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Silvia Baeva, Vasil Shterev, Kristian Yochev, and Nikolay Hinov



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Route Optimization for Long Durability Battery Life During e-Bike Cycling

Silvia Baeva^{a)}, Vasil Shterev^{b)}, Kristian Yochev^{c)} and Nikolay Hinov^{d)}

Technical University of Sofia, Bulgaria

^{a)} sbaeva@tu-sofia.bg

^{b)} vas@tu-sofia.bg

^{c)} kris.yochev@gmail.com

^{d)} hinov@tu-sofia.bg

Abstract. The paper presents the creation of a general mathematical model of a two-step optimization problem which finds the most energy efficient route where the battery and the electric engine of an electric-bicycle are used to minimum. In the first step, a multi-criteria problem is solved which finds the most efficient mode of operation of the battery and the engine in every segment part of the complete route. To solve this task a scalarization method is used (also known as weighted sum method). In the second step, a common most efficient route is found by applying a network optimization model. To find the solution a method of dynamic optimization is used - Bellman's Principle. The major part of the paper is dedicated to the mathematical models, the approaches and the demonstration of the models' workability. For this reason, the generation of data is based on the a priori known information as well as on information which is periodic and stochastic in the time and space.

INTRODUCTION

In recent years, there is growing trend of electric and pedal-assisted bicycles usage as an environment-friendly and healthy alternative for urban mobility. Electric-powered cars are typically much more energy-efficient than fossil-fuelled fuels. The increasing use of electric vehicles, and in particular those powered by renewable energy sources, can play an important role in achieving the EU's goal of reducing greenhouse gas emissions and moving towards a low carbon future. There are many scenarios where the factors that connect to transport systems are optimized with the infrastructure in place - a separate department is brought out (e.g. a separate e-bike) that has the entire transport network with all created. Therefore, in order to optimize the energy consumed by the vehicle, the authors of [6] have also predicted the vehicle speed by using neural network (NN) and deep learning. After generation of the route checkpoints the input data is passed to an "ordinary" NN (usually with one or two hidden layers). After this the data is treated by a deep learning NN (with more than two hidden layers). At the NN output, the predicted speed along the route is obtained. After the initial analysis of the route are determined the stretches available for optimization. The overall efficiency gains by using the proposed optimization is around 4%. The result is further processed by a 5th order filter for noise reduction. A detailed study of the physical aspects of electrical - and pedal-assisted bicycles aiming to improve their interaction with the user is presented in [1]. The authors have concentrated their study on bicycles with rear-mounted electric motor, but recently other configurations have emerged. The influence and cross-dependencies between the subsystems are studied: battery, power electronic converter, electrical motor, mechanical system and the control system. The authors in [7] have described the development of a useful tool for agencies and researchers for clustering of similar transportation patterns with respect to time-based events. The proposed supervision algorithm is conceived to take advantage of background knowledge of the dataset along with the similarity. Compared to analogous methods, this one stands out with scalarization and low computational complexity along with its other advantages. The present study is focused on investigation of the risk during e-bike cycling. Unlike similar works [10] here under "risk" are claimed objects and events directly menacing the cyclist life and not indirect factors influencing the cyclist

health in the long-term. By using the presented method, two-stage problem is solved as an example. Random input data is used for the solved numerical example. For the purposes of this research, a network optimization model has been proposed to find the most energy efficient route in terms of several categories of ascent / descent, obstacles, road surface quality, weather conditions, etc.

The paper presents the creation of a general mathematical model of a two-step optimization problem which finds the most energy efficient route where the battery and the electric engine of an electric-bicycle are used to minimum. In the first step, a multi-criteria problem is solved which finds the most efficient mode of operation of the battery and the engine in every segment part of the complete route. To solve this problem a scalarization method is used (also known as weighted sum method). In the second step, a common most efficient route is found by applying a network optimization model. To find the solution a method of dynamic optimization is used - Bellman's Principle. The major part of the paper is dedicated to the mathematical models, the approaches and the demonstration of the models' workability. For this reason, the generation of data is based on the a priori known information as well as on information which is periodic and stochastic in the time and space. The similar problems are analysed and solved in [8] by using a multi-objective optimization approach. Various approaches to finding optimal routes by different criteria are described in [2], [3], [4], [5], [9], etc.

The purpose of this article is to present a general methodology for finding the optimal route in the sense of the most economical for the battery of an electric bicycle, depending on the factors - obstacles on the road, quality of the road surface, road levelling, weather conditions.

DESCRIPTION OF THE PROBLEM

1st stage

A cyclist is riding a pedal-assisted electric bicycle and is travelling from a certain point of departure to his destination. This can be performed via a number of routes including combination of their sections. The routes can be represented by a network model of an oriented graph $V(G, D)$, where $G = \{G_i\}_{i=1}^k$ are the nodes and $D = d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, are the graph arcs (figure 1).

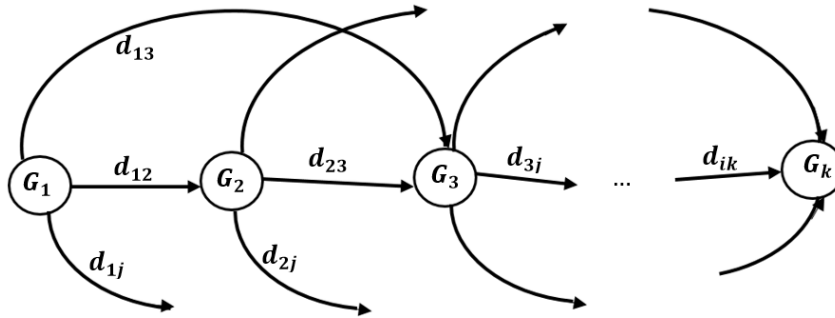


FIGURE 1. Bicycle routes allowed - Network model of the oriented graph $V(G, D)$

The factors such as the number of obstacles, the quality of road surface and weather conditions have a direct impact on the energy capacity of the e-bike battery. The addition to the problem is a factor directly linked to the energy capacity of the e-bike's battery, taking into account the slope and length of a given section of route. It shows the efficiency of using the battery's resource over the slope, distance, presence of sharp turns and other factors affecting the range of the e-bike.

Aim: To determine the probability of a risk of shortening the battery life $Q_{ij} \in [0; 1], i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, for each arc $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, of the possible routes of the cyclist.

2nd stage

Let the arcs of the graph $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, are associated with probability $p_{ij} = 1 - Q_{ij}$, $p_{ij} \in [0; 1], i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, for the absence of risk of shortening the battery life when the cyclist managing the e-bike for each arc $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, of the possible routes.

Aim: Find the route with the least consumption of battery resources.

SOLUTION OF THE PROBLEM

Input data and processing:

Input data for each arc $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, from the cyclist's route:

- number of obstacles;
- quality of the road surface;
- possible weather condition;
- intensity of using the battery.

Let $\omega_{ijr}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j; r = 1, \dots, R$, is combined weight that characterizes the risk of the factors: number of obstacles, quality of the road surface, possible weather condition for each arc of route $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$.

For each arc of route d_{ij} the factor „battery efficiency“ with weight $W_{ij} \in [0; 1], i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, is given and it shows the intensity of using the battery resource over distance, slope, presence of sharp turns and other factors affecting the range of the e-bike.

For a particular state of nature $r, r = 1, \dots, R$:

$$Q_{ijr} = \omega_{ijr} \cdot W_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j. \quad (1)$$

In the first stage, there is solved problem for finding the probability of the occurrence of risk for battery life, at risk being the gradual addition of the factors: obstacles - mobile and stationary; quality of road surface; weather conditions; intensity of using the battery in the different arcs that form the complete route. To solve the problems, an weighted sum method is used.

The probability for the absence of risk of shortening the battery life when the cyclist managing the e-bike for each arc $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, for a particular state of nature $r, r = 1, \dots, R$, is:

$$p_{ijr} = 1 - Q_{ijr}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j, r = 1, \dots, R. \quad (2)$$

A network model of the route is developed (figure 2) where each arc $d_{ij}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j$, is characterized by the probability $p_{ijr}, i = 1, \dots, k - 1; j = 2, \dots, k; i < j, r = 1, \dots, R$, for a particular state of nature $r, r = 1, \dots, R$.

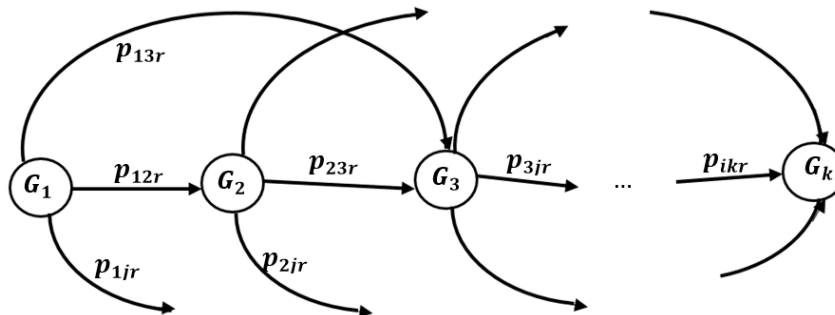


FIGURE 2. Probability network model of the possible routes

The problem of finding the route with the least consumption of battery resources from the starting point to the final destination is modelled as a network problem, but in fact it is also a reliability problem. This complex problem can be solved by a dynamic programming method by decomposing it into sub-problems which are easier to solve. The decomposition consists in dividing the solution into stages and formulation of optimization problems for each stage that are less complex than the global problem. For each stage there is a scalar (control) variable whose value can be optimized and then the results are linked by a recursive algorithm. Therefore, the solution of the global problem is obtained finally after consecutive solution of a number of sub-problems. This method, based on recursive iterations relies on the Bellman optimality principle.

Using the backwards Bellman's principle, the overall optimal route is found from the starting to the destination point where the battery most economical (figure 3) for a particular state of nature r , $r = 1, \dots, R$.



FIGURE 3. Optimal cycling path where the battery most economical

Then:

$$f_{max,r} = \prod_{v=1}^v p_{mvr} \tag{3}$$

Note. In this paper the so-called factor „battery efficiency“ is observed and it does not represent the amount of charge and discharge of the battery from riding a given route, which is an appropriate subject for future observations. This observation focuses on the depletion of the battery from a slope standpoint – intensity of drops and climbs (measure of angle from the horizon).

NUMERICAL EXAMPLE

In order to help the cyclist in the decision making, the above method is developed for finding an optimal route in respect to battery life of an e-bike during its use.

Figure 4 illustrates a testing route for the cyclist from the starting point to the destination with a possibility for choosing different sub-routes in eventual force maverick situations. There are a finite number of routes, each with its own sub-routes.

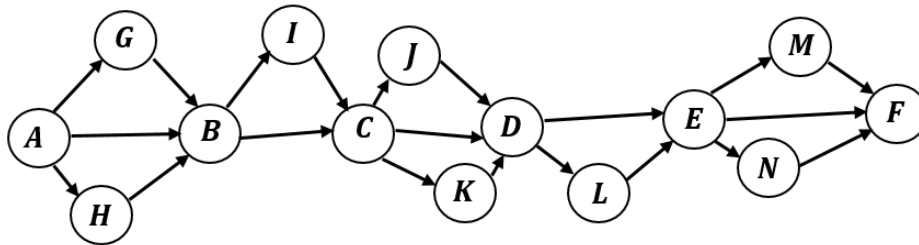


FIGURE 4. Tested routes

Input numerical data and processing:

After the chosen testing routes, there are entered data for the average number of obstacles in each of the sub-routes for ten days and then random data is generated in the interval [0 – 120] (table 1).

TABLE 1. Random assignment for the average number of obstacles and their relative frequencies on the different arc

	<i>average number</i>	<i>relative frequency</i>
<i>AGB</i>	1543	0.08744
<i>AB</i>	1481	0.08396
<i>AHB</i>	1495	0.08472
<i>BC</i>	1447	0.08199
<i>BIC</i>	1429	0.08099
<i>CJD</i>	1442	0.08173
<i>CD</i>	1452	0.08227
<i>CKD</i>	1428	0.08093
<i>DE</i>	1560	0.08840
<i>DLE</i>	1514	0.08579
<i>EMF</i>	1428	0.08090
<i>EF</i>	1407	0.07973
<i>ENF</i>	1449	0.08214
<i>Sum</i>	19075	1

The quality of the road surface for each arc is shown in table 2.

TABLE 2. Weights characterizing the quality of the road surface for each arc of road

<i>AGB</i>	<i>AB</i>	<i>AHB</i>	<i>BC</i>	<i>BIC</i>	<i>CJD</i>			
0.1756	0.75345	0.56482	0.6752	0.56892	0.27531			
<i>CD</i>	<i>CKD</i>	<i>DE</i>	<i>DLE</i>	<i>EMF</i>	<i>EF</i>	<i>ENF</i>		
0.87253	0.24567	0.67289	0.34958	0.75987	0.35746	0.68715		

The influence of the weather conditions on each arc of the road is shown in table 3.

TABLE 3. Weights characterizing the influence of the weather conditions for each arc of the road

	<i>AGB</i>	<i>AB</i>	<i>AHB</i>	<i>BC</i>	<i>BIC</i>	<i>CJD</i>	<i>CD</i>	<i>CKD</i>	<i>DE</i>	<i>DLE</i>	<i>EMF</i>	<i>EF</i>	<i>ENF</i>
<i>S₁</i>	0.8	0.5	1	0.5	0.2	0.1	0.5	0.9	0.6	0.2	0.9	0.8	0.3
<i>S₂</i>	0.8	0.4	0.2	0.1	0.5	0.2	0.1	0.1	0.9	1	0.1	0.4	0.7
<i>S₃</i>	1	0.7	0.9	0.3	0.4	0.4	0.8	1	0.1	0.2	0.2	0.2	0.8
<i>S₄</i>	0.4	0.8	0.2	0.7	1	1	0.3	0.6	1	0.9	0.5	0.7	0.6
<i>S₅</i>	0.5	0.1	0.5	0.2	0.5	0.5	0.3	0.6	0.1	0.6	0.4	0.4	0.8
<i>S₆</i>	1	0.6	0.5	0.4	0.9	0.1	0.4	1	0.2	0.9	0.7	0.4	0.5
<i>S₇</i>	0.2	0.3	0.5	0.4	0.8	0.7	1	0.3	0.8	0.9	0.6	0.5	0.8
<i>S₈</i>	0.5	0.9	1	0.9	0.2	0.4	0.4	0.9	0.6	0.8	0.3	1	0.2
<i>S₉</i>	0.2	0.9	0.2	0.3	0.5	0.2	1	0.2	0.1	0.9	0.4	0.7	0.6
<i>S₁₀</i>	0.7	0.9	0.9	0.1	1	0.1	0.5	0.7	0.5	0.9	0.7	0.1	0.5

For each arc of route d_{ij} the factor „battery efficiency“ with weight $W_{ij} \in [0; 1]$, $i = 1, \dots, k - 1$; $j = 2, \dots, k$; $i < j$, is given in table 4 and it shows the intensity of using the battery resource over distance, slope, presence of sharp turns and other factors affecting the range of the e-bike.

TABLE 4. The factor „battery efficiency“ with weight for each arc of route

<i>d_{ij}</i>	<i>AGB</i>	<i>AB</i>	<i>AHB</i>	<i>BC</i>	<i>BIC</i>	<i>CJD</i>	<i>CD</i>
<i>W_{ij}</i>	0.1100	0.1100	0.1000	0.1091	0.0857	0.4375	0.4375
<i>d_{ij}</i>	<i>CKD</i>	<i>DE</i>	<i>DLE</i>	<i>EMF</i>	<i>EF</i>	<i>ENF</i>	
<i>W_{ij}</i>	0.4375	0.4571	0.3765	0.1647	0.1647	0.1556	

The probability for the absence of risk of shortening the battery life when the cyclist managing the e-bike for each arc d_{ij} , $i = 1, \dots, k - 1$; $j = 2, \dots, k$; $i < j$, for each weather condition, is shown in table 5.

TABLE 5. The probability for the absence of risk of shortening the battery life when the cyclist managing the e-bike for each arc

	<i>AGB</i>	<i>AB</i>	<i>AHB</i>	<i>BC</i>	<i>BIC</i>	<i>CJD</i>	<i>CD</i>
<i>S</i> ₁	0.99929	0.99640	0.99592	0.99821	0.99972	0.99667	0.99241
<i>S</i> ₂	0.99929	0.99712	0.99918	0.99964	0.99929	0.99334	0.99848
<i>S</i> ₃	0.99911	0.99497	0.99632	0.99893	0.99943	0.98669	0.98785
<i>S</i> ₄	0.99964	0.99425	0.99918	0.99750	0.99859	0.96672	0.99544
<i>S</i> ₅	0.99955	0.99928	0.99796	0.99929	0.99929	0.98336	0.99544
<i>S</i> ₆	0.99911	0.99568	0.99796	0.99857	0.99873	0.99667	0.99392
<i>S</i> ₇	0.99982	0.99784	0.99796	0.99857	0.99887	0.97670	0.98481
<i>S</i> ₈	0.99955	0.99353	0.99592	0.99678	0.99972	0.98669	0.99392
<i>S</i> ₉	0.99982	0.99353	0.99918	0.99893	0.99929	0.99334	0.98481
<i>S</i> ₁₀	0.99938	0.99353	0.99632	0.99964	0.99859	0.99667	0.99241
	<i>CKD</i>	<i>DE</i>	<i>DLE</i>	<i>EMF</i>	<i>EF</i>	<i>ENF</i>	
<i>S</i> ₁	0.99329	0.98652	0.99549	0.99140	0.99349	0.99882	
<i>S</i> ₂	0.99926	0.97978	0.97747	0.99904	0.99675	0.99724	
<i>S</i> ₃	0.99254	0.99776	0.99549	0.99809	0.99837	0.99684	
<i>S</i> ₄	0.99552	0.97754	0.97972	0.99522	0.99430	0.99763	
<i>S</i> ₅	0.99552	0.99776	0.98648	0.99618	0.99675	0.99684	
<i>S</i> ₆	0.99254	0.99551	0.97972	0.99331	0.99675	0.99803	
<i>S</i> ₇	0.99776	0.98203	0.97972	0.99427	0.99593	0.99684	
<i>S</i> ₈	0.99329	0.98652	0.98197	0.99713	0.99186	0.99921	
<i>S</i> ₉	0.99851	0.99776	0.97972	0.99618	0.99430	0.99763	
<i>S</i> ₁₀	0.99478	0.98877	0.97972	0.99331	0.99919	0.99803	

The length and travel time of each arc of the e-bike road is shown in table 6.

TABLE 6. The length and travel time of each arc

<i>d</i> _{ij}	<i>AGB</i>	<i>AB</i>	<i>AHB</i>	<i>BC</i>	<i>BIC</i>	<i>CJD</i>	<i>CD</i>
Length (m)	1420	1350	1525	750	977	1045	964
Travel time (min)	5	4.7	5.3	2.7	2.5	3.5	3.2
<i>d</i> _{ij}	<i>CKD</i>	<i>DE</i>	<i>DLE</i>	<i>EMF</i>	<i>EF</i>	<i>ENF</i>	
Length (m)	1220	782	850	975	920	1020	
travel time (min)	4	2.8	2.7	2.9	3	3.4	

The numerical example is implemented and solved by using the software environments MatLab and Maple.

RESULTS

The solution of the risk optimization problem is obtained according to a multiplicative objective function and the result is a product of the sub-problem solutions. It reflects the reliability of a system constructed by a number of separate elements. The numerical solution of the problem is depicted in table 7, 8 and 9 by using this method.

TABLE 7. Optimal rout – the best battery efficiency for each weather condition

<i>S</i> ₁	<i>AGB</i>	<i>BC</i>	<i>CJD</i>	<i>DLE</i>	<i>ENF</i>
<i>S</i> ₂	<i>AGB</i>	<i>BIC</i>	<i>CKD</i>	<i>DE</i>	<i>EMF</i>
<i>S</i> ₃	<i>AGB</i>	<i>BIC</i>	<i>CD</i>	<i>DE</i>	<i>EF</i>
<i>S</i> ₄	<i>AGB</i>	<i>BIC</i>	<i>CKD</i>	<i>DLE</i>	<i>ENF</i>
<i>S</i> ₅	<i>AGB</i>	<i>BC</i>	<i>CKD</i>	<i>DE</i>	<i>ENF</i>
<i>S</i> ₆	<i>AGB</i>	<i>BIC</i>	<i>CJD</i>	<i>DE</i>	<i>ENF</i>
<i>S</i> ₇	<i>AGB</i>	<i>BIC</i>	<i>CKD</i>	<i>DE</i>	<i>ENF</i>
<i>S</i> ₈	<i>AGB</i>	<i>BIC</i>	<i>CD</i>	<i>DE</i>	<i>ENF</i>
<i>S</i> ₉	<i>AGB</i>	<i>BIC</i>	<i>CKD</i>	<i>DE</i>	<i>ENF</i>
<i>S</i> ₁₀	<i>AGB</i>	<i>BC</i>	<i>CJD</i>	<i>DE</i>	<i>ENF</i>

TABLE 8. The length and travel time of optimal rout for each weather condition

	<i>Length (m)</i>	<i>Travel time (min)</i>
<i>S₁</i>	5312	17.1
<i>S₂</i>	5147	17.4
<i>S₃</i>	5063	16.5
<i>S₄</i>	5487	17.6
<i>S₅</i>	5192	17.9
<i>S₆</i>	5244	17.2
<i>S₇</i>	5419	17.7
<i>S₈</i>	5163	16.9
<i>S₉</i>	5419	17.7
<i>S₁₀</i>	5017	17.4
<i>minimum</i>	4766	16

TABLE 9. The remaining charge in the battery on the optimal route in each weather condition

<i>S₁</i>	69.0%
<i>S₂</i>	67.7%
<i>S₃</i>	68.7%
<i>S₄</i>	67.1%
<i>S₅</i>	68.9%
<i>S₆</i>	68.8%
<i>S₇</i>	67.5%
<i>S₈</i>	67.9%
<i>S₉</i>	69.3%
<i>S₁₀</i>	68.3%

Note. It's important to note that in the beginning of each experiment the battery level has been 100%. It is assumed that all factors affecting the e-bike are equal except the ones that are described in the problem.

Discussion of the results

The problem is solved using an iterative method. Through the appropriate elements of the model, the complex problem is decomposed into simpler ones, which are solved almost independently. The solutions found at each stage are optimal and acceptable. This is due to the fact that the problem of the size of the problem is solved by being reduced in stages through the recurrent dependence. And this increases the capabilities of using this method to solve complex problems.

Regarding the results obtained by application of the Bellman's optimization principle, the following observations can be made:

- The shortest path from the numerical example has a length of 4766 m, but as the numerical example is shown, it does not benefit the battery life (figure 5). In the general case, not the shortest route is the most economical for battery operation, but it does not rule out a coincidence, which would be very good, but this is a very rare special case.

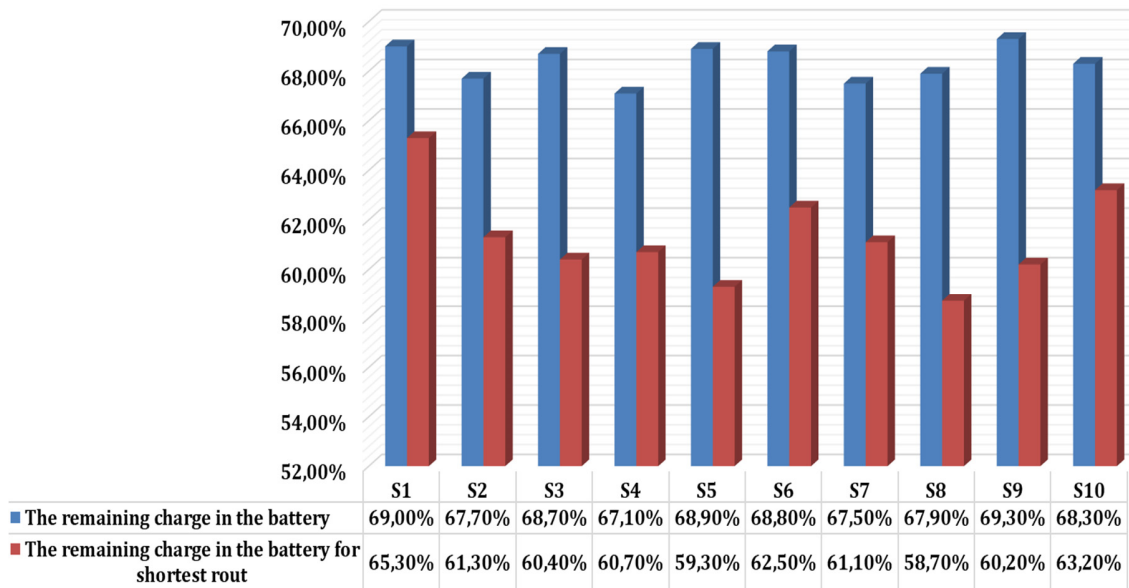


FIGURE 5. Comparative chart between the shortest and the least consumption of battery resources for each weather condition

- The duration of the fastest route is 16 min, but as the numerical example is shown, it does not benefit the battery life (figure 5). In general, not the fastest route is the most economical for battery operation, but it does not rule out a coincidence, which would be very good, but this is a very rare special case.

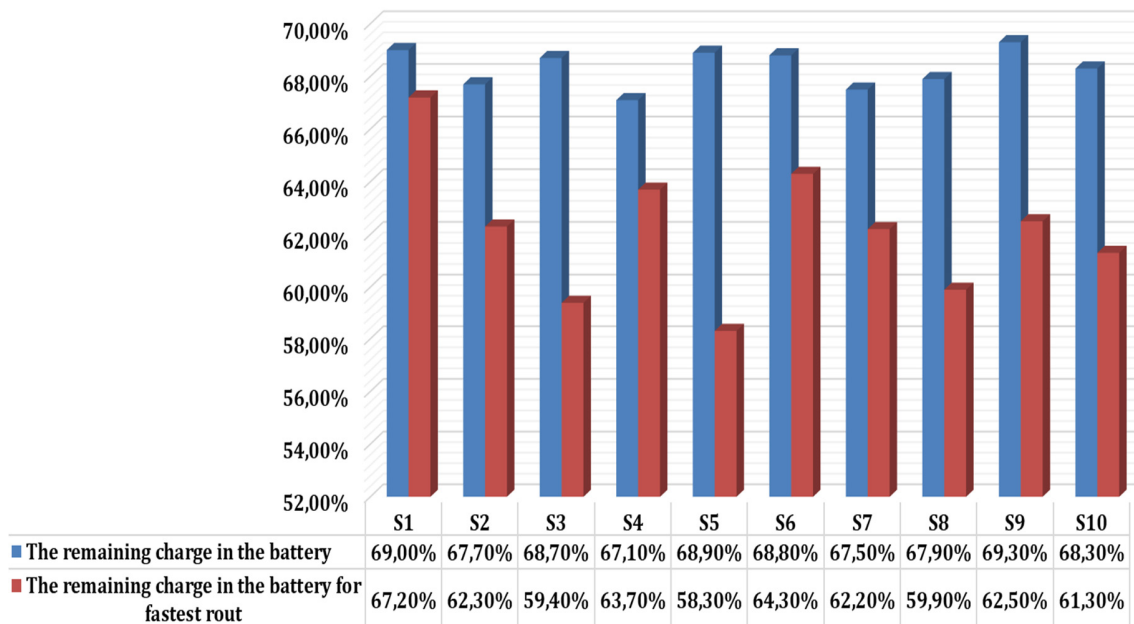


FIGURE 6. Comparative chart between the fastest and the least consumption of battery resources for each weather condition

From the results one can conclude that the selection of route affects the battery discharge rate significantly, which in a real situation would increase its life many times.

CONCLUSION AND FUTURE WORKS

The work proposes a methodology that specifies the parameters of the specific studies proposed, their boundaries, what is the necessary information to be collected and processed so that it is in favour of the cyclist to decide on which route to pass before he left.

Route optimization in this research is based on a dynamic programming approach, adding the factors that characterize the cycling of electric bikes and pedal bicycles in urban environments. Due to the significant urban traffic dynamics, the input data of the optimization problem are highly variable in the daily and hourly scales and are also influenced by weather conditions and other factors or events.

The aim has been to observe the effect of terrain slope and other risk factors, shown above, on the discharge of the battery per unit distance.

The next stages of observations will be focused on optimizing the energy needs of the e-bike, the battery and the route.

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