MULTIPLE REFERENCE FRAMES STATE SPACE MODELS OF BRUSHLESS DC MOTOR FOR FEEDBACK LINEARIZATION

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Abstract: Multiple reference frames orientated state space nonlinear mathematical models for brushless DC motors are derived in the paper using the power invariant dqo transformation in rotating and stationary form. The nonlinear state space models obtained are strictly orientated, decoupled, and linear with respect to the model parameters. This allows convenient application of advanced control theory differential geometric design approaches including multi-input multi-output feedback linearization and nonlinear adaptive systems design. Moreover the decoupling of the currents permits closed-loop nonlinear systems synthesis accomplishing simultaneous elimination of the torque pulsations and copper losses minimization. The nonlinear state space models derived are simulated to investigate their properties.

Key words: Nonlinear systems, Nonlinear transformations, Feedback linearization, Brushless DC motor, Power invariance.

INTRODUCTION

The interest in permanent-magnet motors during the last thirty years is due to the multiple advantages they offer. The brushless DC motors (BDCM) are part of this advantage class. They are the latest choice of researches due to their high efficiency, silent operation, compact size, high reliability and low maintenance requirements. The BDCM have trapezoidal back electromotive force and require rectangular-shaped stator phase currents to produce constant torque [16, 17]. The difference between the conventional brushed DC motor and the BDCM is that the latter has no brushes, commutator, and field winding. The excitation of these motors is achieved by strong rare-earth permanent magnets in the rotor providing very powerful field without the need of excitation current and no electrical losses in the rotor [6, 7]. As a result they are very efficient energetically while generating comparatively high torque. The lack of brushes leads to longer life span of the BDCM. These motors have maximal ratio between generated torque and motor inertia or total mass conditioned by the high power density of the BDCM. The BDCM models in original coordinates are commonly used in practice for analysis and design with various types of control methods [1, 2, 3, 10, 11, 15], because the coordinate transformations of these motors [4] do not reduce the order of the system as in the synchronous motor case. The advantage of the dqo transformation and its partial case the \(c/o\) transformation presented in this article is the decomposition into two virtual motors: a single-phase motor capable to provide pulsating torque only (the homopolar part) and a two-phase motor with rotating field (the dq part). The mathematical equation of the homopolar current \(i_{p}\), which does not exist in original coordinates, is very useful for the torque pulsation elimination. This is achieved by controlling the homopolar current \(i_{o}\), to converge asymptotically to zero. Another advantage is the easy synthesis of copper minimization control via the current \(i_{d}\).

The above mentioned peculiarities prove that the BDCM are highly nonlinear systems and the coefficients of their nonlinear mathematical models are time-variable and some of them are also unknown. These two real circumstances require the use of advanced nonlinear control methods for a high performance closed-loop control system design. Suitable approaches in this context are the Lie algebra multivariable feedback linearization based on differential geometry, adaptive feedback linearization, nonlinear observers synthesis, and adaptive nonlinear control design methods based on Lyapunov stability theory [5, 8, 9, 13, 12, 14]. The joint approach of nonlinear adaptive control design with nonlinear observers can yield the highest performance closed-loop systems implementation if some unsolved yet problems of this design approach are worked out in the near future.

The paper presents a derivation of multiple reference frames nonlinear state space mathematical models for BDCM, using the power invariant dqo transformation and its partial case the \(c/o\) transformation. The models derived are in strictly orientated state space form suitable for multivariable feedback linearization, nonlinear adaptive control and nonlinear observer design. The mathematical equivalence of the nonlinear models is confirmed by simulation.

STATE SPACE MODEL IN ORIGINAL COORDINATES

The derivation of this model is based on the assumptions that the induced by the harmonic fields of the stator currents in the rotor, the iron losses, and the leakage losses are neglected. The initial equation in matrix-vector form is

\[
V_{abc} = RI_{abc} + L\frac{di_{abc}}{dt} + E_{abc},
\]

(1)

\[
V_{abc} = [V_x, V_y, V_z]^T, \quad I_{abc} = [i_x, i_y, i_z]^T, \quad E_{abc} = [e_x, e_y, e_z]^T,
\]

are the vectors of the stator voltages, currents, and EMF respectively. The matrix of the stator resistances and the matrix of the inductances are

\[
R = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \quad L = \begin{bmatrix}
L & -M & 0 \\
M & -L & 0 \\
0 & 0 & L + M
\end{bmatrix}.
\]

(2)

The phase voltage equation is identical with the armature voltage equation of the DC motor. The resemblance to a DC machine and the lack of brushes and commutator are the reasons for which this machine is called permanent magnet brushless DC machine. The electromagnetic torque is

\[
T_e = \frac{e_xi_x + e_yi_y + e_zi_z}{\psi_{in}}.
\]

(3)
The instantaneous induced EMF can be written as
\[ e_a = J_a f_a(t), \quad e_b = J_b f_b(t), \quad e_c = J_c f_c(t) \]
(4)
where the functions \( f_{abc} = [f_a(t), f_b(t), f_c(t)] \) have the same trapezoidal shape as \( e_a, e_b, \) and \( e_c \) with a maximum magnitude \( 1. \) The induced EMF edges are rounded because the EMF are derivatives of the flux linkages, which are continuous functions. The fringing also makes the flux density functions smooth with no abrupt edges. The electromagnetic torque can be rewritten as
\[ T_e = J_p f_{abc} \]
The motion equation for system with inertia \( J, \) friction coefficient \( B, \) and load torque \( T_L \)
\[ \frac{d\omega_m}{dt} = \frac{T_e - T_L - B\omega_m}{J} \]
(5)
The electrical speed and position are related by
\[ \frac{d\Theta_e}{dt} = \frac{1}{J} \]
(6)
The transformed electromagnetic torque (5) is given by
\[ T_{dqo} = j_p f_{dqo} T_{dqo} + T_{dqo} - j_p f_{dqo} I_{dqo} \]
(7)
Taking into account \( T_{dqo}, \) the motion equation (6), the electrical dynamics (12) and choosing the state space vector
\[ x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [\Theta_e, \omega_e, i_d, i_q, i_o]^T \]
(8)
leads to the first state space model in \( dqo \) coordinates
\[ x_1 = \omega_e \]
\[ x_2 = \frac{J_p}{J} f_{dqo} x_3 + f_{dqo} x_4 + f_{dqo} x_5 x_6^1 - c_4 - c_2 x_2 \]
\[ x_3 = -c_4 x_3 - c_2 f_{dqo} x_7 x_8 + c_1 u_1 \]
\[ x_4 = -c_4 x_4 - c_2 f_{dqo} x_7 x_8 + c_1 u_2 \]
\[ x_5 = -c_4 x_5 - c_2 f_{dqo} x_7 x_8 + c_1 u_3 \]
with
\[ c_1 = \frac{P}{J}, \quad c_2 = \frac{J_p}{J}, \quad c_3 = J, \quad c_4 = \frac{T_L}{J}, \quad c_5 = B/J, \quad c_6 = R(J - M), \quad c_7 = \frac{1}{J}/MM, \quad c_8 = 1/(L - M). \]

STATE SPACE MODEL IN \( \alpha/\beta \) COORDINATES

The model of the BDCM in \( \alpha/\beta \) reference frames is derived by the \( \alpha/\beta \) transformation. The \( \alpha/\beta \) transformation is used when the rotating speed of the reference frames is different from zero. The straight and the inverse \( \alpha/\beta \) transformations are
\[ T_{dqo} = \begin{bmatrix} \cos(\alpha) & \cos(\beta) & 2\pi \sin(\alpha)/3 \\ -\sin(\beta) & \sin(\alpha) & 2\pi \sin(\beta)/3 \end{bmatrix} \]
\[ T_{dqo} = \begin{bmatrix} \cos(\beta) & \sin(\alpha) & \sqrt{2}/2 \\ -\sin(\alpha) & \cos(\beta) & \sqrt{2}/2 \end{bmatrix} \]
(11a)
\[ T_{\alpha/\beta} = \begin{bmatrix} \cos(\alpha) & \cos(\beta) & 2\pi \sin(\alpha)/3 \\ -\sin(\beta) & \sin(\alpha) & 2\pi \sin(\beta)/3 \end{bmatrix} \]
\[ T_{\alpha/\beta} = \begin{bmatrix} \cos(\beta) & \sin(\alpha) & \sqrt{2}/2 \\ -\sin(\alpha) & \cos(\beta) & \sqrt{2}/2 \end{bmatrix} \]
(11b)
for reference frames rotating with the electrical rotor speed \( \omega_e \) (rotor reference frames). This transformation is power invariant as \( T_{dqo} = [T_{dqo}] \) and it is applicable for signals with arbitrary shape. Applying the \( \alpha/\beta \) transformation for the matrix-vector equation (1) gives the matrix-vector equation of the BDCM electrical dynamics in rotating \( dqo \) coordinates written in terms of the currents
\[ \frac{dI_{dqo}}{dt} = -(L^T R + T_{dqo} \frac{dT_{dqo}}{dt} P_{dqo}) I_{dqo} + L^T (V_{dqo} - E_{dqo}) \]
(12)

The transformed electromagnetic torque (5) is given by
\[ T_{dqo} = j_p f_{dqo} T_{dqo} + T_{dqo} - j_p f_{dqo} I_{dqo} \]
(13)

STATE SPACE MODEL IN \( \alpha/\beta \) COORDINATES

The straight \( \alpha/\beta \) transformation and its inverse are a partial case of the \( \alpha/\beta \) transformation (11)
\[ T_{\alpha/\beta} = T_{\alpha/\beta}(0), \]
\[ T_{\alpha/\beta} = T_{\alpha/\beta}(0), \]
(16a)
(16b)
This transformation converts the model of the BDCM in new stationary coordinates and it is also power invariant as \( T_{\alpha/\beta} \). The transformation of the original BDCM model in \( \alpha/\beta \) coordinates is easier than in the \( dqo \) coordinates. Applying the transformation (16) via the rule
\[ x_{\alpha/\beta} = T_{\alpha/\beta} x_{dqo} \] to equation (1) leads to
\[ V_{\alpha/\beta} = RI_{\alpha/\beta} + L \frac{dI_{\alpha/\beta}}{dt} + E_{\alpha/\beta}. \]
(17)

The electromagnetic torque (5) in \( \alpha/\beta \) coordinates reads
\[ T_{\alpha/\beta} = j_p f_{\alpha/\beta} T_{\alpha/\beta} T_{\alpha/\beta} = j_p f_{\alpha/\beta} I_{\alpha/\beta}. \]
(18)
Considering (17), (18), the mechanical dynamics (6), and (7) with state space vector
\[ x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [\Theta_e, \omega_e, i_d, i_q, i_o]^T, \]
(19)
and control vector
\[ \mathbf{u} = [u_1, u_2, u_3]^T = [v_{\alpha}, v_{\beta}, v_0]^T \]  
(20)
gives the third state space model in \( \alpha\beta\theta \) coordinates
\[
\begin{align*}
\dot{x}_1 &= c_3x_2 \\
\dot{x}_2 &= c_4f_{\beta}(x_1)x_4 + f_{\beta}(x_1)x_2 + c_5u_1 \\
\dot{x}_3 &= -c_6x_3 - c_7f_\alpha(x_1)x_2 + c_8u_2 \\
\dot{x}_4 &= -c_9x_4 - c_7f_\alpha(x_1)x_2 + c_8u_2 \\
\dot{x}_5 &= -c_6x_5 - c_7f_\alpha(x_1)x_2 + c_8u_2 \\
\end{align*}
\]  
(21)
where \( c_1 = P, \quad c_2 = J_p, \quad c_3 = J, \quad c_4 = T_L / J, \quad c_5 = B / J, \quad c_6 = R / (L - M), \quad c_7 = \gamma_p / (L - M), \quad c_8 = 1 / (L - M). \) The \( \alpha\beta\theta \) transformation yields a simpler state space model than the \( \alpha\beta \) transformation which is an advantage. Both transformations are power invariant and decompose the machine into two virtual motors. The main advantage of these transformations is that they provide a separate mathematical equation of the homopolar current \( i_0 \), which does not exist in original coordinates, but it is useful for the torque pulsation elimination being an essential goal of the control for these motors. All this is possible because the \( \alpha\beta\theta \) current equations are decoupled as in original coordinates which allows independent control of the currents.

**SIMULATION AND SYSTEM TIME RESPONSES**

The three state space models derived are simulated as open loop systems from zero initial conditions and with passive load torque created by the friction included in the models. Figure 1 confirms the equivalence of the three state space models displaying the coincidence of the torques and the angular velocities which are outputs of the simulated systems. The evolution of the currents is shown on figures 2, 3, and 4 in original abc, rotating \( \alpha\beta \), and stationary \( \alpha\beta\theta \) coordinates respectively. The pulsations of the current \( i_0 \) create the torque pulsations seen on figure 1a. The trapezoidal electromotive forces of the BDCM in abc and \( \alpha\beta\theta \) coordinates can be seen on figures 5 and 6. The control input voltages evolve on figure 7 in abc coordinates.
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