# Risk Assessment and Route Optimization for E-bike Cycling

Silvia Baeva [1]
Vasil Shterev [2]
Nikolay Hinov [3]
Hristiyan Kanchev [4]

Technical University of Sofia
Bulgaria

[1] sbaeva@tu-sofia.bg
[2] vas@tu-sofia.bg
[3] hinov@tu-sofia.bg
[4] hkanchev@tu-sofia.bg

**Abstract.** In this paper, investigation of the risk during cycling of an e-bike is presented. The input parameter taken into account is the number of obstacles in each alternative branch of the route. Under obstacle in this study is considered any object that could either threaten the life of cyclist or present an obstacle in his way (potholes on the road or dangerous terrain, stray animals, hooligans, incompetent car drivers etc.). The mathematical model is expressed as a stochastic problem with two stages. The first stage consists of a calculation of the average risk in each branch of the route. In this study the average risk in each branch is expressed by grouped statistical data. The input parameters are stochastic and discretized, preprocessed by statistic methodology. In the second stage the optimal route is found by application of the Bellman's optimality principle.

This research is funded and carried out in the framework of project 'Optimal Control of the Energy Flows in Electric Vehicles by means of Artificial Intelligence', contract № 191ПР0018-07, Scientific and Research Sector of the Technical University of Sofia.

**Keywords:** risk assessment, e-bike cycling, optimal route, mathematical model, stochastic problem

## 1 Introduction

In recent years, there has been considerable scientific interest in the assessment of risk and route optimization of various types of vehicles. The technological developments from the last years have given a new look to classical paradigms, such as the route optimization of personal vehicles. There is growing trend of electric and pedal-assisted bicycles usage as an environment-friendly and healthy alternative for urban mobility. These personal vehicles are becoming an interesting subject for scientific research in various fields of transportation, logistics and traffic safety. Therefore, by application of various mathematical methods and tools the risk assessment and route optimization for this newly emerged category of personal vehicles and their interaction with the urban environment and all the other actors in city traffic should be considered. Electric-powered cars are typically much more energy-efficient than fossil-fueled fuels. The increasing use of electric vehicles, and in particular those powered by renewable energy sources, can play an important role in achieving the EU's goal of reducing greenhouse gas emissions and moving towards a low carbon future.

Research into one direction for assessing the risk to the life and health of the cyclist in terms of pollution and exhaust emissions is done in [11]. There, the authors have made a study on the influence of exhaust gases such as CO, NOx and fine particulate matter of conventional vehicles (i.e. using internal combustion engines) resulting in serious health concerns for the general public. Another example is a research [4], showing that the mortality rate for people living in the most polluted cities can be 29% higher than those living in the least polluted cities based on data in the past several decades.

Research into another direction for assessing the risk to the life and health of a cyclist with regard to its technical characteristics of e-bike and environmental objects have been done in [8]. There are shown several types of stability are introduced and analyzed (average, mean-squared, almost-sure stability etc.) with the aim of widening the optimization scope. After numerous iterations of the optimization, the stability conditions are expressed with several levels of conservatism and feasibility. The authors in [6] have described the development of a useful tool for agencies and researchers for clustering of similar transportation patterns with respect to time-based events. The proposed supervision algorithm is conceived to take advantage of background knowledge of the dataset along with the similarity. Compared to analogous methods, this one stands out with scalarization and low computational complexity along with its other advantages. An intelligent control of the traffic lights is studied in [12]. A feed-forward neural network is adopted to accomplish the traffic signal controller. The proposed solution has several advantages compared to traditional methods and in particular the self-learning ability. The research in [10] is based on the Nash equilibrium for an infinite time horizon. Furthermore, various strategies are defined for the individual players in the competition. Their performance is then compared under equal conditions, with or without a feedback in the informational structure.

In this paper, investigation of the risk during cycling of an e-bike is presented. The input parameter taken into account is the number of obstacles in each alternative branch of the route. Under obstacle in this study is considered any object that could either threaten the life of cyclist or present an obstacle in his way (potholes on the road or dangerous terrain, stray animals, hooligans, incompetent car drivers etc.). The mathematical model is expressed as a stochastic problem with two stages. The first stage consists of a calculation of the average risk in each branch of the route. In this study the average risk in each branch is expressed by grouped statistical data. The input parameters are stochastic and discretized, preprocessed by statistic methodology. In the second stage the optimal route is found by application of the Bellman's optimality principle. The similar problems are analyzed and solved in [7] by using a multi-objective optimization approach. Various approaches to finding optimal routes by different criteria are described in [1], [2], [3], [5], [9], etc.

In the article the research done is specific because it is another point of view of the the assigned problems. The problems arise from the increasing number of people in urban areas (urbanization) and the corresponding increase in human density. On the other hand, transport needs in the urban environment are specific (generally speaking). This means:

- A wide variety of vehicles (from e-bike to stand-alone trucks);
- Extensive amounts of data, such as vehicle technical information, travel profiles (route + driving style + vehicle);
- Share Common Resource - Road infrastructure is used simultaneously by objects that are different in size, weight and speed. Accordingly, the interaction between road users is different from the point of view of the vehicle.

The significance of the research is that the benefit is maximum for cyclists who have to make a decision (related to the route) in an environment of uncertainty. Decision makers would like to assess the risks before they decide to understand the scope of the possible outcomes and the significance of the unwanted consequences.

## 2 Description and Aim of the two-stage problem

### 1st stage: Description of problem 1

A cyclist is riding a pedal-assisted electric bicycle and is travelling from a certain point of departure to his destination. This can be performed via a number of routes including combination of their sections The routes can be represented by a network model of an oriented graph $V(G,D)$, where $G = \{G_i\}_{i=1}^{k}$ are the nodes and $D = d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, are the graph arcs (figure 1).

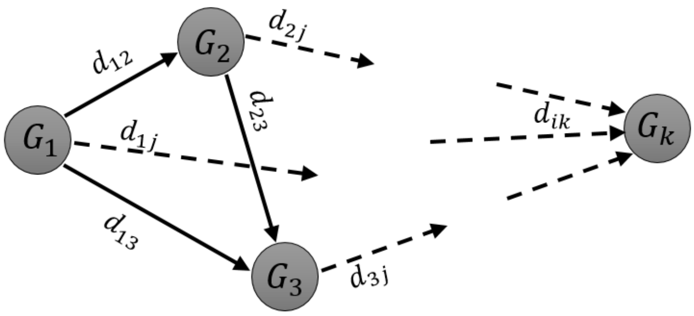


Figure 1: Network model of the oriented graph $V(G,D)$

The risk to the life and health of a cyclist during an electric bicycle management is directly related to the number of obstacles on the e-bike routes.

### Aim of problem 1

The aim of the first stage is to assess the risk (the probability) $q_{ij} \in [0;1], i = 1, \dots, k-1; j = 2, \dots, k; i < j$, of encountering obstacles in each of the sections $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, forming the possible routes of the cyclist from the initial to the final point.

### 2nd stage: Description of problem 2

Let the arcs of the graph $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, are associated with probability $p_{ij} = 1 - q_{ij}$, $p_{ij} \in [0;1], i = 1, \dots, k-1; j = 2, \dots, k; i < j$, for the absence of risk to the life and health of the cyclist when he managing the e-bike for each arc $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, of the possible routes.

### Aim of problem 2

The aim of the optimization problem is to determine a route from the departure to the final destination exposing the cyclist to a minimal the risk of encountering obstacles.

## 3 Solution of the two-stage problem

### Solution of problem 1

**Input data and processing:**

$X$ is a discrete random number characterizing the number of obstacles observed along each of the arcs $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, for each hour of the day $l_t, t = 1, \dots, 24$.

The data is processed statistically and for each of the sections $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, and the frequency of occurrence is expressed for a number of equal intervals $[x_r; x_{r+1}], r = 1, \dots, R$ (table 1).

Table 1: Frequency of occurrence of the random event $X$

| *№* | *Interval* $[x_r; x_{r+1}]$ | *Average Value* $\overline{x_r}^*$ | *Absolute Frequency of occurrence* $L_r$ | *Relative frequency of occurrence* $P_r^*$ | $\overline{x_r}^*.P_r^*$ |
|---|---|---|---|---|---|
| **1** | $[x_1; x_2)$ | $\overline{x_1}^*$ | $L_1$ | $P_1^* = \frac{L_1}{L}$ | $\overline{x_1}^*.P_1^*$ |
| **2** | $[x_2; x_3)$ | $\overline{x_2}^*$ | $L_2$ | $P_2^* = \frac{L_2}{L}$ | $\overline{x_2}^*.P_2^*$ |
| … | … | … | … | … | … |
| **R** | $[x_R; x_{R+1}]$ | $\overline{x_R}^*$ | $L_R$ | $P_R^* = \frac{L_R}{L}$ | $\overline{x_R}^*.P_R^*$ |
| | | | $\sum_{r=1}^{R} L_r = L_{ij} = L$ | $\sum_{r=1}^{R} P_r^* = 1$ | $\sum_{r=1}^{R} \overline{x_r}^*.P_r^* = EX^*$ |

For each arcs $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, the average number of obstacles is expressed:

$$Ex_{ij} = \sum_{r=1}^{R} \overline{x_r}^*.P_r^* = EX^*, i = 1, \dots, k-1; j = 2, \dots, k; i < j. \quad (1)$$

Generally speaking, risk occurs when certain decision has to be taken and the results are uncertain, at the contrary – there is no risk if no uncertainty in the results of an action exist. The risk is more or less subjective, but the uncertainty is impartial. The lack of information (which is also objective and can be assessed) results in a risk. As the uncertainty is a source of risk it can be minimized by obtaining more information (and in an ideal case uncertainty could be eliminated at all). In practice it is rarely possible to reduce all uncertainty. As a result, every decision that has to be taken in an uncertain environment can be treated as a risk assessment problem.

In decision theory risk can be used to quantify uncertainty and is often defined as a deviation from the expected result. Based on this, mathematical methods for estimation of the risk can be implemented. Therefore, the risk for an e-bike cyclist is proportional to the number of obstacles on his route, in other words a route with higher average number of obstacles is riskier for the cyclist.

Each section $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, with an average number of obstacles $Ex_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, is associated to the probability of being risky $q_{ij} \in [0; 1], i = 1, \dots, k-1; j = 2, \dots, k; i < j$, where:

$$q_{ij} = \frac{Ex_{ij}}{\sum_{i=1}^{k-1} \sum_{j=2}^{k} Ex_{ij}}, i = 1, \dots, k-1; j = 2, \dots, k; i < j. \quad (2)$$

The solution of the first stage consists in expressing the probability $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, of encountering obstacles such as: pedestrians, animals, vehicles in each section $q_{ij} \in [0; 1], i = 1, \dots, k-1; j = 2, \dots, k; i < j$, of the route. These obstacles represent a risk for the cyclist, because the probability of an accident is proportional to the number of obstacles in the e-bike cyclist path.

## Solution of problem 2

The probability of a risk-free ride (the lack of risk) for each arc $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, is:

$$p_{ij} = 1 - q_{ij} = 1 - \frac{Ex_{ij}}{\sum_{i=1}^{k-1} \sum_{j=2}^{k} Ex_{ij}}, i = 1, \dots, k-1; j = 2, \dots, k; i < j. \quad (3)$$

A network model of the route is developed (figure 2) where each arc $d_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$, is characterized by the probability $p_{ij}, i = 1, \dots, k-1; j = 2, \dots, k; i < j$.

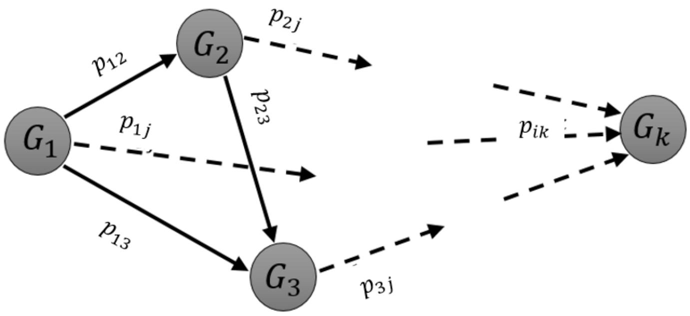


Figure 2: Network model of the oriented graph $V(G, P)$

The problem of finding the less risky route from the starting point to the final destination is modeled as a network problem, but in fact it is also a reliability problem. This complex problem can be solved by a dynamic programming method by decomposing it into sub-problems which are easier to solve. The decomposition consists in dividing the solution into stages and formulation of optimization problems for each stage that are less complex than the global problem. For each stage there is a scalar (control) variable whose value can be optimized and then the results are linked by a recursive algorithm. Therefore, the solution of the global problem is obtained finally after consecutive solution of a number of sub-problems. This method, based on recursive iterations relies on the Bellman optimality principle that states: 'The optimal strategy is composed of optimal sub-strategies'.

The objective function if a generalized characteristic of the decisions taken and the results obtained by solving the problem. It reflects the way in which the global problem is decomposed in less complex sub-problems. In the problem of optimal path, the objective function is multiplicative and the global result is a product of the results obtained each stage. It is similar to the reliability of a system built by consecutive addition of a number of elements (building blocks).

The number of stages after decomposition of the main problem is $k$. At each stage $E_n, n = 1, \dots, k$, the problem of finding the less risky path between nodes $G_1$ and $G_n, n = 1, \dots, k$. A Bellman's function $f_N, N = 0,1, \dots, k$, is introduced. It gives a quantitative measure of the less risky way from the initial point to the $n^{\text{th}}$ $(n = 1, \dots, k)$ and is defined by a recursive dependence:

$$f_j = \max_{i<j} \{p_{ij} . f_i\}, i = 1, \dots, k-1; j = 2, \dots, k; \quad (4)$$

$$f_0 \equiv 1, f_1 = 1. \quad (5)$$

The optimal value is then obtained at the final stage $E_k$ and it is:

$$f_k = f_{max}. \quad (6)$$

By inversion of this principle, the less risky for the cyclist path from his initial point of departure to his destination is obtained (figure 3).

Figure 3: Optimal cycling path

Then

$$f_{max} = \prod_{V=1}^{v} p_{mV}. \quad (7)$$

In the second stage of the problem, the optimal path is found. The objective function is minimization of the risk for the cyclist during his way from the starting point to the destination.

Problem 2 is solved using an iterative method. Through the appropriate elements of the model, the complex problem is decomposed into simpler ones, which are solved almost independently. The solutions found at each stage are optimal and acceptable. This is due to the fact that the problem of the size of the problem is solved by being reduced in stages through the recurrent dependence. And this increases the capabilities of using this method to solve complex problems.

## 4 Numerical example

The proposed method is applied to this particular problem. In reality the e-bike cyclist is choosing his route just before or during the cycling, but when he arrives he can recapitulate his path and analyze if it has been the less risky from all possible routes. In order to assist the cyclist in choosing his route is developed the presented method for optimization of the risk for electric bike cyclists. In figure 4 is depicted an example of departure and destination of an e-bike cyclist and the possible routes that he can choose. The number of routes is finite and includes combinations of their arcs.

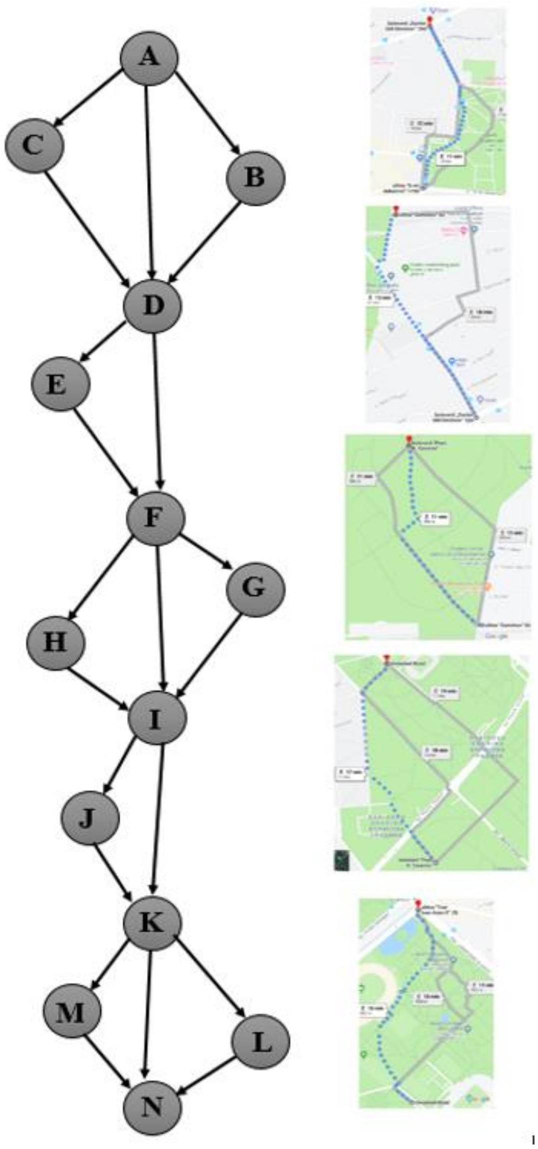


Figure 4: Routes considered for the numerical example

**Input numerical data and processing:**

After determination of the possible routes, input data of the hourly number of obstacles (entire number - integer) is collected for each arc of the routes. In this numerical example random data is generated and each arc of the routes includes an hourly number of obstacles in the range from 0 to 120 (table 2).

Table 2: Random generated hourly number of obstacles for each arc

| | *0:00* | *1:00* | *2:00* | *3:00* | *4:00* | *5:00* | *6:00* | *7:00* |
|---|---|---|---|---|---|---|---|---|
| ***ABD*** | **11** | **63** | **19** | **114** | **75** | **52** | **45** | **54** |
| ***AD*** | **81** | **110** | **25** | **35** | **7** | **17** | **58** | **60** |
| ***ACD*** | **86** | **27** | **6** | **27** | **121** | **4** | **7** | **99** |
| ***DF*** | **95** | **72** | **83** | **73** | **95** | **122** | **23** | **106** |
| ***DEF*** | **116** | **82** | **37** | **83** | **96** | **39** | **86** | **13** |
| ***FGI*** | **128** | **4** | **70** | **54** | **8** | **38** | **43** | **23** |
| ***FI*** | **100** | **80** | **90** | **27** | **112** | **43** | **117** | **47** |
| ***FHI*** | **76** | **47** | **65** | **123** | **121** | **61** | **15** | **7** |
| ***IK*** | **121** | **6** | **70** | **11** | **128** | **84** | **128** | **68** |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ***IJK*** | **75** | **64** | **58** | **14** | **112** | **3** | **70** | **44** |
| ***KLN*** | **2** | **25** | **16** | **18** | **102** | **109** | **92** | **23** |
| ***KN*** | **16** | **16** | **64** | **22** | **67** | **73** | **130** | **27** |
| ***KMN*** | **112** | **27** | **111** | **81** | **23** | **111** | **37** | **118** |
| | ***8:00*** | ***9:00*** | ***10:00*** | ***11:00*** | ***12:00*** | ***13:00*** | ***14:00*** | ***15:00*** |
| ***ABD*** | **88** | **9** | **39** | **107** | **90** | **104** | **43** | **78** |
| ***AD*** | **61** | **32** | **8** | **44** | **72** | **95** | **85** | **103** |
| ***ACD*** | **119** | **7** | **39** | **38** | **52** | **7** | **97** | **48** |
| ***DF*** | **14** | **57** | **6** | **97** | **8** | **9** | **76** | **27** |
| ***DEF*** | **97** | **2** | **66** | **1** | **101** | **12** | **96** | **11** |
| ***FGI*** | **96** | **117** | **99** | **6** | **44** | **104** | **31** | **100** |
| ***FI*** | **73** | **26** | **82** | **87** | **79** | **123** | **96** | **27** |
| ***FHI*** | **24** | **12** | **12** | **78** | **96** | **89** | **126** | **50** |
| ***IK*** | **78** | **40** | **11** | **68** | **14** | **17** | **113** | **72** |
| ***IJK*** | **39** | **59** | **101** | **95** | **17** | **94** | **11** | **30** |
| ***KLN*** | **17** | **13** | **118** | **92** | **71** | **14** | **48** | **83** |
| ***KN*** | **28** | **129** | **69** | **102** | **63** | **15** | **48** | **63** |
| ***KMN*** | **116** | **43** | **14** | **37** | **116** | **83** | **89** | **20** |
| | ***16:00*** | ***17:00*** | ***18:00*** | ***19:00*** | ***20:00*** | ***21:00*** | ***22:00*** | ***23:00*** |
| ***ABD*** | **102** | **56** | **35** | **1** | **41** | **41** | **20** | **59** |
| ***AD*** | **13** | **90** | **5** | **55** | **41** | **22** | **50** | **55** |
| ***ACD*** | **38** | **99** | **88** | **85** | **28** | **81** | **21** | **47** |
| ***DF*** | **31** | **56** | **56** | **94** | **33** | **128** | **99** | **73** |
| ***DEF*** | **69** | **85** | **59** | **69** | **116** | **22** | **113** | **97** |
| ***FGI*** | **12** | **14** | **79** | **14** | **91** | **34** | **46** | **55** |
| ***FI*** | **53** | **121** | **8** | **82** | **72** | **52** | **89** | **56** |
| ***FHI*** | **14** | **24** | **41** | **16** | **24** | **10** | **38** | **16** |
| ***IK*** | **15** | **35** | **100** | **17** | **28** | **89** | **69** | **3** |
| ***IJK*** | **102** | **104** | **91** | **13** | **10** | **52** | **108** | **38** |
| ***KLN*** | **38** | **63** | **16** | **18** | **119** | **128** | **78** | **41** |
| ***KN*** | **78** | **100** | **17** | **22** | **92** | **52** | **44** | **85** |
| ***KMN*** | **125** | **51** | **12** | **26** | **73** | **81** | **39** | **124** |

The input data given in table 2 is processed and characterized by frequency of occurrence of obstacles for each arc of the route (table 3).

Table 3: Frequency analysis for the arc FHI

| № | ***Interval*** $[x_r; x_{r+1}]$ | ***Average value*** $\overline{x_r}^*$ | ***Absolute frequency*** $L_r$ | ***Relative frequency*** $P_r^*$ | $\overline{x_r}^* . P_r^*$ |
|---|---|---|---|---|---|
| **1** | **[0;10)** | **5** | **0** | **0** | **0** |
| **2** | **[10;20)** | **15** | **2** | **0.0244** | **0.3659** |
| **3** | **[20;30)** | **25** | **6** | **0.0732** | **1.8293** |
| **4** | **[30;40)** | **35** | **3** | **0.0366** | **1.2805** |
| **5** | **[40;50)** | **45** | **1** | **0.0122** | **0.5488** |
| **6** | **[50;60)** | **55** | **3** | **0.0366** | **2.0122** |
| **7** | **[60;70)** | **65** | **0** | **0** | **0** |
| **8** | **[70;80)** | **75** | **2** | **0.0244** | **1.8293** |
| **9** | **[80;90)** | **85** | **2** | **0.0244** | **2.0732** |
| **10** | **[90;100)** | **95** | **1** | **0.0122** | **1.1585** |
| **11** | **[100;110)** | **105** | **1** | **0.0122** | **1.2805** |
| **12** | **[110;120]** | **115** | **0** | **0** | **0** |
| | | | | $\sum P_r^* = 1$ | $EX_{FHI}$ =**12.3781** |

For each arc of the route the average number of obstacles is associated with a probability of risk. In table 4 are presented the average number of obstacles and the probabilities of risky and risk-free ride in each arc.

Table 4: Risk associated to each section of the route

| | ***ABD*** | ***AD*** | ***ACD*** | ***DF*** | ***DEF*** | ***FGI*** |
|---|---|---|---|---|---|---|
| ***E*** | **53.0488** | **17.8049** | **14.8780** | **18.2927** | **17.5000** | **15.6098** |
| ***q*** | **0.2201** | **0.0739** | **0.0617** | **0.0759** | **0.0726** | **0.0648** |
| **p** | **0.7799** | **0.9261** | **0.9383** | **0.9241** | **0.9274** | **0.9352** |

| | ***FI*** | ***FHI*** | ***IK*** | ***IJK*** | ***KLN*** | ***KN*** | ***KMN*** |
|---|---|---|---|---|---|---|---|
| ***E*** | **17.6829** | **12.3781** | **13.0488** | **18.4756** | **14.5732** | **16.8293** | **10.9756** |
| ***q*** | **0.0734** | **0.0514** | **0.0541** | **0.0767** | **0.0605** | **0.0698** | **0.0455** |
| ***p*** | **0.9266** | **0.9486** | **0.9459** | **0.9233** | **0.9395** | **0.9302** | **0.9545** |

## 5 Results

The solution of the risk optimization problem is obtained according to a multiplicative objective function and the result is a product of the sub-problem solutions. It reflects the reliability of a system constructed by a number of separate elements. By using this method, the numerical solution of the problem is depicted in figure 5.

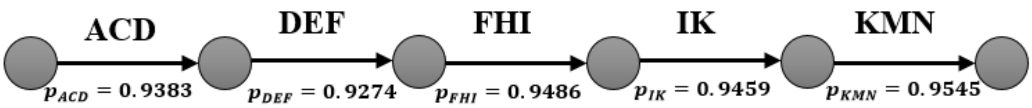


Figure 5: Optimal cycling route

The probability characterizing the least risky route is 75% which is not far away from the theoretical maximum: 100% absence of risk. The result is adequate and reflects the real situation, taking into account the inherent incertitude due to the urban environment in combination with arcs situated in a large park including a dense forest.

The numerical example is implemented and solved by using the software environments MatLab and Maple.

## 6 Discussion of the results

Regarding the results obtained by application of the Bellman’s optimization principle, the following observations can be made:

The shortest path from the numerical example has a length of 5.15 km and the length of the least risky one is 5.6 km. This means that in this particular situation an 8% increase in the distance would reduce the risk by 17% (figure 6).

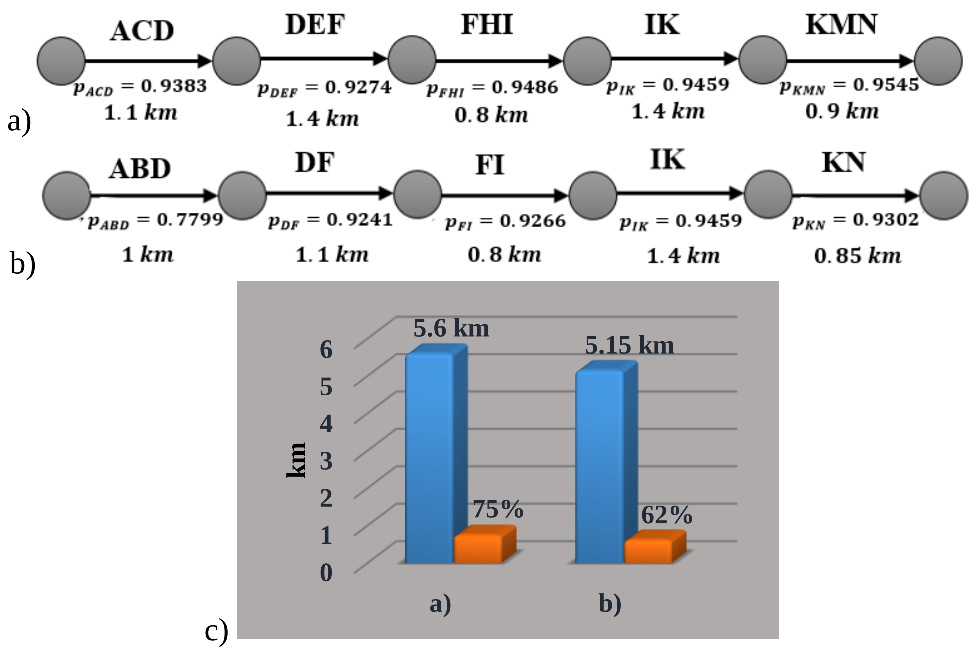

Figure 6: a) The least risky route; b) Shortest route;
c) Comparison between the length and the safety of these the routes

The duration of the fastest route is 39 min and the time needed for travelling along the least risky route is 46 min, which points out that an increase by 7 min would reduce the risk by 25% (figure 7).

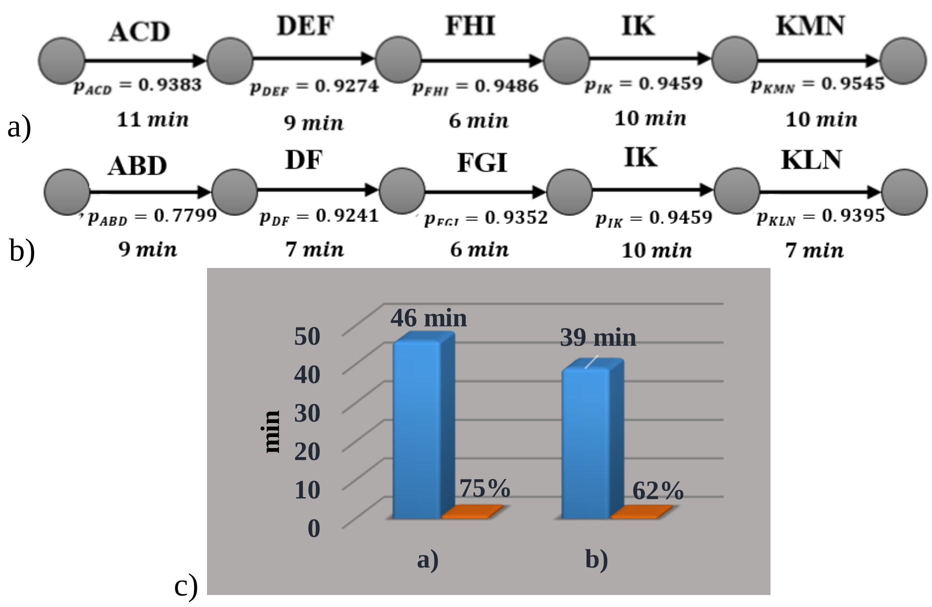

Figure 7: a) The least risky route; b) The fastest route;
c) Comparison between both routes

## 7 Conclusion and future works

This paper presents a study on risk assessment and route optimization for electric bike cycling in urban environment. A typical cycling route in the city of Sofia, comprising arcs in residential districts and in the city park is used for a numerical example illustrating the optimization efficiency. The objective of the route optimization is minimization of the average number of obstacles and overall difficulty of the path.

The aim of this study is verification of the mathematical modeling and the optimization procedure by using randomly generated statistical data of the hourly number of obstacles in the route sections. Due to the dynamics of urban traffic and the lifestyle of its citizens, the number of obstacles should also depend not only on the time of the day, but also on the season and the day of the week (weekend or working day) and the weather conditions. Therefore, in further studies the authors plan to implement a more sophisticated optimization procedure by using aggregated statistical data for various days and seasons.

## 8 Acknowledgement

This research is funded and carried out in the framework of project ‘Optimal Control of the Energy Flows in Electric Vehicles by means of Artificial Intelligence’, contract № 191ПР0018-07, Scientific and Research Sector of the Technical University of Sofia.

## References

[1] Dr Pradeep J Jha, *Operations Research*, L J Institute of engineering and Technology, Ahmedabad, India, 2014.

[2] G. Srinivasan, *Operations Research – Principles and Applications*, Third Edition, Delhi – 110092, 2017.

[3] H. A. Eiselt, Carl-Louis Sandblom, *Operations Research – A model – Based Approach*, Second Edition, Springer, e-book, 2012.

[4] J. O. Anderson, J. G. Thundiyil, and A. Stolbach, *Clearing the air: a review of the effects of particulate matter air pollution on human health*, Journal of Medical Toxicology, vol. 8, no. 2, pp. 166–175, 2012.

[5] Jan A. Snyman, Daniel N. Wilke, *Practical Mathematical Optimization – Basic Optimization Theory and Gradient – Based Algorithms*, Second Edition, Springer International Publishing, part of Springer Nature, e-book, 2018.

[6] Mohamed Haitam Laarabi, *Multi-criteria route optimization for dangerous goods transport using fuzzy risk assessment and agent-based traffic simulation*, Business administration, Ecole Nationale Supérieure des Mines de Paris, 2014.

[7] Mohammed H. Almannaa, Mohammed Elhenawy, and Hesham A. Rakha, Senior Member, *A Novel Supervised Clustering Algorithm for Transportation System Applications*; Transactions on intelligent transportation systems, IEEE, 2019.

[8] P. Bolzern, P. Colaneri, G. De Nicolao, *Stochastic stability of positive Markov jump linear systems*, Automatica, 50:1181-1187, 2014.

[9] S. I. Gass & M. Fu, eds, *Encyclopedia of Operations Research and Management Science, Springer*, New York, 2013.

[10] T. Azevedo-Perdicoulis, G. Jank, *Linear Quadratic Nash Games on Positive Linear Systems*, European Journal of Control, 11:1-13, 2005.

[11] Yingqi Gu, Mingming Liu, Matheus Souza and Robert N. Shorten, *On the Design of an Intelligent Speed Advisory System for Cyclists*, 21st International Conference on Intelligent Transportation Systems (ITSC), Maui, Hawai, USA, November 4-7, 2018.

[12] Yujie Dai, Jinzong Hu, Dongbin Zhao, Member and Fenghua Zhu; *Neural Network Based Online Traffic Signal Controller Design with Reinforcement Training*; 14th International IEEE Conference on Intelligent Transportation Systems, Washington, DC, USA, October 5-7, IEEE, 2011.