-	-	590	1180	1770	2360
4	-	-	-	590	1180	1770
5	-	-	-	-	590	1180
6	-	-	-	-	-	590

Machine 11

Year of replacement / *Year of buying*	*2*	*3*	*4*	*5*	*6*	*7*
1	*1200*	*2400*	*3600*	*5100*	*6300*	*7500*
2	-	*1200*	*2400*	*3600*	*5100*	*6300*
3	-	-	*1200*	*2400*	*3600*	*5100*
4	-	-	-	*1200*	*2400*	*3600*
5	-	-	-	-	*1200*	*2400*
6	-	-	-	-	-	*1200*

Machine 13

Year of replacement / *Year of buying*	*2*	*3*	*4*	*5*	*6*	*7*
1	*3255*	*6510*	*9765*	*14420*	*17675*	*20930*
2	-	*3255*	*6510*	*9765*	*14420*	*17675*
3	-	-	*3255*	*6510*	*9765*	*14420*
4	-	-	-	*3255*	*6510*	*9765*
5	-	-	-	-	*3255*	*6510*
6	-	-	-	-	-	*3255*

Machine 14, Machine 16

Year of replacement / *Year of buying*	*2*	*3*	*4*	*5*	*6*	*7*
1	*250*	*500*	*750*	*1000*	*1250*	*1500*
2	-	*250*	*500*	*750*	*1000*	*1250*
3	-	-	*250*	*500*	*750*	*1000*
4	-	-	-	*250*	*500*	*750*
5	-	-	-	-	*250*	*500*
6	-	-	-	-	-	*250*

Machine 15

Year of replacement / *Year of buying*	*2*	*3*	*4*	*5*	*6*	*7*
1	*840*	*1680*	*2520*	*3360*	*4200*	*5040*
2	-	*840*	*1680*	*2520*	*3360*	*4200*
3	-	-	*840*	*1680*	*2520*	*3360*
4	-	-	-	*840*	*1680*	*2520*
5	-	-	-	-	*840*	*1680*
6	-	-	-	-	-	*840*

TABLE 2. Net expenses (bgn) for each machine

So entered data were processed with software lingo: [1]

Machine 1

LINGO - [LINGO Model - M1]

File Edit LINGO Window Help

```
Model:
       Sets:
       V/1..7/:path;
       Arcs(V,V)/1,2 1,3 1,4 1,5 1,6 1,7 2,3 2,4 2,5 2,6 2,7 3,4 3,5 3,6 3,7 4,5 4,6 4,7 5,6 5,7 6,7/:dist;
       Endsets
       Max=path(7)-path(1);
       @For(Arcs(i,j):path(j)<path(i)+dist(i,j));
       Data:
         dist=220,440,750,970,1190,3500,220,440,750,970,1190,220,440,750,970,220,440,750,220,440,220;
       Enddata
End
```

Machine 2, Machine 8

LINGO - [LINGO Model - M2,M8]

File Edit LINGO Window Help

```
Model:
       Sets:
       V/1..7/:path;
       Arcs(V,V)/1,2 1,3 1,4 1,5 1,6 1,7 2,3 2,4 2,5 2,6 2,7 3,4 3,5 3,6 3,7 4,5 4,6 4,7 5,6 5,7 6,7/:dist;
       Endsets
       Max=path(7)-path(1);
       @For(Arcs(i,j):path(j)<path(i)+dist(i,j));
       Data:
         dist=60438,120876,211514,273952,334390,425028,60438,120876,211514,273952,334390,60438,120876,
211514,273952,60438,120876,211514,60438,120876,60438;
       Enddata
End
```

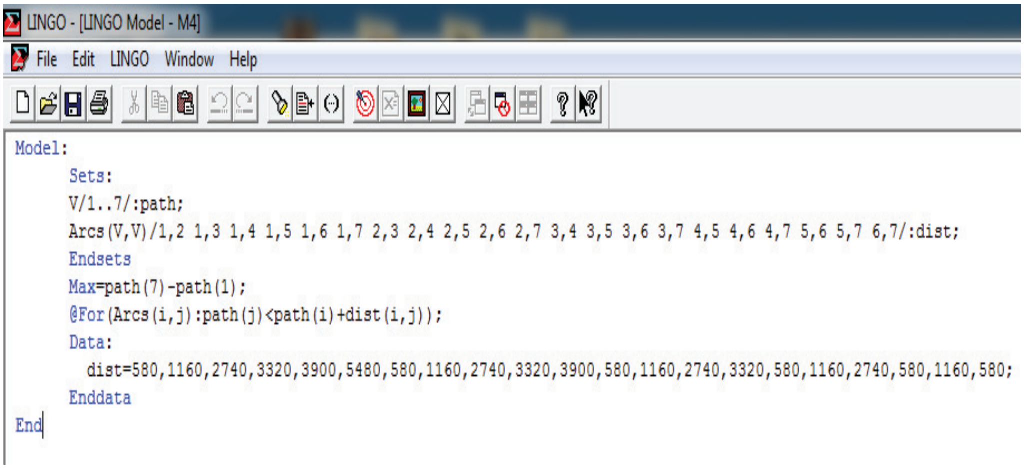
Machine 3

LINGO - [LINGO Model - M3]

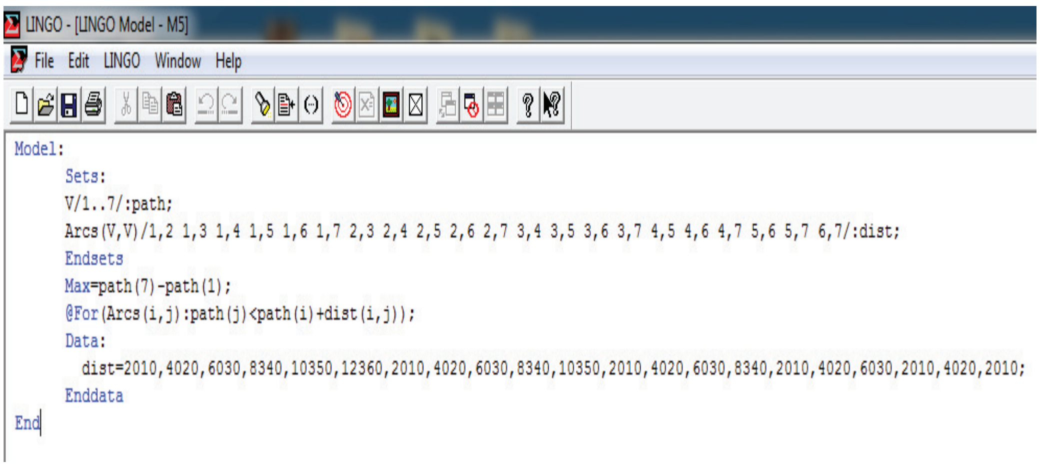
File Edit LINGO Window Help

```
Model:
       Sets:
       V/1..7/:path;
       Arcs(V,V)/1,2 1,3 1,4 1,5 1,6 1,7 2,3 2,4 2,5 2,6 2,7 3,4 3,5 3,6 3,7 4,5 4,6 4,7 5,6 5,7 6,7/:dist;
       Endsets
       Max=path(7)-path(1);
       @For(Arcs(i,j):path(j)<path(i)+dist(i,j));
       Data:
         dist=260,520,870,1130,1390,3740,260,520,870,1130,1390,260,520,870,1130,260,520,870,260,520,260;
       Enddata
End
```

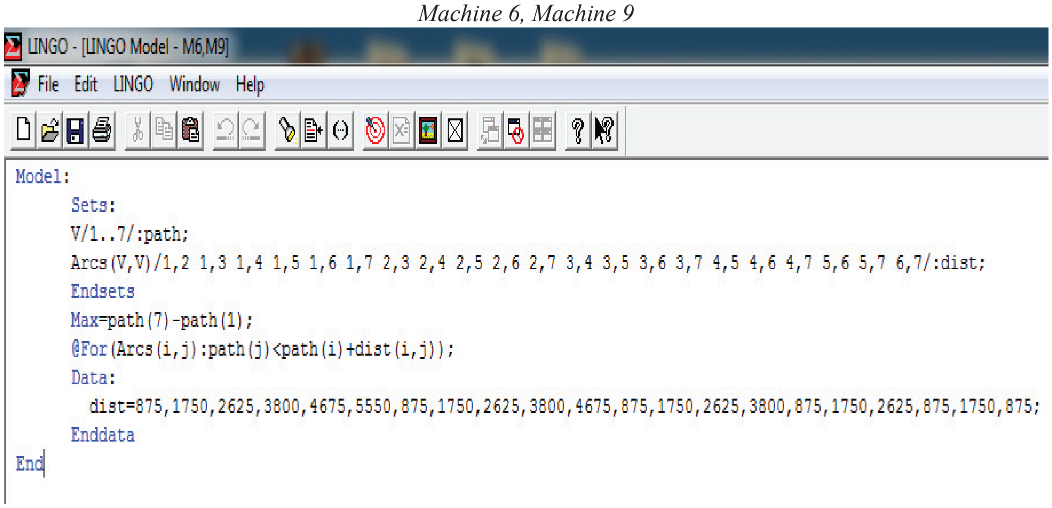
Machine 4



Machine 5



Machine 6, Machine 9



Machine 7, Machine 10, Machine 12

```
LINGO - [LINGO Model - M7,M10,M12]
File  Edit  LINGO  Window  Help

Model:
      Sets:
      V/1..7/:path;
      Arcs(V,V)/1,2 1,3 1,4 1,5 1,6 1,7 2,3 2,4 2,5 2,6 2,7 3,4 3,5 3,6 3,7 4,5 4,6 4,7 5,6 5,7 6,7/:dist;
      Endsets
      Max=path(7)-path(1);
      @For(Arcs(i,j):path(j)<path(i)+dist(i,j));
      Data:
        dist=590,1180,1770,2360,2950,3540,590,1180,1770,2360,2950,590,1180,1770,2360,590,1180,1770,590,1180,590;
      Enddata
End
```

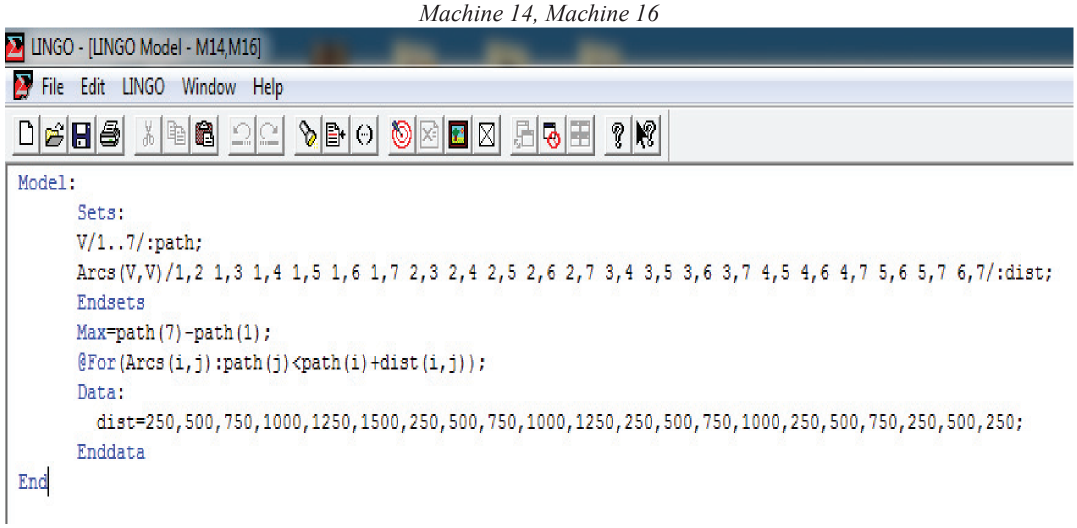
Machine 11

```
LINGO - [LINGO Model - M11]
File  Edit  LINGO  Window  Help

Model:
      Sets:
      V/1..7/:path;
      Arcs(V,V)/1,2 1,3 1,4 1,5 1,6 1,7 2,3 2,4 2,5 2,6 2,7 3,4 3,5 3,6 3,7 4,5 4,6 4,7 5,6 5,7 6,7/:dist;
      Endsets
      Max=path(7)-path(1);
      @For(Arcs(i,j):path(j)<path(i)+dist(i,j));
      Data:
        dist=1200,2400,3600,5100,6300,7500,1200,2400,3600,5100,6300,1200,2400,3600,5100,1200,2400,3600,1200,2400,1200;
      Enddata
End
```

Machine 13

```
LINGO - [LINGO Model - M13]
File  Edit  LINGO  Window  Help

Model:
      Sets:
      V/1..7/:path;
      Arcs(V,V)/1,2 1,3 1,4 1,5 1,6 1,7 2,3 2,4 2,5 2,6 2,7 3,4 3,5 3,6 3,7 4,5 4,6 4,7 5,6 5,7 6,7/:dist;
      Endsets
      Max=path(7)-path(1);
      @For(Arcs(i,j):path(j)<path(i)+dist(i,j));
      Data:
        dist=3255,6510,9765,14420,17675,20930,3255,6510,9765,14420,17675,3255,6510,9765,14420,3255,6510,9765,3255,6510,3255;
      Enddata
End
```

Machine 14, Machine 16



Machine 15

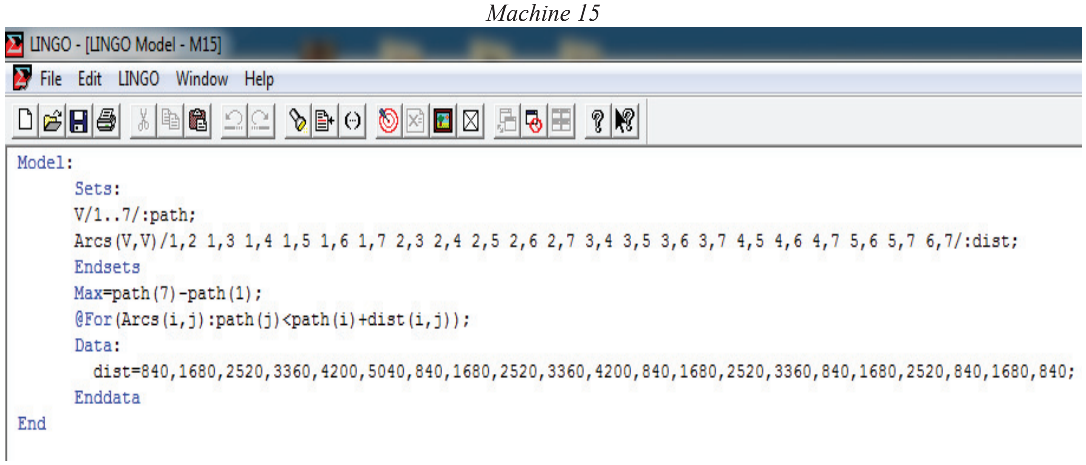


FIGURE 3. Solution of the problem in lingo

Results

Machine	*Min expenses*	*Years of replacement*	*Machine*	*Min expenses*	*Years of replacement*
M1	*1320*	*1→3→5→7*	*M9*	*5250*	*1→4→7*
M2	*362628*	*1→3→5→7*	*M10*	*3540*	*1→7*
M3	*1560*	*1→3→5→7*	*M11*	*7200*	*1→4→7*
M4	*3480*	*1→3→5→7*	*M12*	*3540*	*1→7*
M5	*12060*	*1→4→7*	*M13*	*19530*	*1→4→7*
M6	*5250*	*1→4→7*	*M14*	*1500*	*1→7*
M7	*3540*	*1→7*	*M15*	*5040*	*1→7*
M8	*362628*	*1→3→5→7*	*M16*	*1500*	*1→7*

TABLE 3. The optimal replacement of equipment

The overall minimum expenses are 799 566 (bgn).

CONCLUSION

The mathematical model can be applied for the replacement of various types of equipment in companies from different sectors of the economy. The problem could be described by a model of linear optimization, but the specific structure makes reasonable use of special networking methods and algorithms to solve - due to the large number of constraints and variables in them.

REFERENCES

1. B. Cheshankov*, Operation Research, part 1*, Sofia, Bulgaria, 2004.
2. B. Gartner, J. Matonsek, *Understanding and Using Linear Programing*, Berlin, Germany, 2006.
3. G. Gatev, *Operations Research*, Sofia, Bulgaria, 2003.
4. H. Karloff, *Linear Programing*, Atlanta, USA, 2008.
5. H. A. Taha, *Operations Research, An Introduction*, New York, London 1982.
6. M. Fiedler, J. Nedoma, J. Ramik, J. Rohn, K. Zimmermann, *Linear Optimization Problem with Inexact Data*, Berlin, Germany, 2006.
7. S. I. Gass, *Linear Programing. Methods and Applications*, New York, USA, 2003.
8. http://academy.telerik.com/algoacademy/season-2012-2013/algorythms-and-graphs
9. https://en.wikipedia.org/wiki/Graph_theory
10. http://e-maxx.ru/algo/ford_bellman