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Multistage self-orientating gauge measuring system best fitting parameters for circularity estimation

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Abstract. The deviation of circularity estimation of large-scaled details in many cases is a serious metrological problem, which can be solved using a measuring system based on a multistage self-orientating gauge. The purpose of this paper is to study the influence of the gauge parameters on the accuracy of measurement of the circularity deviation and optimization thereof in order to ensure the maximum correspondence between the real and the measured profile. The accuracy of the re-creation of individual and group of harmonic components of the profile is examined. Analytical and experimental research results are provided.

1. Introduction

The deviation of circularity measurement of large-scaled details is a serious metrological problem, determined by the specificity of the measured objects - their huge dimensions and mass, and the associated difficulties of a design and exploitative nature.

This problem can be solved successfully by means of a measuring system based on the effect of realization of the median circle of the cross-section profile, where the measuring device is established on a multi-stage self-orientating gauge [2, 4].

The principle scheme of such a system is shown in Figure 1 [4]. It contains the following basic functional elements: a base roller gauge 1 based on the measured detail 2 and a measuring clamp 3. The measuring clamp is mounted on the measured detail 2 on the multi-stages self-orientating gauge. A measuring head 4 is mounted on the clamp. With relative rotation of the clamp to the workpiece due to the aforesaid effect, each point of the housing describes a circle equivalent to the average approximating circle of the profile on which the prism is based. This circle is at the same time the datum of measurement.

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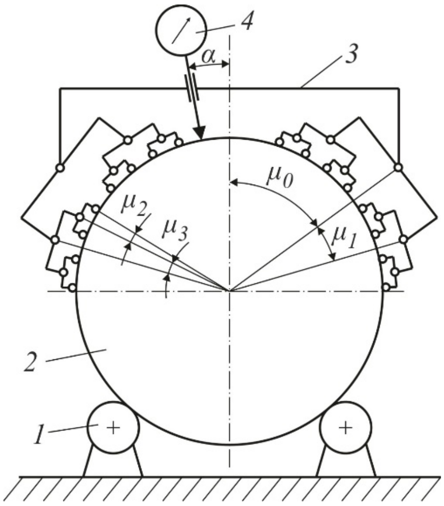

Figure 1. Principal diagram of the system

The number of contact points depends on the number of the gauge grades t /on the Figure t = 3/ and is defined by the expression $k = 2^{t+1}$. Angle α is the angle between the measuring line and the symmetry of the clamp, μ_0 - the angle between the symmetry of the yoke and the symmetry of the first degree, and μ_1, μ_2 and μ_3 - the angles between the symmetries of the respective self-orientating degrees.

The subject of this paper is the optimization of the basic parameters of the system - the angles μ0, μ_i and α, aiming increasing the accuracy of the realization of the datum in determining the shape of the measured profiles of the details.

2. Mathematical model of recreation of the measured profile when measured by means of a multistage self-orientating gauge

The mathematical model allows clarification of the kinematics of the process of transferring the shape of the measured from the gauge profile L of the workpiece onto the trajectory l of the p. M of the body of the gauge as well as on the obtained L_p profile as a realization of the distance between the p. M and p. K of the measured profile L (Figure 2).

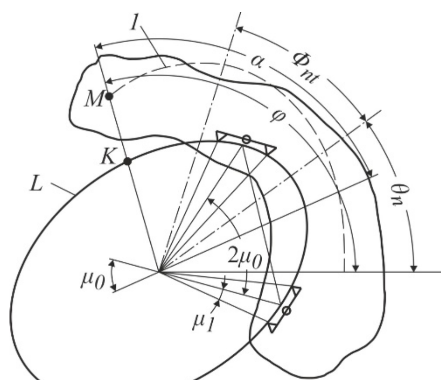

Figure 2. Multistage self-orientating gauge measuring system's body point trajectory

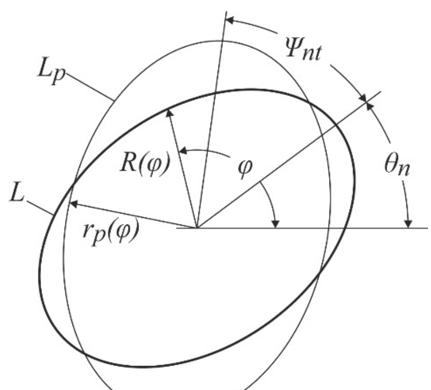

Figure 3. Multistage self-orientating gauge measuring system's body point trajectory

Measured profile L in polar coordinates can be described by Fourier order:

$$\left| \begin{array}{l} R(\varphi) = R_o + \sum_{n=2}^{\infty} \Delta R_n(\varphi) \\ \Delta R_n(\varphi) = a_n \cos n(\varphi - \theta_n) \end{array} \right. , \tag{1}$$

where:

$R(\varphi)$ - radius-vector of the measured profile L; R_o - constant; a_n and θ_n – amplitude and initial phase of the nth harmonic; φ - polar coordinate.

Generally, when using a t-stage gauge, the equation of the trajectory l at point M in polar coordinates is the following [1]:

$$\left| \begin{array}{l} r_l(\varphi) = r_{lo} + \sum_{n=2}^{\infty} \Delta r_{ln}(\varphi) \\ \Delta r_{ln}(\varphi) = \lambda_{nt} . a_n \cos n(\varphi - \theta_n - \Phi_{nt}) \end{array} \right. , \tag{2}$$

where:

$r_l(\varphi)$ – radius-vector of trajectory l of the p. M; r_{lo} - constant; λ_{nt} - copying coefficient of the amplitude of the n-th harmonic of the measured profile ($\lambda_{nt}>0$) determined by the expression:

$$\lambda_{nt} = \prod_{i=1}^{t} \frac{\cos n\mu_i}{\cos \mu_i} \sqrt{\frac{\cos^2 n\mu_0}{\cos^2 \mu_0} \cos^2 \alpha + \frac{\sin^2 n\mu_0}{\sin^2 \mu_0} \sin^2 \alpha} , \tag{3}$$

Φ_{nt} – angle of phase shifting of the n-th harmonic from the measured profile when it is reproduced on the trajectory l at point M, i.e., the angle between the radius vectors $R_{n\ max}$ and $r_{ln\ max}$:

$$\Phi_{nt} = \begin{cases} \alpha - q_n, & if\ \prod_{i=1}^{t} \cos n\mu_i \geq 0 \\ \frac{\pi}{n} + \alpha - q_n, & if\ \prod_{i=1}^{t} \cos n\mu_i < 0 \end{cases} , \tag{4}$$

where q_n is determined by: $\mathrm{tg}\, nq_n = \mathrm{tg}\alpha . \mathrm{ctg}\mu_o . \mathrm{tg} n\mu_o$,

by following the signs rules:

$$\left| \begin{array}{l} \mathrm{sgn}(\sin nq_n) = \mathrm{sgn}(\sin \alpha \sin n\mu_0) \\ \mathrm{sgn}(\cos nq_n) = \mathrm{sgn}(\cos \alpha \cos n\mu_0) \end{array} \right. .$$

The radius-vector $r_p(\varphi)$ of profile L_p is [2]:

$$\left| \begin{array}{l} r_p(\varphi) = r_{p0} + \sum_{n=2}^{\infty} \Delta r_{pn}(\varphi) \\ \Delta r_{pn}(\varphi) = a_{pn} . \cos n(\varphi - \theta_n + \psi_{nt}) \end{array} \right. , \tag{5}$$

where: r_{po} - constant; a_{pn} – amplitude of the profile of the n-th harmonic; $a_{pn} = K_{nt}.a_n$; K_{nt}– coefficient of re-creation of the amplitude of the n-th harmonic ($K_{nt} \geq 0$);

$$K_{nt} = \sqrt{1 - 2\lambda_{nt}\cos n\Phi_{nt} + \lambda^2_{nt}} \tag{6}$$

ψ_{nt} – phase shifting difference of the n-th harmonics of L and L_p

$$\mathrm{tg}n\psi_{nt} = \frac{\lambda_{nt}\sin n\Phi_{nt}}{1 - \lambda_{nt}\cos n\Phi_{nt}},$$

according to the signs rules:

$$\left| \begin{array}{l} \mathrm{sgn}(\sin n\psi_{nt}) = \mathrm{sgn}(\sin n\Phi_{nt}) \\ \mathrm{sgn}(\cos n\psi_{nt}) = \mathrm{sgn}(1 - \lambda_{nt}\cos n\Phi_{nt}) \end{array} \right.$$

The measured profile will be re-created on the profile without distortion and phase shifting at K_{nt} = 1 и ψ_{nt} = 0.

3. Research methodology

The study aims to minimize the function $|K_{nt}-1|$, reflecting the accuracy of the recreation of the measured profile on the profile diagram for a separately collected harmonics and for a group of harmonics. In the second case, the maximum value of $|K_{nt} - 1|$ of the harmonics group must be minimal.

For this purpose, the programs have been developed on the basis of the software "Maple V". It varies with the following parameters: t = 1÷4, α = 0÷90°, μ_0=30°÷80°, μ_1 = 15°÷45°, μ_2 = 7°÷25°, μ_3 = 4°÷16°, μ_4 = 3°÷8°. These limits are in accordance with the possibilities for constructive realization of the gauge.

The studied harmonics (from 2nd to 7th) are dominant in the real technological processes.

4. Study results

Research for a particular harmonic

The analytical examination shows that the necessary and sufficient condition for complete and accurate reflection of the measured profile L, respectively for equal amplitudes and absence of phase shift of the profile Lp, is:

$$\Delta r_{pn}(\varphi) = \Delta R_n(\varphi)$$

or

$$a_{pn}.\cos n(\varphi - \theta_n + \psi_{nt}) = a_n \cos n(\varphi - \theta_n)$$

To be true, the above identity must:

$$\left| \begin{array}{l} a_{pn} = a_n \\ \cos n(\varphi - \theta_n + \psi_{nt}) = \cos n(\varphi - \theta_n) \end{array} \right.$$

$a_{pn} = a_n$, when the coefficient of recreation K_{nt} is equal to 1, i.e. when

$$\left| \begin{array}{l} \lambda_{nt} = 2\cos n\Phi_{nt} \\ \lambda_{nt} = 0 \end{array} \right.$$

Hence, in order to reproduce fully and accurately /without distortion/ n-th harmonic, it is necessary and sufficient in the gauge to have an angle μ_i of any i-th stage that satisfies the condition:

$$n\mu_i = (2k+1).\pi/2 \quad \text{or}$$

$$\left| \begin{array}{lll} n\mu_o = (2k+1).\pi/2 & \text{at} & \alpha = 0, \pi \\ n\mu_o = k.\pi & \text{at} & \alpha = \pm\pi/2 \end{array} \right.$$

$$k = 0, 1, 2, \ldots$$

<u>Harmonic Group Study</u>

The results of the numerical experiment are represented in Table 1, where $\Delta = \left|\frac{K_{nt} - 1}{1}\right|$ is the relative error of recreation of the measured profile. It follows that Δ decreases as the number of degrees t increases, with t = 3 and t = 4 not exceeding 2.4%. The larger value of Δ at t = 4 compared to Δ at t = 3 is explained by the additional limitations of constructive character / possibility of constructive realization of the gauge/.

Table 1. Parameters of t-stage optimized at K_{nt}, gauge for n=2÷7 .

t	μ_i, α [°]	n	K_{nt}	Δ, [%]	ψ_{nt} [°]
1	μ_0 = 56, μ_1 = 16, α = 78	2	0.968	3.2	30
		3	1.035	3.5	6
		4	0.873	12.7	6
		5	1.111	11.1	2
		6	0.939	6.1	0
		7	0.883	11.7	-2
2	μ_0 = 57, μ_1 = 19, μ_2 = 13, α = 82	2	1.041	4.1	24
		3	1.040	4.0	3
		4	0.958	4.2	2
		5	0.967	3.3	-1
		6	0.960	4.0	0
		7	1.008	0.8	0
3	μ_0 = 77, μ_1 = 30, μ_2 = 18, μ_3 = 10, α = 89	2	0.983	1.7	6
		3	1.000	0.0	0
		4	1.022	2.2	-2
		5	1.000	0.0	0
		6	0.995	0.5	2
		7	1.002	0.2	0
4	μ_0 = 77, μ_1 = 30, μ_2 = 18, μ_3 = 8, μ_4 = 3, α = 89	2	0.983	1.7	6
		3	1.000	0.0	0
		4	1.024	2.4	-2
		5	1.000	0.0	0
		6	1.000	0.0	3
		7	1.003	0.3	0

Table 2. Parameters of t-stage optimized at K_{nt}, gauge for n=2÷7 at α= 0.

t	μ_i, [°]	n	K_{nt}	Δ, [%]	ψ_{nt} [°]
1	μ_0 = 36, μ_1 = 18	2	0.675	32.5	0
		3	1.236	23.6	0
		4	1.325	32.5	0
		5	1.000	0.0	0
		6	0.675	32.5	0
		7	0.764	23.6	0
2	μ_0 = 41, μ_1 = 24, μ_2 = 15	2	0.879	12.1	0
		3	1.179	17.9	0
		4	0.925	7.5	0
		5	0.824	17.6	0
		6	1.000	0.0	0
		7	0.889	11.1	0
3	μ_0 = 48, μ_1 = 26, μ_2 = 14, μ_3 = 5	2	1.096	9.6	0
		3	1.208	20.8	0
		4	0.786	21.4	0
		5	0.829	17.1	0
		6	1.044	4.4	0
		7	0.821	17.9	0
4	μ_0 = 49, μ_1 = 26, μ_2 = 14, μ_3 = 4, μ_4 = 3	2	1.131	13.1	0
		3	1.219	21.9	0
		4	0.785	21.5	0
		5	0.852	14.8	0
		6	1.059	5.9	0
		7	0.808	19.2	0

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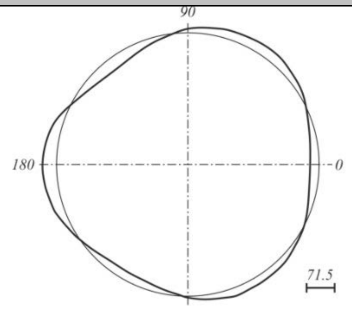
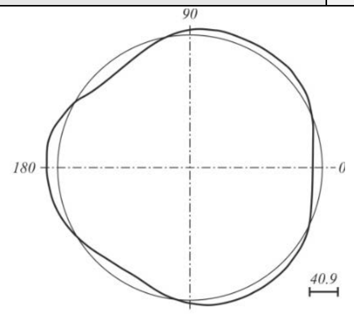
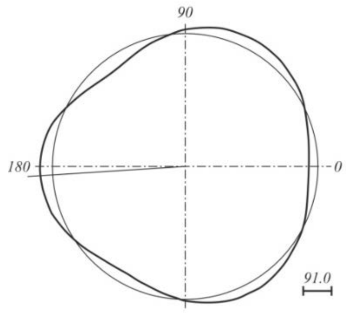
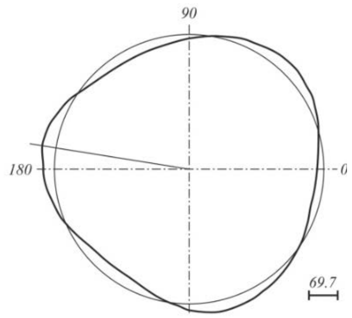
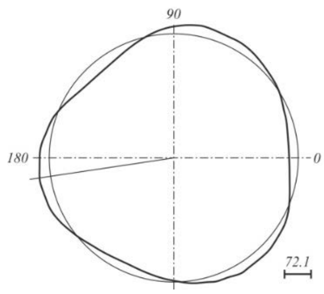
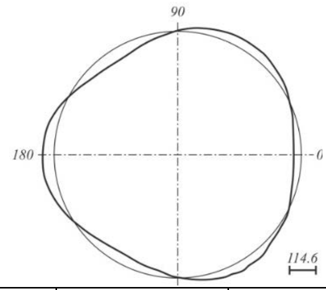
As can be seen from Table 2, the relative error at fixed angle α=0 is several times greater. This case, however, is appropriate not only from a constructive point of view, but also because the phase shifting of the individual harmonics is zero. This allows a clearer reflection of the character of the profile.

Table 3. Experimental results of multistage self-orientating gauge measuring system (**MSSOGMS**)

CMMS-2			**MSSOGMS**		α = 40˚
					
EFK_a = 71.5			$K_{ntт}$ = 0.546 K_{nte} = 0.572 Δ_K = 4.8 %	$\psi_{ntт}$ = 0˚ ψ_{nte} = 0˚ Δ_ψ = 0˚	EFK_{meas} = 40.9 EFK = 74.9
MSSOGMS		α = 0˚	**MSSOGMS**		α = 60˚
					
$K_{ntт}$ = 1.296 K_{nte} = 1.273 Δ_K = 1.8 %	$\psi_{ntт}$ = 0˚ ψ_{nte} = -1˚ Δ_ψ = -1˚	EFK_{meas} = 91.0 EFK = 70.2	$K_{ntт}$ = 1.003 K_{nte} = 0.975 Δ_K = 2.8 %	$\psi_{ntт}$ = 11˚ ψ_{nte} = 10˚ Δ_ψ = 1˚	EFK_{meas} = 69.7 EFK = 69.5
MSSOGMS		α = 20˚	**MSSOGMS**		α = 90˚
					
$K_{ntт}$ = 1.018 K_{nte} = 1.008 Δ_K = 1.0 %	$\psi_{ntт}$ = -6˚ ψ_{nte} = -7˚ Δ_ψ = -1˚	EFK_{meas} = 72.1 EFK = 70.8	$K_{ntт}$ = 1.611 K_{nte} = 1.603 Δ_K = 0.5 %	$\psi_{ntт}$ = 0˚ ψ_{nte} = 0˚ Δ_ψ = 0˚	EFK_{meas} = 114.6 EFK = 71.1

The factors mentioned (permissible measurement error, phase harmonics shifting, possibility of constructive realization) are decisive in choosing one or another schematic solution.

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Experimental studies have been carried out with a multi-stage self-orientating measuring gauge system [6] with parameters: $\mu_0=40^o$, $\mu_1=20^o$, $\mu_2=10^o$ and $\mu_3=5^o$.

Table 3 shows the results (profiles and deviations from circularity) from the measurement with MSSOGMS at different values of α and those from the measurement of the measured profile with CMMS-2 roundness measurement system. The theoretical $K_{nt\tau}$ and experimentally predetermined K_{nte} coefficients of profile recreation and the corresponding shifting angles $\psi_{nt\tau}$ and ψ_{nte} are presented. The correspondence between the theoretical and experimental results is assessed by the difference $\Delta_\psi = \psi_{nt\tau} - \psi_{nte}$ and the relative Δ_K difference.

$$\Delta_K = (K_{nt\tau} - K_{nte})/ K_{nt\tau}$$

$$K_{nte} = EFK_{meas} /EFK_a ,$$

where EFK_a is the deviation from circularity at calibration, but EFK_{meas} is the deviation from circularity measured by MSSOGMS /without correction/.

It can be seen from the table that the relative difference between the $K_{nt\tau}$ and K_{nte} values in the worst case $\alpha = 40^\circ$ is 4.8% and in the other cases is in the range of 0.4 ... 2.8%.

These results confirm the adequacy of the mathematical model. This allows, with a pronounced harmonic of the profile and given parameters of the clamp, the error of the inaccurate recreation of the profile to be corrected by dividing the yield value of the deviation from circularity EFK_{meas} with the recreation coefficient K_{nt}, i.e.

$$EFK = EFK_{meas} / K_{nt} ,$$

where EFK is the value of measured deviation from circularity with correction.

5. Conclusion

The conducted study allows to be made the following conclusions:

- With a pronounced character of the profile (dominant harmonic), a combination of parameters of the measuring system can be selected, ensuring complete and accurate profile recreation.
- For the harmonics group (from 2-nd to 7-th) studied, the maximum error of profile recreation at the optimum combination of the measuring system parameters is 12.7 percent for single-stage , 4.1% for two-stage, 2.2% for three-stage and 2.4% for four-stage self-orientating gauge.
- With a known profile nature, with a given harmonic, it is possible to correlate the measurement result by dividing it with the corresponding K_{nt} recreation factor.

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