Modeling of Series Resonant DC-DC Power Converters

Nikolay HINOV
Faculty of French Electrical Engineering Education
Technical University of Sofia,
Sofia, Bulgaria
E-mail: hinov@tu-sofia.bg

Bogdan GILEV
Faculty of Applied Mathematics and Informatics
Technical University of Sofia,
Sofia, Bulgaria
E-mail: b_gilev@tu-sofia.bg

Abstract—In the work are realized and presented models of the most commonly used series resonant DC-DC converters: a serial and a parallel loaded capacitor. Models are synthesized using differential equations and switching functions. In this way, it is convenient to formalize the power circuits in question and complex mathematical software does not have to be used to solve the system of differential equations. The obtained models allow to synthesize robust control of the converters and perform optimization on certain criteria. The proposed modeling approach is very suitable for use in power electronics training because it allows the description of various electronic energy converters where there is a switch between two or more states.

Keywords—dc-dc resonant converters; hybrid systems; modeling; series resonant

I. INTRODUCTION

Resonant DC power converters are used in a variety of industries and households: green energy, industrial technology, electrical transport, energy storage, and many other applications [1, 3, 4, 5]. Series power converters are more convenient for practical realization because they are powered by a voltage source, unlike the parallel ones [6, 7, 8, 9, 10]. The purpose of the work is to present mathematical models of two of the most commonly used topologies of series resonant circuits: a pure series and parallel loaded capacitor.

The power circuits of these two converters are shown in Figure 1 (series) and Figure 2 (s parallel loaded capacitor).

II. MODELING METHOD

The switching function method is used to modeling the converters [2, 3]. The complex system of differential, algebraic and logical equations describing the work of the power scheme is transformed into a generalized system. This greatly facilitates the modeling process and makes it suitable for the purposes of power training.

From the analysis of the principle of action of the power scheme, it was found that the converter is a system of a changing structure. In the primary circuit of the transformer, the structure changes twice within one period. From a mathematical point of view, this can be seen as a change of the power supply $U_d$ sign, which is modeled by appropriate multiplication by sign($i_2$).
III. MODELING METHOD AND REALIZATION

The mathematical model for the serial resonance converter of Figure 1 is based on the switching functions and the following:

\[
\begin{pmatrix}
L_1 + L & L_m \\
L_m & L_2
\end{pmatrix}
\begin{pmatrix}
di_1 \\
di_2
\end{pmatrix}
+ \begin{pmatrix}
R_1 & R_m \\
R_m & R_2
\end{pmatrix}
\begin{pmatrix}
i_1 \\
i_2
\end{pmatrix}
= \begin{pmatrix}
control(t) U_d - u_C \\
-\text{sign}(i_2) u_{CF}
\end{pmatrix}
\]

\[
C \frac{du_{CF}}{dt} = i_1
\]

\[
C_f \frac{du_{CF}}{dt} + \frac{u_{CF}}{R} = \text{sign}(i_2) i_2
\]

where

\[control(t) = \text{sign}(\sin(\frac{2\pi t}{T}))\],

- \(L_1\) - inductance of the primary winding of the transformer;
- \(L_2\) - inductance of the secondary winding of the transformer;
- \(L\) - resonant inductance;
- \(L_m = k\sqrt{L_1 L_2}\) - mutual inductance;
- \(k\) - magnetic coupling coefficient;

- \(R_1\) - resistance of the primary winding of the transformer;
- \(R_2\) - resistance of the secondary winding of the transformer;
- \(R_m = k\sqrt{R_1 R_2}\) - resistance of the magnetic connection;
- \(U_d\) - voltage of the DC power supply;
- \(C\) - resonator capacitor;
- \(C_f\) - output capacitor;
- \(R\) - load resistance;
- \(i_1\) - current through the primary winding of the transformer;
- \(i_2\) - current through the secondary winding of the transformer;
- \(u_{CF}\) - voltage across the resonant capacitor;
- \(u_{CF}\) - voltage on the filter capacitor.

With the help of the operator „sign“, the rectifier operation is modeled in the secondary winding of the transformer.

The presented model is implemented in the Matlab/Simulink programming environment. The realization of model of the serial resonant DC-DC converter is shown in Fig. 3.

Converter control is modeled using an Pulse Generator, as shown in Fig. 3. This paper explores two of the most popular power circuits of series converters shown in Figure 1. It is seen that the use of visual programming tools greatly facilitates the realization of the model. On the other hand, the resulting code

![Fig. 3. Serial resonant DC-DC converter model](image-url)
is not optimal in the sense of programming itself, but allows for work with products and people who do not have specific education and software developer competencies.

Similarly, were created the mathematical model and the other power circuit - that of the parallel-loaded capacitor converter.

From the analysis of the principle of action of the power scheme, it was found that the converter is a system of a changing structure. In the primary circuit of the transformer, the structure changes twice within a period, the change occurs in the change of the \( U_d \) sign. This process is modeled by multiplying the \( U_d \) with the function \( control(t) \), where:

\[
control(t) = \begin{cases} 
-1, & \text{for } 0 \leq t < T / 2 \\
1, & \text{for } T / 2 \leq t < T 
\end{cases}
\]

In the secondary circuit of the transformer, the structure changes four times over the period, and also the magnitude of the voltage \( u_2 \). The change of \( u_2 \) depends on the magnitude of the currents \( i_2 \) and \( i_{LT} \). Thus, the voltage model \( u_2 \) is modeled according to the following formula:

\[
u_2 = \begin{cases} 
-500(i_2 + i_{LT}), & \text{for } i_2 \geq i_{LT} \\
0, & \text{for } i_{LT} > i_2 > -i_{LT} \\
-500(i_2 - i_{LT}), & \text{for } -i_{LT} \geq i_2 
\end{cases}
\]

The following mathematical model system is finally obtained:

\[
\begin{pmatrix}
L_1 + L & L_m \\
L_m & L_2
\end{pmatrix} \begin{pmatrix}
\frac{di_1}{dt} \\
\frac{di_2}{dt}
\end{pmatrix} + \begin{pmatrix}
R_1 & R_m \\
R_m & R_2
\end{pmatrix} \begin{pmatrix}
i_1 \\
i_2
\end{pmatrix} = \begin{pmatrix}
u_c \\
u_c
\end{pmatrix}
\]

\[
\frac{di}{dt} = \frac{-u_c + U_c \cdot control(t)}{L}
\]

\[
\frac{du}{dt} = \frac{(i_3 + i)}{C}
\]

\[
L \frac{di_3}{dt} = \frac{(-u_2 - u_c)}{L_i}
\]

\[
\frac{du_c}{dt} = \frac{u_c + i_{LT}}{C_i}
\]

When compiling the model of the scheme, similar meanings have been used as in the first scheme under consideration. In addition, the following is inserted – \( i_{LT} \) - current through the filter inductance.

Based on the above equations in MATLAB/Simulink environment, the model shown in Fig. 4 is compiled. Using this model, the numerically obtained system is solved. In this way an intuitive and visual representation of the model is obtained.

Fig. 4. A serial resonant DC-DC converter with a parallel load capacitor model
by easily tracking the individual equations describing the state variables.

IV. RESULTS

Numerous simulations were implemented with the proposed models. The results of these are presented in Fig. 5 and Fig. 6. In the simulation of the sequential transducer of Fig. 1, the following input data are used: \( U_d = 195 \) V; \( R_1 = 0.21 \) \( \Omega \); \( L_1 = 280 \times 10^{-6} \) H; \( R_2 = 0.035 \) \( \Omega \); \( L_2 = 5 \times 10^{-6} \) H; \( R_m = 0.0849 \) \( \Omega \); \( L_m = 37.042 \times 10^{-6} \) H; \( L = 55 \times 10^{-6} \) H; \( C = 24 \times 10^{-9} \) F; \( R = 0.48 \) \( \Omega \); \( C_f = 100 \times 10^{-6} \) F; \( f = 135 \) kHz.

Fig. 5. Results from the simulation of the serial resonant DC-DC converter

The figure is represented successively from top to bottom as follows: current through resonant inductance, voltage across resonant capacitor and output voltage.

Similar studies were carried out with the resonant circuit model with a parallel loaded capacitor of Fig. 2. The following input data are used: \( U_d = 195 \) V; \( R_1 = 0.21 \) \( \Omega \); \( L_1 = 280 \times 10^{-6} \) H; \( R_2 = 0.035 \) \( \Omega \); \( L_2 = 5 \times 10^{-6} \) H; \( R_m = 0.0849 \) \( \Omega \); \( L_m = 37.042 \times 10^{-6} \) H; \( L = 55 \times 10^{-6} \) H; \( C = 24 \times 10^{-9} \) F; \( R = 0.48 \) \( \Omega \); \( C_f = 100 \times 10^{-6} \) F; \( f = 200 \) kHz.

Fig. 6. Results from the simulation of the serial resonant DC-DC converter with parallel loaded capacitor

V. CONCLUSION

In the work were proposed and realized in suitable program environment models of the most common serial resonance converters: a serial and parallel loaded capacitor. With their help optimization of the schemes under consideration could be realized according to a certain criterion or a given target function. In addition, they find application in power electronics training as well as in model-based design of power electronic devices.

ACKNOWLEDGMENT

The carried out research is realized in the frames of the project "Design of resonant DC-DC converters with guaranteed parameters", contract № 172ПД0029-03, Scientific and Research Sector at the Technical University of Sofia.

REFERENCES


