

# Supercapacitor Sizing for Power Defined Loads

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**Abstract**— The paper presents an analytical model for supercapacitor loading when used in a power defined load scenario. The needed load power is defined as a combination of constant power and linearly increasing or decreasing segments. The analytical expression for the supercapacitor voltage and current under this scenario are presented along with some simulations and experimental results to verify the presented formulas.

**Keywords**— energy storage, supercapacitor, high efficiency, synchronous dc-dc converter, power conversion.

## I. INTRODUCTION

Hybrid energy storage is a very popular method for powering loads that require both very high peak power bursts and large energy stored as no single power source exists that can handle both jobs. Applications are very diverse converting renewable energy sources, electric vehicles or trains [2], [3]. Although the presented results in the paper are general, the application considered is electric vehicles.

From a designer perspective a propulsion system for an electric vehicle and the energy storage can be a very daunting task. The designer must model the required propulsion power from the mechanical perspective, convert it to electrical requirements for the electric drive system and then calculated the required energy storage for a specified driving range. But, from an electrical engineer perspective an estimation of the required propulsion power is enough to model the system and choose the required converters and size the energy storage elements. Simulation of the system is the preferred approach usually [4]. However, if the power profile is adequately approximated an analytical solution can be found and this is highly appreciated as it allows for optimization without significant computational resources. This paper presents such an analytical approach, by modeling the supercapacitor as a combination of elementary cells with the simplest model for each given in [5], while abstracting the load by its power requirements over time. This data is easily obtained or calculated for the vehicle, from its speed-versus-time profile [6].

Using this approach formulas are obtained for two cases with practical significant – linearly increasing/decreasing power demand (typical for vehicle acceleration mode/braking) and constant power. The obtained model is verified by means of simulation and experimental results.

The paper is structured as follows: In the next section shows the block diagram of the system that includes the supercapacitor as the main energy storage element. The section summarizes the equations for the supercapacitor voltage and current for the two practical cases of constant and linearly changing load power. The obtained formulas are than verified with simulation results in section III.

Section IV presents experimental results and a comparison is made in section V. Section VI concludes the paper.

## II. ANALYZED SYSTEM

Figure 1 presents the analyzed system of an energy storage incorporating a supercapacitor. The system is charged through a constant current source. The equations for the system, assuming equal capacitors and neglecting the ESR are presented first in [1], but a more generalized version, allowing for any type of linearly changing load is used here.

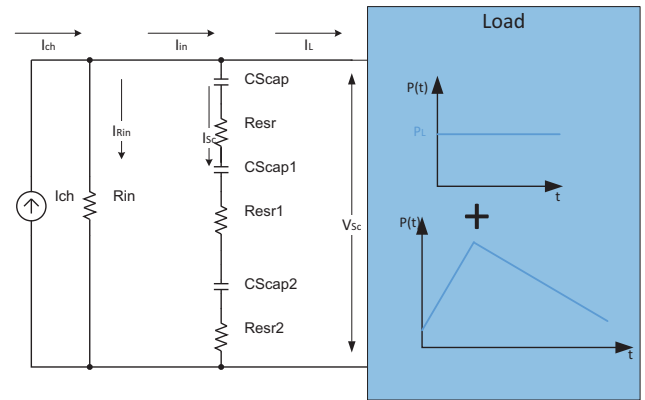


Figure 1. Analyzed System

These equations can be solved for two practical cases presented in figure 2 –constant power and linearly increasing or decreasing load power.

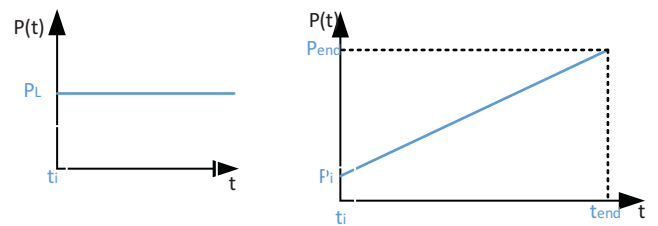


Figure 2. Considered Power profile cases

When the load power is constant it can be represented by the equation:

$$P_L(t) = P_L \quad (1)$$

It is assumed that the time starts at  $t_i$ . It is assumed that the initial voltage on the supercapacitor is  $U_{init}$ .

TABLE I OBTAINED SOLUTIONS FOR THE SUPERCAPACITOR PARAMETERS

Case Par	Constant Power	Linearly Increasing/Decreasing Power
Capacitor Current	$I_{ch} - \frac{P_L}{U_{init}} \frac{\sqrt{C U_{init}^2}}{\sqrt{C U_{init}^2 - 2(t - t_i) P_L}}$	$I_{ch} - \frac{\sqrt{C}(kt + P_i)}{\sqrt{\frac{C}{I_{L0}^2} (kt_{end} + P_i)^2 + k(t_{end}^2 - t^2) + 2P_i(t_{end} - t)}}$
Capacitor Voltage	$I_{ch} t - U_{init} \sqrt{1 - \frac{2(t - t_i) P_L}{C U_{init}^2}}$	$I_{ch} t - i \frac{\sqrt{\frac{C U_{send}^2}{(P_i + kt_{end})^2} (k^2 t_{end}^2 + 2kP_i t_{end} - P_i^2) - (t - t_{end})(P_i(t + t_{end}) + 2P_i)}}{\sqrt{C}}$

When the power is linearly increasing or decreasing its equation can be written as follows:

$$P_L(t) = P_i + \frac{P_{end} - P_i}{t_{end} - t_i} t = P_i + kt \quad (2)$$

The power starts at  $P_i$  (at moment  $t_i$ ) and increases or decreases to  $P_{end}$  (at moment  $t_{end}$ ). Its slope is  $k$ . In this case the resulting differential equation can only be solved by imposing a final condition that sets the supercapacitor voltage at the end of the period (at  $t_{end}$ ) equal to  $U_{send}$ .

For these two practical cases the solutions obtained for the supercapacitor current and voltage are shown in table I. These are encoded in a Matlab function. It accepts a user file that defines the power demand of the load. It separates the power profile into intervals that are either linear or constant and solves the equations for every interval. As the supercapacitor is a state variable it is used to combined the different intervals.

### III. SIMULATIONAL RESULTS

The parameters for the used supercapacitor, constant current charging and its initial voltage are shown in table II.

TABLE II. CIRCUIT PARAMETERS

Value	Parameter	
	Description	Name
2	Charge Current [A]	$I_{ch}$
58/3	Supercapacitor value [F]	$C$
26	Supercapacitor initial voltage[V]	$U_{init}$

Simulation for a short cycle that combines all of the presented cases is shown in figure 3. The supercapacitor voltage and current can be seen on the figure.

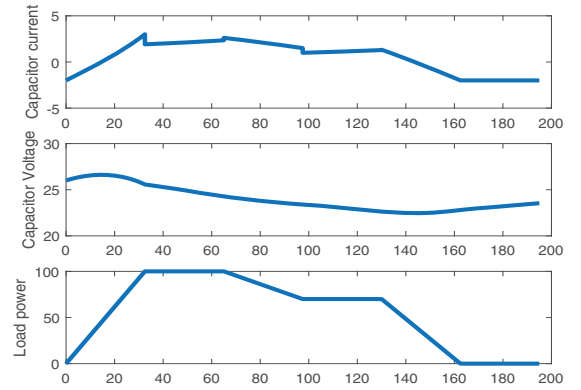


Figure 3. Simulation results for a short cycle

A more detailed simulation can be made for example taking a load profile that mimics a NEDC driving cycle. The results are shown in figure 3. Using the created methodology any power profile can be simulated with relative ease

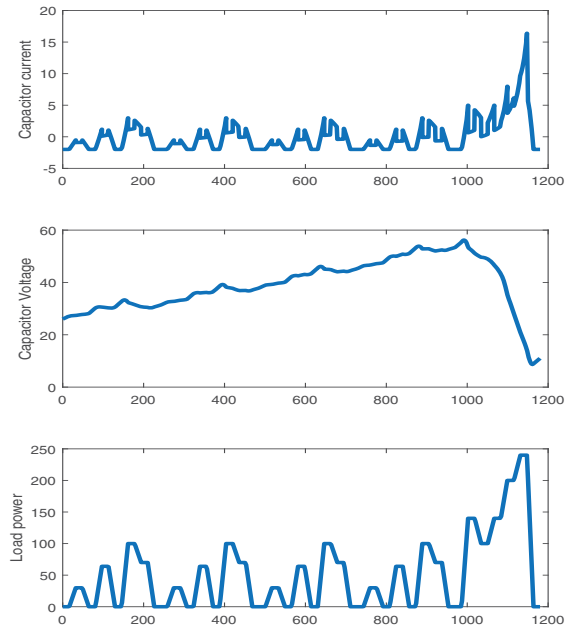


Figure 4. Simulation results for power load based on a driving cycle.

#### IV. EXPERIMENTAL RESULTS

The experimental setup used for the verification of the data is shown in figure 5.

The supercapacitor is a serial combination of three BMOD0058 E016 B02 from Maxwell Technology. Each one is rated at 58F with nominal voltage of 16V. The ESR of the capacitor is 22mOhm. They are charged through a dc/dc converter that is programmed to give constant current of 2A.



Figure 5. Experimental Setup

The electronic load is programmed with parameters given in table III. The supercapacitor current and voltage are measured through a MonoDaq Device, connected to the PC.

TABLE III. ELECTRONIC LOAD PARAMETERS

Time period, s	Parameter	
	Type	Value
0- 28s	Lineary increasing Power	3.51 W/s
28 – 67s	Constant Power	100W
67 – 95s	Lineary decreasing Power	- 1.05 W/s
95 – 133s	Constant Power	70W
133 – 161s	Lineary decreasing Power	-2.45 W/s
161-195s	Constant power	0 W

The results from the experimental model are given in figure 5.

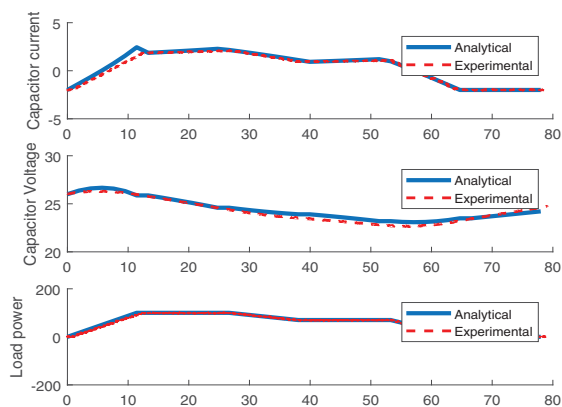


Figure 5. Comparison between experiment and analytical solution

#### V. DISCUSSION

As can be seen from figure 5 the analytical and experimental waveforms for the supercapacitor current and voltage are very close. The maximum difference is observed between the curves of the supercapacitor voltage is 0.5V, which is below 3%.

#### VI. CONCLUSION

The paper presented a mathematical model for a supercapacitor as part of an energy storage system, charged by a constant current source and with load is defined by its power requirements over time and t. The obtained equations allow for estimation of the supercapacitor voltage and current over time, when the load profile can be modeled presented by combining constant power and linearly increasing/decreasing segments. The analytical expressions for the voltage and current are given, and a Matlab script is written that accepts a load profile segments it into the two cases and translates the supercapacitor voltage between periods. The formulas are tested with a power profile and then an experimental setup is also presented that verifies the results. The provided formulas can be used to design any energy storage system that includes a supercapacitor as its element.

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