

# Risk Optimization for E-bike Cycling in Urban Area

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**Abstract.** In this study a common mathematical model of a two-step stochastic problem of the optimal risk-free route from the starting point to the final destination for e-bike cycling. Under risk in this study is considered any obstacle in the route (potholes or bad terrain, pedestrians, pets, hooligans, incompetent drivers etc.). The aim of this paper is the implementation a mathematical model and approach for solving the problem and not particularly the type of hazards taken into account by the risk assessment. For this reason, the input is based on ungrouped statistical data with periodical and stochastic condition in time and space. The mathematical model is expressed as a stochastic problem with two stages. In the first stage, the problem of finding the average risk during e-bike cycling for each section of the route is solved. The input parameter is the number of obstacles in each branch. In the second stage, the problem of finding the optimal risk-free route is solved. A network optimization model of the problem is constructed and the solution is found by application of the Bellman's Principle of optimality.

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**Keywords:** e-bike cycling, risk assessment, optimal risk-free route, Bellman's Principle of optimality, stochastic problem

## 1 Introduction

In recent years, there has been considerable scientific interest in the assessment of risk and route optimization of various types of vehicles. The technological developments from the last years have given a new look to classical paradigms, such as the route optimization of personal vehicles. There is growing trend of electric and pedal-assisted bicycles usage as an environment-friendly and healthy alternative for urban mobility. These personal vehicles are becoming an interesting subject for scientific research in various fields of transportation, logistics and traffic safety. Therefore, by application of various mathematical methods and tools the risk assessment and route optimization for this newly emerged category of personal vehicles and their interaction with the urban environment and all the other actors in city traffic should be considered. Electric-powered cars are typically much more energy-efficient than fossil-fueled fuels. The increasing use of electric vehicles, and in particular those powered by renewable energy sources, can play an important role in achieving the EU's goal of reducing greenhouse gas emissions and moving towards a low carbon future.

Along with the growing trend of electric and pedal-assisted bicycles usage as an environment-friendly and healthy alternative for urban mobility, these personal vehicles are

becoming an interesting subject for scientific research in various fields of transportation, logistics and traffic safety. Therefore, by application of various mathematical methods and tools the risk assessment and route optimization for this newly emerged category of personal vehicles and their interaction with the urban environment and all the other actors in city traffic should be considered. In [8] and [9] are detailed the factors that led to the present state and issues related to urban and suburban traffic in the modern world. The research in [9] is focused on the Braess paradox and the Nash theory of traffic equilibrium in simple two-way road branches. The transport modalities include pedestrians, bicycle, mass urban transport, transit traffic etc. In order to optimize the energy consumed by the vehicle, the authors of [6] have also predicted the vehicle speed by using neural network (NN) and deep learning. After generation of the route checkpoints the input data is passed to an “ordinary” NN (usually with one or two hidden layers). After this the data is treated by a deep learning NN (with more than two hidden layers). At the NN output, the predicted speed along the route is obtained. After the initial analysis of the route are determined the stretches available for optimization. The overall efficiency gains by using the proposed optimization is around 4%. The result is further processed by a 5<sup>th</sup> order filter for noise reduction. In [1] an analysis is presented, and the performance of linear models is compared for two types of adaptive Neural networks. For specific conditions the improved Neural network hybrid model using the look-up table has the best performance. The improved parameters are average absolute voltage error and maximum peak power on test tracks. The authors in [7] have described the development of a useful tool for agencies and researchers for clustering of similar transportation patterns with respect to time-based events. The proposed supervision algorithm is conceived to take advantage of background knowledge of the dataset along with the similarity. Compared to analogous methods, this one stands out with scalarization and low computational complexity along with its other advantages. In [9] several types of stability are introduced and analyzed (average, mean-squared, almost-sure stability etc.) with the aim of widening the optimization scope. After numerous iterations of the optimization, the stability conditions are expressed with several levels of conservatism and feasibility. [5] presents cutting-edge predictive techniques that can be used in shared bicycle use systems. As a major drawback in the work, the large RMSE value due to the choice of a scale for the individual input parameters and the sensitivity of the applied method to the input data can be indicated. Various approaches to finding optimal routes by different criteria are described in [10]. For the purposes of this paper, a network optimization model is proposed to find the most risk-free path for several categories of obstacles: road quality, moving obstacles, etc. The mathematical model is expressed as a stochastic problem with two stages. In the first stage, the problem of finding the average risk during e-bike cycling for each section of the route is solved. The input parameter is the number of obstacles in each branch. In the second stage, the problem of finding the optimal risk-free route is solved. A network optimization model of the problem is constructed and the solution is found by application of the Bellman's Principle of optimality.

The similar problems are analyzed and solved in [8] by using a multi-objective optimization approach. Various approaches to finding optimal routes by different criteria are described in [2], [3], [4], [11], etc.

In the article the research done is specific because it is another point of view of the the assigned problems. The problems arise from the increasing number of people in urban areas (urbanization) and the corresponding increase in human density. On the other hand, transport needs in the urban environment are specific (generally speaking). This means:

- A wide variety of vehicles (from e-bike to stand-alone trucks);
- Extensive amounts of data, such as vehicle technical information, travel profiles (route + driving style + vehicle);

- Share Common Resource - Road infrastructure is used simultaneously by objects that are different in size, weight and speed. Accordingly, the interaction between road users is different from the point of view of the vehicle.

The significance of the research is that the benefit is maximum for cyclists who have to make a decision (related to the route) in an environment of uncertainty. Decision makers would like to assess the risks before they decide to understand the scope of the possible outcomes and the significance of the unwanted consequences.

## 2 Description and Aim of the two-stage problem

### 1<sup>st</sup> stage: Description of problem 1

A cyclist is riding a pedal-assisted electric bicycle and is travelling from a certain point of departure to his destination. This can be performed via a number of routes including combination of their sections. The routes can be represented by a network model of an oriented graph  $V(G, D)$ , where  $G = \{G_i\}_{i=1}^k$  are the nodes and  $D = d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , are the graph arcs (figure 1).

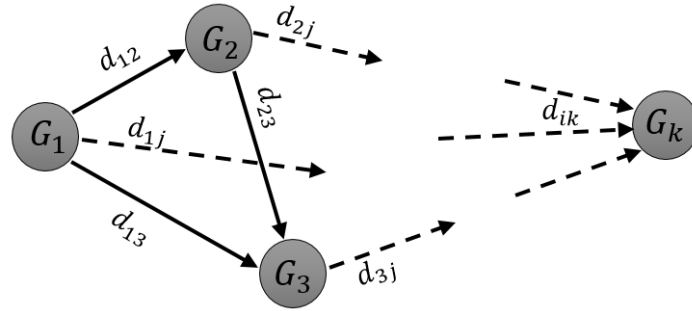


Figure 1: Network model of the oriented graph  $V(G, D)$

The risk to the life and health of the cyclist during the driving of an electric bicycle is directly dependent on the number of obstacles on the bike path and the quality of the road surface.

### Aim of problem 1

To determine the probability of occurrence of risk  $Q_{ij} \in [0; 1], i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , for each arc  $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , from the possible routes of the cyclist in depending on the number of obstacles and the quality of the road surface.

### 2<sup>nd</sup> stage: Description of problem 2

Each of the arcs  $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , is associated with a given risk of encountering obstacles and bad road surfaces and vice-versa with the probability of risk-free travel over the arc:  $p_{ij} = 1 - Q_{ij}, p_{ij} \in [0; 1], i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ .

### Aim of problem 2

The aim of the optimization problem is to determine a route from the departure to the final destination exposing the cyclist to a minimal the risk of encountering obstacles and bad road surfaces.

### 3 Solution of the two-stage problem

#### Solution of problem 1

##### Input data and processing:

Let  $X$  be a discrete random value characterizing the hourly number of obstacles, counted by discrete observations for  $S$  days, in each arc of the route:  $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$  (table 1).

Table 1: Data obtained by observation

Day	Hour			
	00:00	01:00	...	23:00
	<i>Number of obstacles</i>			
1	$x_{1,1}$	$x_{1,2}$	...	$x_{1,24}$
2	$x_{2,1}$	$x_{2,2}$	...	$x_{2,24}$
...	...	...	...	...
S	$x_{S,1}$	$x_{S,2}$	...	$x_{S,24}$
$EX_t \rightarrow$ $t = 1, \dots, 24$	$EX_1 =$ $= \frac{1}{S} \sum_{s=1}^S x_{s,1}$	$EX_2 =$ $= \frac{1}{S} \sum_{s=1}^S x_{s,2}$	...	$EX_{24} =$ $= \frac{1}{S} \sum_{s=1}^S x_{s,24}$

Then for each arc  $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , the average number of obstacles is:

$$EX_{ij} = \frac{1}{24} \sum_{t=1}^{24} EX_t = \frac{1}{24 \cdot S} \sum_{t=1}^{24} \sum_{s=1}^S x_{s,t} \quad (1)$$

And for each arc of the route the average number of obstacles  $EX_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , is associated to the probability of being risky due to the number of obstacles that can appear by cycling through it. Therefore, the probability is expressed by  $q_{ij} \in [0; 1], i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , where

$$q_{ij} = \frac{EX_{ij}}{\sum_{i=1}^{k-1} \sum_{j=2}^k EX_{ij}}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j. \quad (2)$$

For each arc of the route  $d_{ij}$  the road quality (for cycling)  $b_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , is rated by  $b_{ij} = 1, \dots, U$ . A higher value of  $b_{ij}$  means a worse quality of the pavement. The value of  $d_{ij}$ , is associated with a corresponding weight  $w_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$ , characterizing the road pavement:

$$w_{ij} = \frac{b_{ij}}{U}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j. \quad (3)$$

Generally speaking, risk occurs when certain decision has to be taken and the results are uncertain, at the contrary – there is no risk if no uncertainty in the results of an action exist. The risk is more or less subjective, but the uncertainty is impartial. The lack of information (which is also objective and can be assessed) results in a risk. As the uncertainty is a source of risk it can be minimized by obtaining more information (and in an ideal case uncertainty could be eliminated at all). In practice it is rarely possible to reduce all uncertainty. As a result, every decision that has to be taken in an uncertain environment can be treated as a risk assessment problem.

In decision theory risk can be used to quantify uncertainty and is often defined as a deviation from the expected result. Based on this, mathematical methods for estimation of the risk can be implemented. Therefore, the risk for an e-bike cyclist is proportional to the number of obstacles and depend from road surface on his route, in other words a route with higher average number of obstacles and bad road surface is riskier for the cyclist.

The risk to the life and health of the cyclist during the driving of an electric bicycle is directly dependent on the number of obstacles on the bike path and the quality of the road surface, is characterized by probability

$$Q_{ij} = q_{ij} \cdot w_{ij} = \frac{Ex_{ij}}{\sum_{i=1}^{k-1} \sum_{j=2}^k Ex_{ij}} \cdot \frac{b_{ij}}{\sum_{u=1}^U u}, i = 1, \dots, k-1; j = 2, \dots, k; i < j. \quad (4)$$

## Solution of problem 2

The probability of the absence of risk (the inverse of risk occurrence) for each arc of the route  $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$ , is

$$p_{ij} = 1 - Q_{ij} = 1 - q_{ij} \cdot w_{ij} = \frac{Ex_{ij}}{\sum_{i=1}^{k-1} \sum_{j=2}^k Ex_{ij}} \cdot \frac{b_{ij}}{U}, i = 1, \dots, k-1; j = 2, \dots, k; i < j. \quad (5)$$

A network model of the route is developed (figure 2) where each arc  $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$ , is characterized by the probability  $p_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$ .

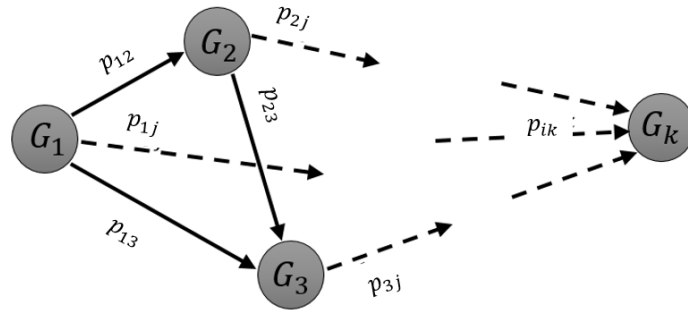


Figure 2: Network model of the oriented graph  $V(G, P)$

The problem of finding the less risky route from the starting point to the final destination is modeled as a network problem, but in fact it is also a reliability problem. This complex problem can be solved by a dynamic programming method by decomposing it into sub-problems which are easier to solve. The decomposition consists in dividing the solution into stages and formulation of optimization problems for each stage that are less complex than the global problem. For each stage there is a scalar (control) variable whose value can be optimized and then the results are linked by a recursive algorithm. Therefore, the solution of the global problem is obtained finally after consecutive solution of a number of sub-problems. This method, based on recursive iterations relies on the Bellman optimality principle that states: ‘The optimal strategy is composed of optimal sub-strategies’.

The objective function is a generalized characteristic of the decisions taken and the results obtained by solving the problem. It reflects the way in which the global problem is decomposed in less complex sub-problems. In the problem of optimal path, the objective function is multiplicative and the global result is a product of the results obtained each stage. It is similar to the reliability of a system built by consecutive addition of a number of elements (building blocks).

The number of stages after decomposition of the main problem is  $k$ . At each stage  $E_n, n = 1, \dots, k$ , the problem of finding the less risky path between nodes  $G_1$  and  $G_n, n = 1, \dots, k$ . A Bellman’s function  $f_N, N = 0, 1, \dots, k$ , is introduced. It gives a quantitative measure of the less risky way from the initial point to the  $n^{\text{th}}$  ( $n = 1, \dots, k$ ) and is defined by a recursive dependence:

$$f_j = \max_{i < j} \{p_{ij} \cdot f_i\}, i = 1, \dots, k-1; j = 2, \dots, k; f_0 \equiv 1, f_1 = 1. \quad (6)$$

The optimum is achieved at the final stage  $E_k$ :

$$f_k = f_{\max}. \quad (7)$$

And then:

$$f_{max} = \prod_{v=1}^v p_{mv} \tag{8}$$

By the recursive Bellman method is obtained the altogether least risky route from the departure to the final destination, as depicted in figure 3.



Figure 3: Optimal cycling route

In the second stage of the problem, the optimal route is found. The objective function is minimization of the risk for the cyclist during his way from the starting point to the destination.

Problem 2 is solved using an iterative method. Through the appropriate elements of the model, the complex problem is decomposed into simpler ones, which are solved almost independently. The solutions found at each stage are optimal and acceptable. This is due to the fact that the problem of the size of the problem is solved by being reduced in stages through the recurrent dependence. And this increases the capabilities of using this method to solve complex problems.

#### 4 Numerical example

The proposed method is applied to this particular problem. In reality the e-bike cyclist is choosing his route just before or during the cycling, but when he arrives he can recapitulate his path and analyze if it has been the less risky from all possible routes. In order to assist the cyclist in choosing his route is developed the presented method for optimization of the risk for electric bike cyclists. In figure 4 is depicted an example of departure and destination of an e-bike cyclist and the possible routes that he can choose. The number of routes is finite and includes combinations of their arcs.

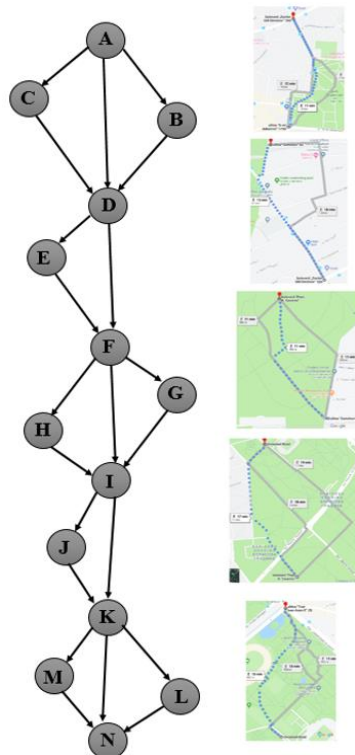


Figure 4: Routes considered for the numerical example

## Input numerical data and processing:

After determination of the possible routes, input data of the hourly number of obstacles (entire number - integer) is collected for each arc of the routes. In this numerical example random data is generated and each section of the routes includes an hourly number of obstacles in the range from 0 to 120 (table 2).

Table 2: Randomly generated distribution of the number of obstacles in some arcs

<i>ABD</i>	<u>00:00</u>	<u>01:00</u>	<u>02:00</u>	<u>03:00</u>	<u>04:00</u>	<u>05:00</u>	<u>06:00</u>	<u>07:00</u>
<i>1</i>	12	85	109	64	99	15	1	113
<i>2</i>	72	83	6	6	112	31	35	92
<i>3</i>	120	27	1	89	52	23	66	70
<i>4</i>	82	114	35	54	51	15	38	9
<i>5</i>	12	98	76	84	44	56	46	40
<i>6</i>	59	72	115	74	34	67	115	119
<i>7</i>	35	119	53	100	50	39	80	64
<i>8</i>	98	75	69	3	108	111	116	23
<i>9</i>	9	6	112	22	33	49	48	58
<i>10</i>	97	36	4	5	106	58	54	36
<i>ABD</i>	<u>08:00</u>	<u>09:00</u>	<u>10:00</u>	<u>11:00</u>	<u>12:00</u>	<u>13:00</u>	<u>14:00</u>	<u>15:00</u>
<i>1</i>	23	97	108	100	110	111	95	109
<i>2</i>	60	56	1	106	20	104	39	50
<i>3</i>	33	87	22	12	31	66	29	49
<i>4</i>	41	55	27	105	40	55	41	65
<i>5</i>	95	29	105	27	12	24	59	119
<i>6</i>	16	1	36	65	26	94	52	100
<i>7</i>	103	40	57	33	9	78	56	78
<i>8</i>	22	31	100	47	2	90	33	95
<i>9</i>	43	59	23	3	91	53	114	30
<i>10</i>	64	48	19	43	118	65	98	92
<i>ABD</i>	<u>16:00</u>	<u>17:00</u>	<u>18:00</u>	<u>19:00</u>	<u>20:00</u>	<u>21:00</u>	<u>22:00</u>	<u>23:00</u>
<i>1</i>	3	89	97	62	4	39	80	87
<i>2</i>	26	52	86	29	17	21	8	116
<i>3</i>	36	29	101	6	49	45	61	105
<i>4</i>	8	92	13	18	86	30	55	54
<i>5</i>	12	103	37	53	62	27	74	24
<i>6</i>	3	67	5	62	64	77	58	116
<i>7</i>	119	52	113	90	57	14	6	93
<i>8</i>	98	74	47	48	99	75	45	117
<i>9</i>	34	54	91	56	10	57	25	74
<i>10</i>	92	80	92	73	44	59	82	79
<i>FI</i>	<u>00:00</u>	<u>01:00</u>	<u>02:00</u>	<u>03:00</u>	<u>04:00</u>	<u>05:00</u>	<u>06:00</u>	<u>07:00</u>
<i>1</i>	76	53	86	115	54	115	33	115
<i>2</i>	49	47	65	40	29	106	97	58
<i>3</i>	12	105	3	63	27	38	95	25
<i>4</i>	39	112	108	22	47	28	29	15
<i>5</i>	105	66	36	66	19	106	21	71
<i>6</i>	57	81	5	39	91	44	59	15
<i>7</i>	4	33	38	101	112	25	95	40
<i>8</i>	41	27	11	93	4	24	105	13
<i>9</i>	97	60	112	98	94	93	7	104
<i>10</i>	112	43	57	99	48	52	99	56
<i>FI</i>	<u>08:00</u>	<u>09:00</u>	<u>10:00</u>	<u>11:00</u>	<u>12:00</u>	<u>13:00</u>	<u>14:00</u>	<u>15:00</u>
<i>1</i>	2	103	20	25	43	112	51	118
<i>2</i>	102	60	20	2	69	58	101	84
<i>3</i>	12	113	98	118	114	103	55	105
<i>4</i>	55	18	57	45	101	64	119	25
<i>5</i>	58	120	66	36	5	61	56	54
<i>6</i>	86	69	28	68	117	80	91	17
<i>7</i>	4	27	72	71	97	59	103	66
<i>8</i>	61	119	40	22	69	64	44	81
<i>9</i>	2	75	54	40	25	90	6	4
<i>10</i>	77	111	0	90	35	91	112	26
<i>FI</i>	<u>16:00</u>	<u>17:00</u>	<u>18:00</u>	<u>19:00</u>	<u>20:00</u>	<u>21:00</u>	<u>22:00</u>	<u>23:00</u>
<i>1</i>	57	96	61	104	11	89	48	10
<i>2</i>	76	87	109	26	15	58	95	24
<i>3</i>	100	109	61	45	101	110	68	56

4	116	70	35	1	90	49	111	99
5	76	110	95	86	95	14	102	67
6	21	73	112	107	62	39	81	112
7	44	65	93	77	23	24	39	61
8	45	52	52	38	92	63	65	60
9	98	26	62	83	118	20	51	21
10	101	110	7	36	36	3	91	5

For each arc of road, the average number of obstacles is associated with the probability of a risk area. Table 2 contains the average number of obstacles, the probability of risk and non-risk area.

For each section of the road input data are processed according to the methodology described above, and the results are presented in table 3.

Table 3: Results after input data processing:  $q_{ij}$  – probability for risky section because of an obstacle;  $w_{ij}$  – weight characterizing the quality of the road pavement;  $Q_{ij}$  – probability for introducing an obstacle because of the pavement quality

	$q_{ij}$	$w_{ij}$	$Q_{ij} = q_{ij} \cdot w_{ij}$	$p_{ij} = 1 - Q_{ij}$
ABD	0.08096	0.17560	0.01422	0.98578
AD	0.08173	0.75345	0.06158	0.93842
ACD	0.08170	0.56482	0.04615	0.95385
DF	0.08187	0.67520	0.05528	0.94472
DEF	0.08255	0.56892	0.04696	0.95304
FGI	0.08453	0.27531	0.02327	0.97673
FI	0.08680	0.87253	0.07574	0.92426
FHI	0.08523	0.24567	0.02094	0.97906
IK	0.08190	0.67289	0.05511	0.94489
IJK	0.08550	0.34958	0.02989	0.97011
KLN	0.08286	0.75987	0.06296	0.93704
KN	0.08235	0.35746	0.02944	0.97056
KMN	0.08453	0.68715	0.05809	0.94191

## 5 Results

The solution of the risk optimization problem is obtained according to a multiplicative objective function and the result is a product of the sub-problem solutions. It reflects the reliability of a system constructed by a number of separate elements. By using this method, the numerical solution of the problem is depicted in figure 5.

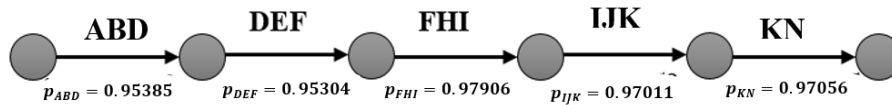


Figure 5: The optimal route for cycling

The most risk-free route is 87%, which is not very far from the hypothetical 100%, but taking into account the terrain and the obstacles, the result is very good.

Problem 2 is solved using an iterative method. Through the appropriate elements of the model, the complex problem is decomposed into simpler ones, which are solved almost independently. The solutions found at each stage are optimal and acceptable. This is due to the fact that the problem of the size of the task is solved by being reduced in stages through the recurrent dependence. And this increases the capabilities of using this method to solve complex problems.

The numerical example was implemented and solved by using the software environments MatLab and Maple.



## 6 Discussion of the results

Regarding the results obtained by application of the Bellman's optimization principle, the following observations can be made:

The shortest path from the numerical example has a length of 5.15 km and the length of the least risky one is 5.75 km. This means that an 10% increase in the distance reduces the risk by 9% (figure 6).

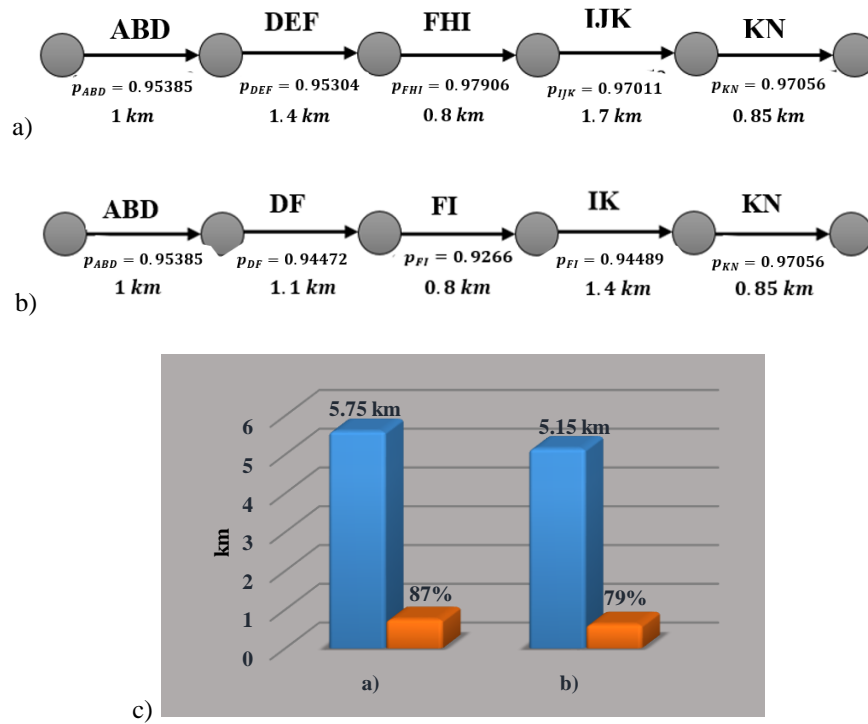
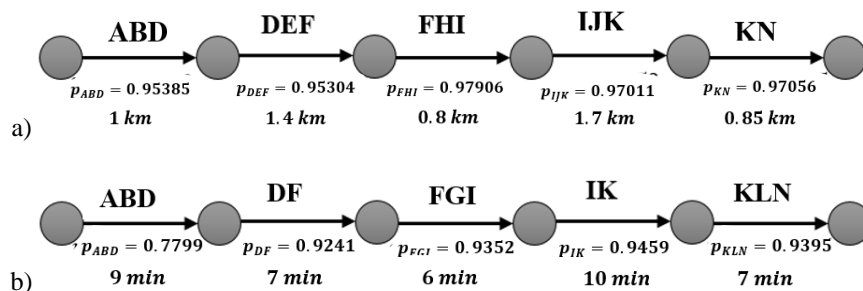


Figure 6. a) The least risky route; b) Shortest route; c) Comparison between the length and the safety of these the routes

The fastest route in this example has a duration of 39 min, and the least risky – e 43 min, which means that an increase of 4 min in the duration of the trip would decrease the risk by 7% (figure 7).



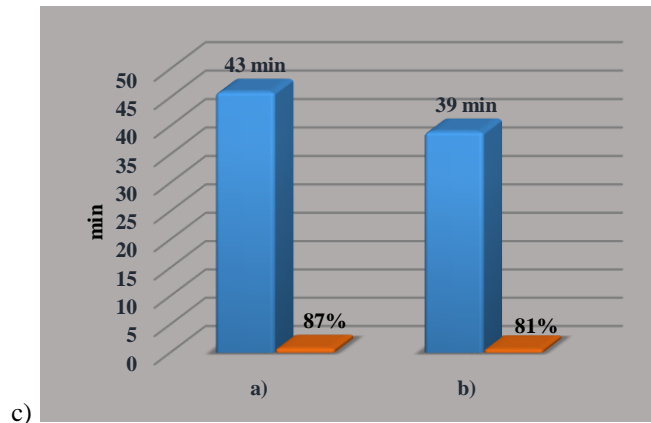


Figure 7. a) The least risky route; b) The fastest route;  
c) Comparative diagram of both routes

## 7 Conclusion and future works

The risk assessment and route optimization for electric bicycle in urban environment is presented in this paper. The route optimization in this study is based on a dynamic programming approach with the addition of supplementary factors characterizing the cycling of electric and pedal-assisted bicycles in urban environment. The capability of the implemented approach for solving the problem and the liability of the developed mathematical model are studied by a numerical example composed of random input data. Due to the considerable dynamics of urban traffic, the input data of the optimization problem is highly volatile in daily and hourly time scale. It is also influenced by the weather conditions and other factors or events. Based on this study, an even more detailed mathematical model and optimization approach can be implemented in future works.

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