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Cite as: AIP Conference Proceedings **2172**, 080015 (2019); <https://doi.org/10.1063/1.5133573>
Published Online: 13 November 2019

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Stochastic Approach to Control of Storage Stocks in Commercial Warehouses

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Abstract. Stochastic programming is an approach for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problems are formulated with known parameters, real world problems almost invariably include parameters which are unknown at the time a decision should be made. When the parameters are uncertain, but assumed to lie in some given set of possible values, one might seek a solution that is feasible for all possible parameter choices and optimizes a given objective function. Such an approach is applied to mathematical model of optimization problem for storage stocks in commercial warehouses. Here probability distributions (e.g., of demand) could be estimated from data that have been collected over time. The aim is to find some policy that is feasible for all (or almost all) the possible parameter realizations and optimizes the expectation of some function of the decisions and the random variables.

INTRODUCTION

In present-day global economics, logistics plays a key role in facilitating trade. Also, by extension, ensuring the success of business operations. Logistics managers have seen increasing challenges to create and keep efficient and effective logistics and supply chain methods.

In [12] is discuss five of the biggest logistics challenges faced on a daily basis.

- Customer Service: Logistics management is all about providing the right product in the right quantity to the right place at the right time. Customers want full transparency into where their delivery is at all times. In this day and age, the location of a customer's shipment is as interconnected as your social network.
- Transportation Cost Control: One of the highest costs contributing to the 'cutting transportation cost' concern is fuel prices. Higher fuel prices are likely to increase transportation costs by pushing up fuel surcharges.
- Planning and Risk Management: In order to stay as efficient and effective as possible, periodic assessments and redesigns of each business sector are necessary. These adjustments are put in place in response to changes in the market, such as new product launches, global sourcing, credit availability and the protection of intellectual property. Managers must identify and quantify these risks in order to control and moderate them.
- Supplier/Partner Relationships: It is important to create, understand and follow mutually agreed upon standards to better understand not only current performance but also opportunities for improvement. Thus, having two different methods for measuring and communicating performance and results in time and effort wasted.
- Government and Environmental Regulations: Carriers face significant compliance regulations imposed by state and local authorities. As well as state regulations, environmental issues such as the anti-idling and other emission reduction regulations brought about by state and local governments have created concern that the compliance costs could exceed their benefits.

In this sense, researches have been carried out and various optimization problems related to the logistics network, the vehicles used to transport the goods and products, their storage in warehouses, etc. have been solved.

In [6] a multiple period replenishment problem based on (s, S) policy is investigated for a supply chain (SC) comprising one retailer and one manufacturer with uncertain demand. Novel mixed-integer linear programming (MILP) models are developed for centralized and decentralized decision-making modes using two-stage stochastic programming. To compare these decision-making modes, a Monte Carlo simulation is applied to the optimization models' policies. To deal with demand uncertainty, scenarios are generated using Latin Hypercube Sampling method and their number is reduced by a scenario reduction technique. In large test problems, where CPLEX solver is not able to reach an optimal solution in the centralized model, evolutionary strategies (ES) and imperialist competitive algorithm (ICA) are applied to find near optimal solutions. Sensitivity analysis is conducted to show the performance of the proposed mathematical models. Moreover, it is demonstrated that both ES and ICA provide acceptable solutions compared to the exact solutions of the MILP model. Finally, the main parameters affecting difference between profits of centralized and decentralized SCs are investigated using the simulation method.

The authors at [11] are resolved the problems of deciding the optimal warehouse location for multiple markets and determining warehouse configuration design against stochastic demands. An appropriate inventory policy with owned and rented warehouses for deteriorating items is further designed. The proposed model maximizes the profit under rent warehouse incentives decreasing over time and price-sensitive demands. Furthermore, there is proposed a solution algorithm to solve the problem effectively. Sensitivity analysis is conducted to examine the effects of the parameters for the model of the algorithm.

In [5] is proposed an extended relocation model for warehouses configuration in a supply chain network, in which uncertainty is associated to operational costs, production capacity and demands whereas, existing researches in this area are often restricted to deterministic environments. In real cases, usually is deal with stochastic parameters and this point justifies why the relocation model under uncertainty should be evaluated. Albeit the random parameters can be replaced by their expectations for solving the problem, but sometimes, some methodologies such as two-stage stochastic programming works more capable. Thus, in this paper, for implementation of two stage stochastic approach, the sample average approximation (SAA) technique is integrated with the Bender's decomposition approach to improve the proposed model results. Moreover, this approach leads to approximate the fitted objective function of the problem comparison with the real stochastic problem especially for numerous scenarios. The proposed approach has been evaluated by some hypothetical numerical examples and the results show that the proposed approach can find better strategic solution in an uncertain environment comparing to the mean-value procedure (MVP) during the time horizon.

The aim of [4] is to provide a comprehensive review of studies in the fields of SCND and reverse logistics network design under uncertainty. The authors are organized in two main parts to investigate the basic features of these studies. In the first part, planning decisions, network structure, paradigms and aspects related to SCM are discussed. In the second part, existing optimization techniques for dealing with uncertainty such as recourse-based stochastic programming, risk-averse stochastic programming, robust optimization, and fuzzy mathematical programming are explored in terms of mathematical modeling and solution approaches. Finally, the drawbacks and missing aspects of the related literature are highlighted and a list of potential issues for future research directions is recommended.

In [10] a commercial problem of enterprise logistics nature called multi-product warehouse sizing problem is formulated and solved. The optimal warehouse inventory level and the order points are determined by minimizing the total inventory ordering and holding costs for a specific time period. The problem is formulated as an appropriate mathematical model of NLP nature and solved using appropriate numerical optimization techniques (successive quadratic programming procedures). The entire mathematical background for formulating the problem and finding the optimal solution to it is presented and appropriately addressed. The model is also illustrated with the help of a numerical case study where the sensitivity of the model parameters involved is given through appropriate figures.

The authors in [9] are proposed a stochastic programming model and solution algorithm for solving supply chain network design problems of a realistic scale. Existing approaches for these problems are either restricted to deterministic environments or can only address a modest number of scenarios for the uncertain problem parameters. The solution methodology integrates a strategy, the Sample Average Approximation scheme, with an accelerated Benders decomposition algorithm to quickly compute high quality solutions to large-scale stochastic supply chain design problems with a huge (potentially infinite) number of scenarios. A computational study involving two real supply chain networks are presented to highlight the significance of the stochastic model as well as the efficiency of the proposed solution strategy.

Various logistics optimization problems and approaches to solving them can also be found in [2], [3], [7], [8] and etc.

Stochastic programming is an approach for modeling optimization problems that involve uncertainty. Whereas deterministic optimization problems are formulated with known parameters, real world problems almost invariably

include parameters which are unknown at the time a decision should be made. When the parameters are uncertain, but assumed to lie in some given set of possible values, one might seek a solution that is feasible for all possible parameter choices and optimizes a given objective function. Such an approach is applied to mathematical model of optimization problem for storage stocks in commercial warehouses. Following some basic ideas of stochastic optimization as in [1], in this paper probability distributions (e.g., of demand) could be estimated from data that have been collected over time. The aim is to find some policy that is feasible for all (or almost all) the possible parameter realizations and optimizes the expectation of some function of the decisions and the random variables.

The research carried out is specific, as problems arise from the increasing use of logistics of products and goods and their storage. On the other hand, the storage of a product or product is a cost to the warehouse manager.

The significance of the research done in the article is that the benefit is maximized for warehouse managers who need to make a decision (related to the storage of goods and products) in an uncertain environment. Decision makers would like to evaluate the risks before deciding to understand the scope of the possible outcomes and the significance of the undesirable effects.

DESCRIPTION OF THE PROBLEM FOR A PRODUCT

Suppose that a company has to decide an order quantity x of a certain product to satisfy demand d . The cost of ordering is $c > 0$ per unit. If the demand d is bigger than x , then a back order penalty of $b \geq 0$ per unit is incurred. The cost of this is equal to $b(d - x)$ if $d > x$, and is zero otherwise. On the other hand if $d < x$, then a holding cost of $h(x - d) \geq 0$ is incurred. The total cost is then

$$G(x, d) = cx + b[d - x]_+ + h[x - d]_+, \quad (1)$$

where $[a]_+$ denotes the maximum of a and 0. We assume that $b > c$, i.e., the back order cost is bigger than the ordering cost. We will treat x and d as continuous (real valued) variables rather than integers. This will simplify the presentation and makes sense in various situations.

The objective is to minimize the total cost $G(x, d)$. Here x is the decision variable and the demand d is a parameter. Therefore, if the demand is known, the corresponding optimization problem can be formulated in the form

$$\min_{x \geq 0} G(x, d). \quad (2)$$

The nonnegativity constraint $x \geq 0$ can be removed if a back order policy is allowed. The objective function $G(x, d)$ can be rewritten as

$$G(x, d) = \max\{(c - b)x + bd, (c + h)x - hd\}, \quad (3)$$

which is piecewise linear with a minimum attained at $\bar{x} = d$. That is, if the demand d is known, then (no surprises) the best decision is to order exactly the demand quantity d .

Consider now the case when the ordering decision should be made *before* a realization of the demand becomes known. One possible way to proceed in such situation is to view the demand D as a *random variable* (denoted here by capital D in order to emphasize that it is now viewed as a random variable and to distinguish it from its particular realization d). We assume, further, that the probability distribution of D is *known*. This makes sense in situations where the ordering procedure repeats itself and the distribution of D can be estimated, say, from historical data.

Let D be a discrete random variable, evenly distributed in the range $[0, M]$, with the distribution law given in Table 1:

TABLE 1. Law of distribution of the random variable D

D	d_1	d_2	...	d_n
p	p_1	p_2	...	p_n

Then the mathematical expectation of D is:

$$E(D) = \sum_{i=1}^n d_i \cdot p_i, \quad (4)$$

and the distribution function of D is:

$$G(D) = \begin{cases} 0, & 0 < d < d_1 \\ p_1, & d_1 \leq d < d_2 \\ p_1 + p_2, & d_2 \leq d < d_3 \\ \dots & \dots \\ \sum_{i=1}^n p_i = 1, & d \geq d_n \end{cases} \quad (5)$$

The aim of this problem is to find such x quantities to minimize the overall cost of storing the product.

Then it makes sense to talk about the expected value, denoted $\mathbb{E}[G(x, D)]$, of the total cost and to write the corresponding optimization problem

$$\min_{x \geq 0} \mathbb{E}[G(x, D)]. \quad (6)$$

The above formulation approaches the problem by optimizing (minimizing) the total cost *on average*. What would be a possible justification of such approach? If the process repeats itself then, by the Law of Large Numbers, for a given (fixed) x , the average of the total cost, over many repetitions, will converge with probability one to the expectation $\mathbb{E}[G(x, D)]$. Indeed, in that case a solution of problem (6) will be optimal on average.

The above problem gives a simple example of a *recourse action*. At the first stage, before a realization of the demand D is known, one has to make a decision about ordering quantity x . At the second stage after demand D becomes known, it may happen that $d > x$. In that case the company can meet demand by taking the recourse action of ordering the required quantity $d - x$ at a penalty cost of $b > c$.

The next question is how to solve the optimization problem (6). In the present case problem (6) can be solved in a closed form. Consider the cumulative distribution function (cdf) $F(z) := \text{Prob}(D \leq z)$ of the random variable D . Note that $F(z) = 0$ for any $z < 0$. This is because the demand cannot be negative. It is possible to show that

$$\mathbb{E}[G(x, D)] = b\mathbb{E}[D] + (c - b)x + (b + h) \int_0^x F(z) dz, \quad (7)$$

i.e.

$$\mathbb{E}[G(x, D)] = \begin{cases} b\mathbb{E}[D] + (c - b)x, & 0 < x < d_1 \\ b\mathbb{E}[D] + (c - b)x + (b + h) \cdot p_1 \cdot \int_{d_1}^x dz, & d_1 \leq x < d_2 \\ b\mathbb{E}[D] + (c - b)x + (b + h) \cdot \left(p_1 \int_{d_1}^{d_2} dz + (p_1 + p_2) \int_{d_2}^x dz \right), & d_2 \leq x < d_3 \\ \dots \dots \dots \\ b\mathbb{E}[D] + (c - b)x + (b + h) \cdot \left(p_1 \int_{d_1}^{d_2} dz + (p_1 + p_2) \int_{d_2}^{d_3} dz + \dots + \sum_{i=1}^n p_i \cdot \int_{d_n}^x dz \right), & x \geq d_n \end{cases} \quad (8)$$

Therefore, by taking the derivative, with respect to x , of the right hand side of (7) and equating it to zero we obtain that optimal solutions of problem (6) are defined by the equation $(b + h)F(x) + c - b = 0$, and hence an optimal solution of problem (6) is given by the quantile

$$\bar{x} = F^{-1}(k), \quad (9)$$

where $k := \frac{b-c}{b+h}$.

DESCRIPTION OF THE PROBLEM FOR MANY PRODUCTS

For a multi-product task, the play is as follows:

Consider N of products P_1, P_2, \dots, P_N . For each product $P_j, j = 1, \dots, N$, the set-up described above is valid.

The aim of the problem is to find the quantities $x_j, j = 1, \dots, N$, of product $P_j, j = 1, \dots, N$, for which

$$\min_{x_j \geq 0} \sum_{j=1}^N \mathbb{E}[G_j(x_j, D_j)], \quad (10)$$

where:

D_j is discrete random variable, evenly distributed in the range $[0, M_j]$, with the distribution law given in Table 2:

TABLE 2. Law of distribution of the random variable D_j

D_j	d_{j1}	d_{j2}	...	d_{jn}
p_j	p_{j1}	p_{j2}	...	p_{jn}

Then the mathematical expectation of D_j is:

$$\mathbb{E}(D_j) = \sum_{j=1}^N \sum_{i=1}^n d_{ji} \cdot p_{ji}, \quad (11)$$

and the distribution function of D_j is:

$$G(D_j) = \begin{cases} 0, & 0 < d_j < d_{j1} \\ p_{j1}, & d_{j1} \leq d_j < d_{j2} \\ p_{j1} + p_{j2}, & d_{j2} \leq d_j < d_{j3} \\ \dots \dots \dots \\ \sum_{i=1}^n \sum_{l=1}^n p_{ji} = 1, & d_j \geq d_{jn} \end{cases} \quad (12)$$

and

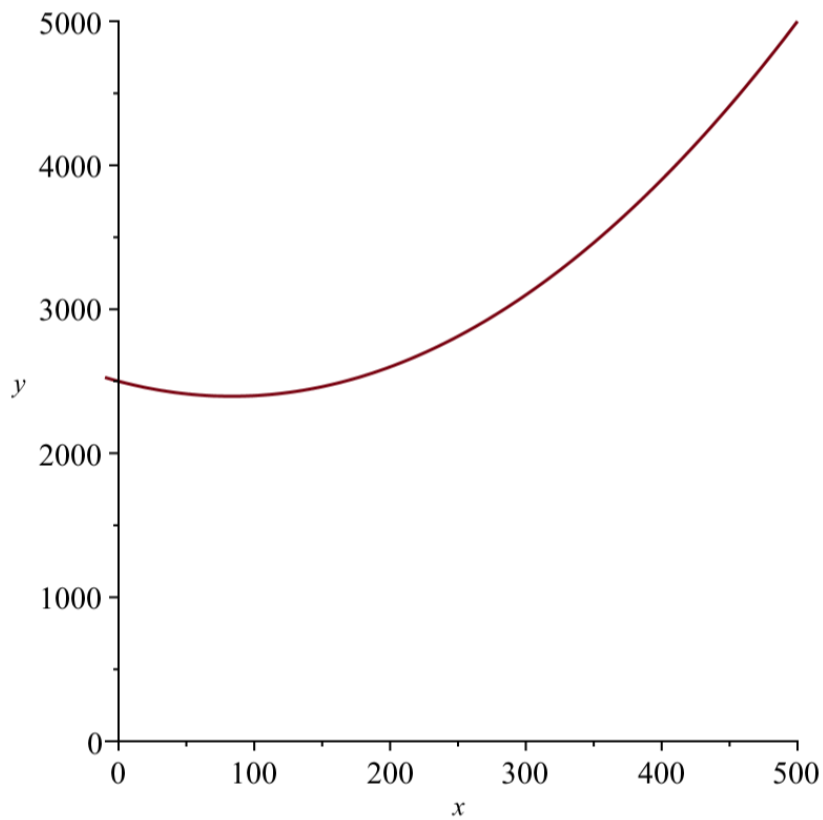


FIGURE 1 Continuous case, where the axis x is demanded quantity (kg), the axis y is storage costs of first product (BGN)

Discrete case in two, three, four and five points:

TABLE 4. Law of distribution of the random variable D_1 in two, three, four and five points

D_1	100	400			
p_1	$\frac{1}{2}$	$\frac{1}{2}$			
D_1	100	250	400		
p_1	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$		
D_1	50	200	350	500	
p_1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	
D_1	50	150	250	350	450
p_1	$\frac{10}{50}$	$\frac{5}{50}$	$\frac{20}{50}$	$\frac{5}{50}$	$\frac{10}{50}$

Continuous and discrete case is shown in figure 2:

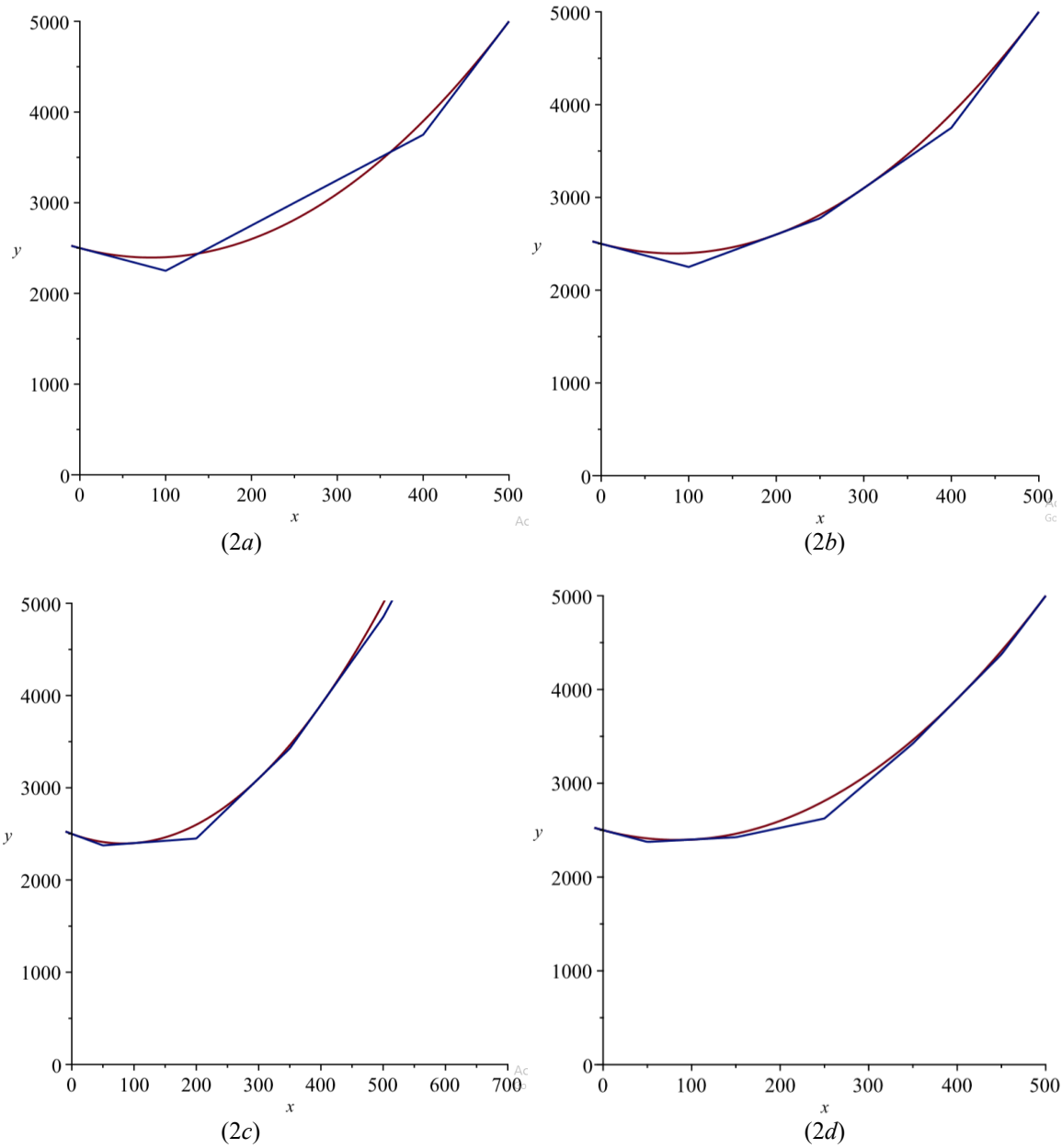


FIGURE 2 Continuous and discrete case of two (2a), three (2b), four (2c) and three (2d) points, where the axis x is demanded quantity (kg), the axis y is storage costs of first product (BGN)

Second product: Donkey milk

Continuous case is shown in figure 3: $x \in [0; 150]$

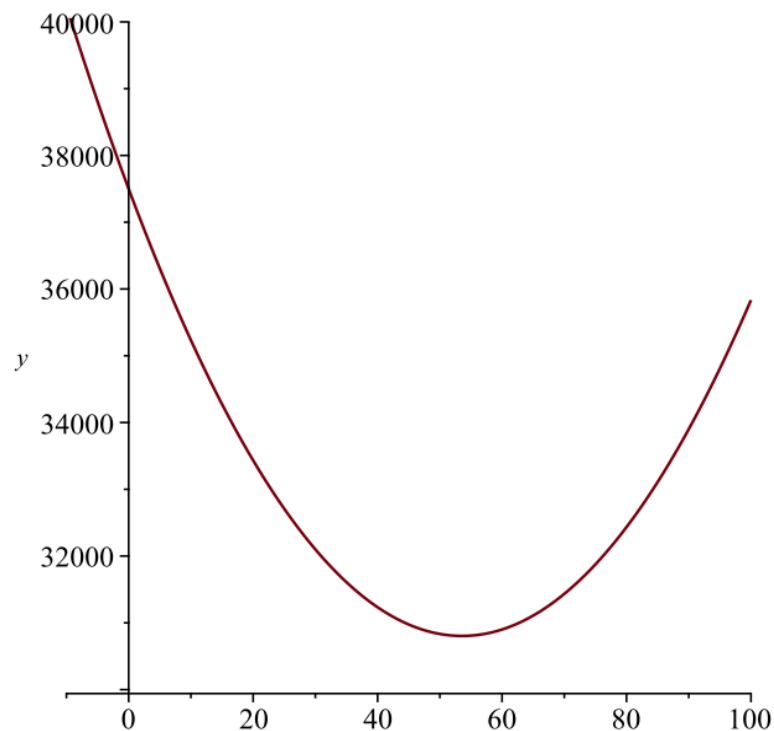


FIGURE 3 Continuous case, where the axis x is demanded quantity (kg), the axis y is storage costs of second product (BGN)

Discrete case in two, three, four and five points:

TABLE 5. Law of distribution of the random variable D_2 in two, three, four and five points

D_2	25	125			
p_2	$\frac{1}{2}$	$\frac{1}{2}$			
D_2	25	75	125		
p_2	$\frac{17}{50}$	$\frac{16}{50}$	$\frac{17}{50}$		
D_2	20	60	100	140	
p_2	$\frac{11}{40}$	$\frac{9}{40}$	$\frac{13}{40}$	$\frac{7}{40}$	
D_2	20	50	80	110	140
p_2	$\frac{8}{30}$	$\frac{4}{30}$	$\frac{8}{30}$	$\frac{4}{30}$	$\frac{6}{30}$

Continuous and discrete case is shown in figure 4:

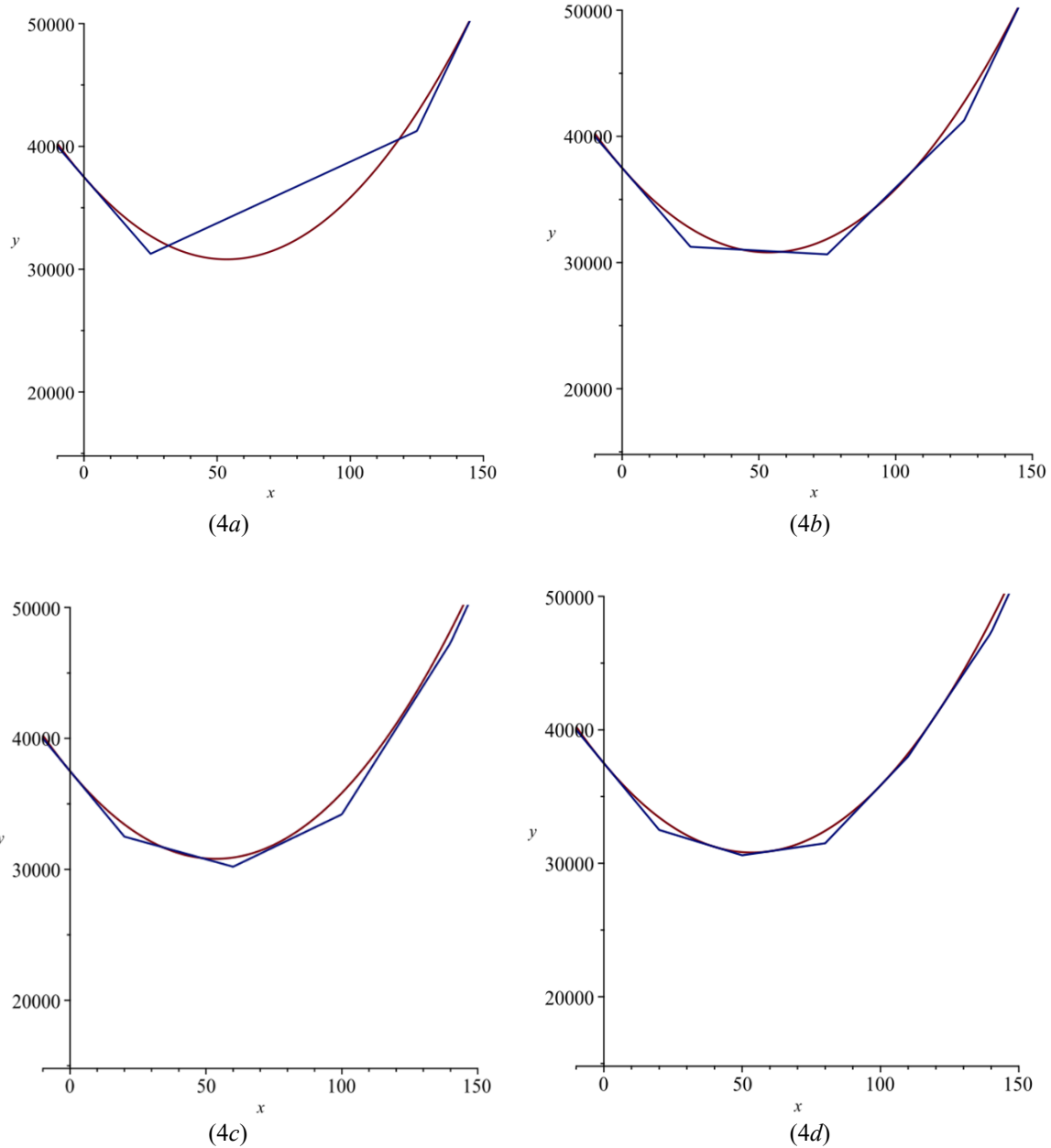


FIGURE 4 Continuous and discrete case of two (4a), three (4b), four (4c) and three (4d) points, where the axis x is demanded quantity (kg), the axis y is storage costs of second product (BGN)

Third product: Butter

Continuous case is shown in figure 5: $x \in [0; 200]$

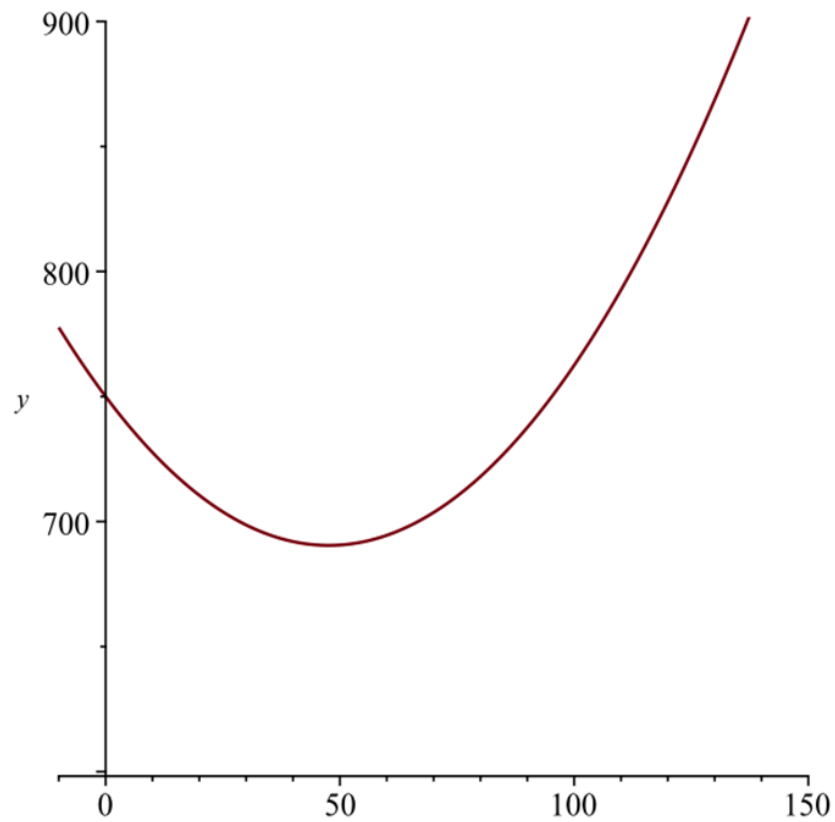


FIGURE 5 Continuous case, where the axis x is demanded quantity (kg), the axis y is storage costs of third product (BGN)

Discrete case in two, three, four and five points:

TABLE 6. Law of distribution of the random variable D_3 in two, three, four and five points

D_3	50	150			
p_3	$\frac{1}{2}$	$\frac{1}{2}$			
D_3	40	100	160		
p_3	$\frac{24}{60}$	$\frac{11}{60}$	$\frac{25}{60}$		
D_3	30	80	130	180	
p_3	$\frac{15}{50}$	$\frac{10}{50}$	$\frac{14}{50}$	$\frac{11}{50}$	
D_3	20	60	100	140	180
p_3	$\frac{8}{40}$	$\frac{8}{40}$	$\frac{7}{40}$	$\frac{10}{40}$	$\frac{7}{40}$

Continuous and discrete case is shown in figure 6:

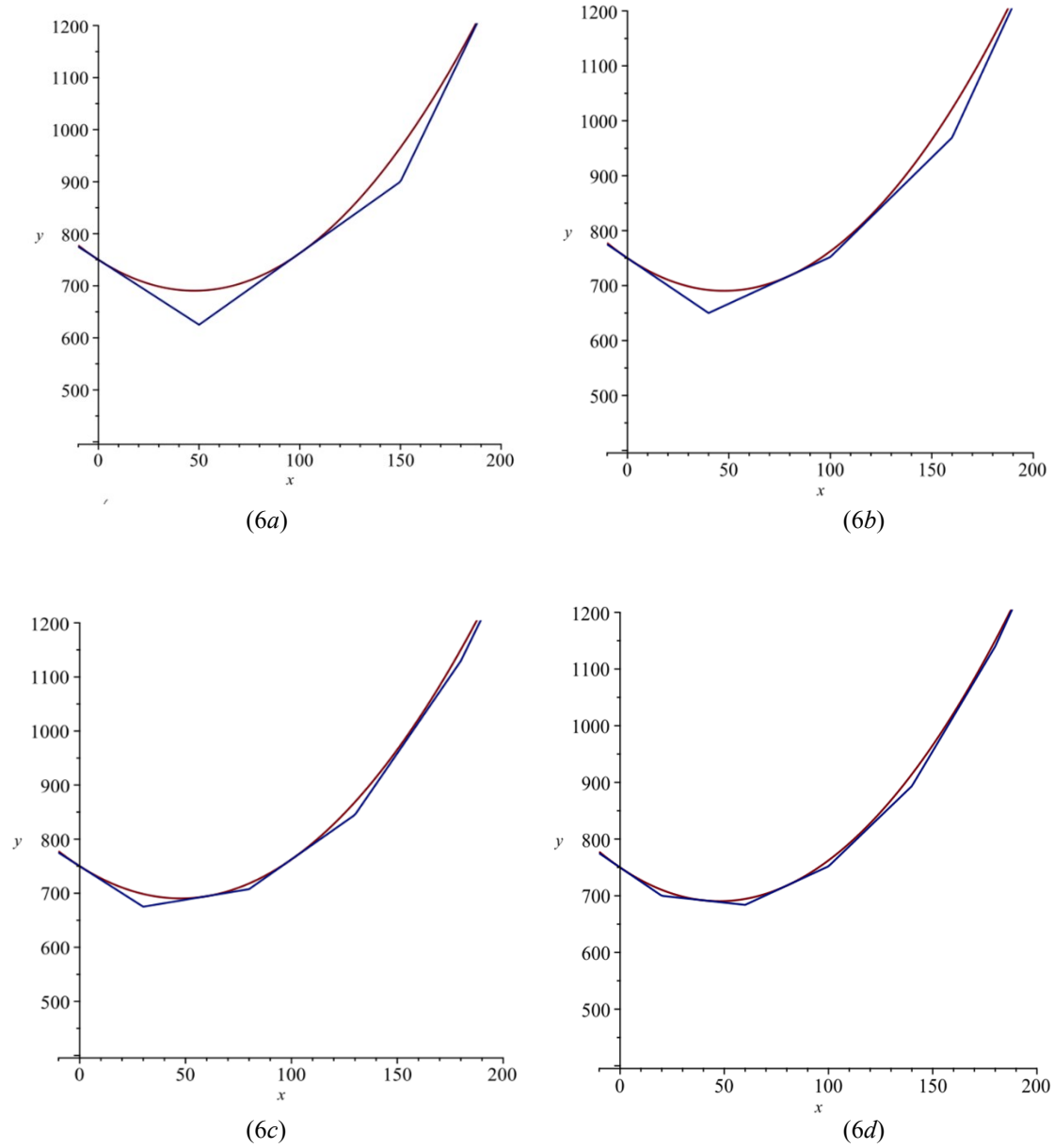


FIGURE 6 Continuous and discrete case of two (6a), three (6b), four (6c) and three (6d) points, where the axis x is demanded quantity (kg), the axis y is storage costs of third product (BGN)

The numerical realization is implemented in Python, Maple, MatLab software environments.

Numerical results

The numerical realization' results of the problem are presented in a table 7.

TABLE 7. Numerical results

Cost (BGN) in case Product	Continuous case	Discreet case with two points	Discreet case with tree points	Discreet case with four points	Discreet case with five points
Cheese	$\mathbb{E}(83, D_1) = 2\,396.835$	$\mathbb{E}(100, D_1) = 2\,250$	$\mathbb{E}(100, D_1) = 2\,250$	$\mathbb{E}(100, D_1) = 2\,400$	$\mathbb{E}(100, D_1) = 2\,400$
Donkey milk	$\mathbb{E}(54, D_2) = 30\,804$	$\mathbb{E}(25, D_2) = 31\,250$	$\mathbb{E}(58, D_2) = 30\,854$	$\mathbb{E}(49, D_2) = 30\,832.5$	$\mathbb{E}(60, D_2) = 30\,900$
Butter	$\mathbb{E}(48, D_3) = 690.48$	$\mathbb{E}(50, D_3) = 625$	$\mathbb{E}(80, D_3) = 718$	$\mathbb{E}(60, D_3) = 694$	$\mathbb{E}(40, D_3) = 692$
$\sum_{j=1}^3 \mathbb{E}(x_j, D_j) =$	33 891.315	34 125	33 822	33 927	33 992

Discussion of results

In the continuous case, the best solution is obtained, but it is practically inapplicable and requires discretion. A sampling of each of the three products is made at two, tree, four and five points. From the above graphs (figures 2, 4, 6) it is seen that with the increase of the number of discrete points the deviation decreases.

In conclusion, it can be said that increasing the number of discrete points results in a better approximation, which leads to an increase in the accuracy of the desired end results, but the discrete points in a closed interval must be the final count.

CONCLUSION AND FUTURE WORKS

The stochastic approach used to solve the problem of stockpiling can be applied in other cases. For example, building an investment portfolio for maximum return. Here probability distributions of the returns on the financial instruments being considered are assumed to be known, but in the absence of data from future periods, these distributions will have to be elicited from some accompanying model, which in its simplest form might derive solely from the prior beliefs of the decision maker. Another complication in this setting is the choice of objective function: maximizing expected return becomes less justifiable when the decision is to be made once only, and the decision-maker's attitude to risk then becomes important.

Future research will be focus on appling and studing stochastic programming' models two-stage (linear) problems. The interest stems from the fact that the decision maker is often required takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision and in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision.

REFERENCES

1. Alexander Shapiro, Andy Philpott, *A Tutorial on Stochastic Programming*, e-paper, March 2007
2. E. Çevikcan, İ. U. Sarı, C. Kahraman, *Multi-objective Assessment of Warehouse Storage Policies in Logistics and a Fuzzy Information Axiom Approach*, Springer, London, e-book, October 2013.
3. John J. Bartholdi, Steven T. Hackman, *Warehouse & Distribution Science*, Atlanta, GA 30332-0205 USA, January 2011.
4. Kannan Govindan, Mohammad Fattahi, Esmail Keyvanshokooh, *European Journal of Operational Research*, Elsevier, Volume 263, Issue 1, Pages 108-141, November 2017.
5. M. Bashiri, H. R. Rezaei, *Reconfiguration of Supply Chain: A Two Stage Stochastic Programming*, Internattiionall Journall off Industriiall Engineeeriing & Producttiion Research, Volume 24, Number 1, pp. 47-58, March 2013.

6. Mohammad Fattahi, Esmacil Keyvanshokoo, Masoud Mahootchi, *Investigating replenishment policies for centralised and decentralised supply chains using stochastic programming approach*, [Article in International Journal of Production Research](http://dx.doi.org/10.1080/00207543.2014.922710), <http://dx.doi.org/10.1080/00207543.2014.922710>, June 2014.
7. Pawel Zajac, *Evaluation Method of Energy Consumption in Logistic Warehouse Systems*, Springer International Publishing, Switzerland, e-book, 2015.
8. Timm Gudehus, Herbert Kotzab, *Comprehensive Logistics*, Springer – Verlag Berlin Heidelberg, Second Edition, 2012.
9. Tjendera Santoso, Shabbir Ahmed, Marc Goetschalckx, Alexander Shapiro, *A stochastic programming approach for supply chain network design under uncertainty*, School of Industrial & Systems Engineering, Georgia Institute of Technology, 765 Ferst Drive, Atlanta, GA 30332, 2003.
10. V. V. Petinis, C D Tarantilis, C. T. Kiranoudis, *Warehouse sizing and inventory scheduling for multiple stock-keeping products*, [International Journal of Systems Science](#), Volume 36, Issue 1, 2005.
11. Yu-SiangLin, Yu-SiangLin, *A two-stage stochastic optimization model for warehouse configuration and inventory policy of deteriorating items*, Computers & Industrial Engineering, Elsevier, Volume 120, Pages 83-93, June 2018.
12. <https://www.plslogistics.com/blog/5-challenges-that-logistics-managers-face-every-day/>