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The use of Finite Differences in the Time Domain numerical method for calculation of sound diffusion coefficient

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Abstract: *Sound scattering coefficients are very important and used in room acoustics task. The goal in this article is to investigate sound scattering phenomenon and especially sound scattering coefficients, the well-established as parameters in room acoustics design. The investigation has been conducted by using standard staggered grid leapfrog Finite Difference in the Time Domain (FDTD) numerical scheme in two dimensions. The shape under investigation is single plane Schroeder sound diffuser. The calculations of sound diffusion coefficients with the proposed method are simulated in Matlab developing a suitable algorithm for FDTD. The results from the simulations have been presented and compared against simulations using Boundary Element Method and measured data.*

Keywords: *FDTD; sound diffusion; Schroeder diffuser; simulation*

I. Introduction

Using computer algorithms for calculation of various acoustic parameters is nowadays common. Currently various implementations of such algorithms are used, such as *raytracing* and *image source* methods [1]. All those methods possess limitations and are mainly used for frequencies above several hundred Hertz.

This article is a continuation of previous work and describes a *MATLAB* developed calculation algorithm based on the Finite Differences in the Time Domain (FDTD) numerical scheme for calculation of acoustic scattering. Sound diffusion *d* coefficient have been calculated following the requirements of the ISO standard [2]. The results from the simulation have been compared against data of commercial *Quadratic Residue Diffuser (QRD)* as well as calculations by the *Boundary Element Method (BEM)* method.

II. Description of the proposed numerical method for calculation of sound diffusion coefficiens

A. The brief theory of FDTD method

A thorough description of the method used has been presented in [3]; here only a brief introduction has been considered. As a starting point for acoustic modelling, by the use of FDTD, is the discretization of the two fundamental equations – the *Euler equation*:

$$\Delta p = -\rho_0 \frac{\partial u}{\partial t} \tag{1}$$

and *Continuity equation*:

$$\Delta u = -\frac{1}{c^2 \rho_0} \frac{\partial p}{\partial t} \tag{2}$$

Here *p* denotes sound pressure, *u* denotes particle velocity vector, ρ is the propagation media density, and $k = \rho_0 c^2$ is the compressibility of the medium. Because of the interdependence of pressure and velocity a staggered grid must be used, as initially described in [4]. In this arrangement the discretisation in space is achieved by calculating both pressure and velocity in fixed positions, each spaced one step Δx for the *x* direction and Δy for the *y* direction apart. Pressure and velocity are spaced ½ spatial step apart and are calculated on each time iteration.

In the two dimensional case, which is of concern in this article, we need to define three grids – one for the pressure derivative, one for *x* and one for *y* direction of the velocity vector. Using a fixed discrete time interval Δt the resulting grid update equations are as follows, [3]:

$$u_x^{n+1/2}(i+1/2, j) = u_x^{n+1/2}(i+1/2, j) - \frac{\Delta t}{\rho}\left(\frac{p^n(i+1, j) - p^n(i, j)}{\Delta x}\right), \quad (3)$$

$$u_x^{n+1/2}(i+1/2, j) = u_y^{n+1/2}(i+1/2, j) - \frac{\Delta t}{\rho}\left(\frac{p^n(i+1, j) - p^n(i, j)}{\Delta y}\right), \quad (4)$$

$$p^n(i, j) = p^{n-1}(i, j) - \frac{1}{k}\Delta t\left(\frac{u_x^{n-1/2}(i+1/2\, j) - u_x^{n-1/2}(i-1/2\, j)}{\Delta x} + \frac{u_y^{n-1/2}(i+1/2\, j) - u_y^{n-1/2}(i-1/2\, j)}{\Delta y}\right). \quad (5)$$

In equations 3 to 5 *n* denotes the time step and *i* and *j* are spatial indexes. The notation *n+½* is deliberate to the velocity calculation and corresponds to he staggered spatial grid. Equation 5 shows that the pressure is actually split in two parts for *x* and *y* directions, which of course has no physical meaning. The sum of both parts corresponds to the actual pressure value. In order to guarantee numerical stability for the discretised mesh, the relation between time and spatial step should comply with the so called Courant number, described in [5].

B. Definition of Freefield conditions

In order to emulate free field conditions either very big discretisation domain should be used, or a proper absorbing boundary should be implemented. *Perfectly Matched Layer (PML)* is the most widely used and accepted boundary conditions used for wave truncation. This boundary condition forms a medium with impedance equal to the homogeneous lossless sound propagation media implemented in the discretised domain, but with significant absorption properties. In order to include this absorption in the update algorithm, two absorption coefficients are added to equations (1) and (2), which are then written as

$$\Delta p = -\rho_0 \frac{\partial u}{\partial t} - \alpha' p \quad (7)$$

$$\Delta u = -\frac{1}{k}\frac{\partial p}{\partial t} - \alpha p \quad (8)$$

Here α is the usual compressibility absorption coefficient and α' is density absorption coefficient, which is analogous to mass-proportional damping in mechanics. When condition (9) is fulfilled, there will be no reflection back when a plane wave is travelling from lossless medium to PML.

$$\alpha' = \alpha \rho k \quad (9)$$

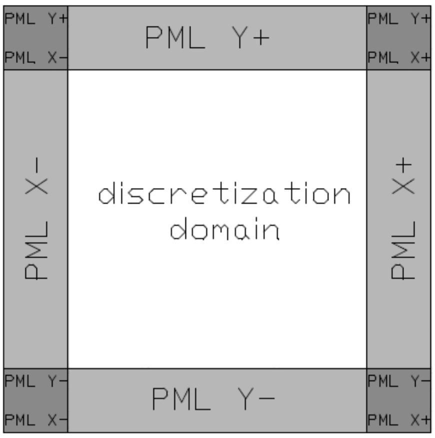
The idea behind this boundary conditions is to define a set of non-physical equations by using the assumption that α is anisotropic and split in two parts for p_x, p_y. While the velocity is a vector quantity, splitting the pressure in two components with *x* and *y* directions has no physical meaning. When the two absorption coefficients are equal, the physical pressure is equal the sum of the two components.

In regions where α coefficients are zero there is no absorption for the propagating sound wave. Figure 1 shows the PML regions around the discretization domain. Both absorption coefficients have been applied for the update equations in the two directions x and y in a region of 10 discretization points. For the corner regions absorption is present for both directions.

C. Simulated diffuser shape

A well-known and widely investigated shape has been chosen (Figure 1) for the purposes of calculation validation – the QRD or Schroeder diffuser shape. The motivation behind this choice is that this is essentially a two dimensional shape, i.e. scatters the sound in a hemi cylinder way. The exact shape and dimensions has been discussed in [6].

Figure 1 Perfectly matched layer setup with discretization domain



D. The way of freefield sound diffusion coefficient estimation

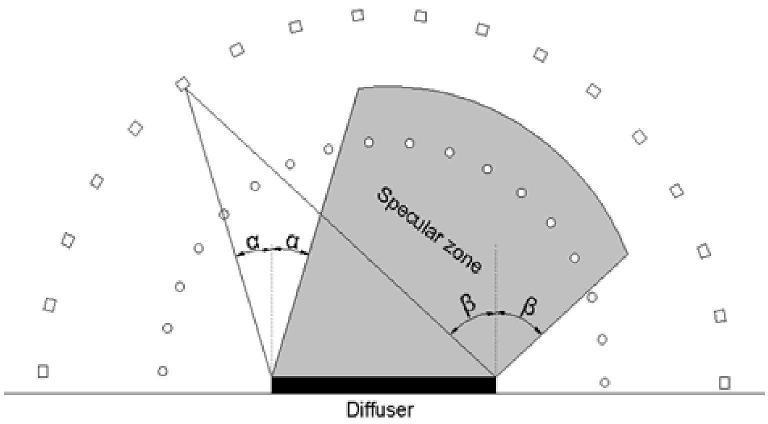
For calculating the diffusion coefficient, [2] calls for free field measurement; thus the need for emulating those conditions. The free field condition is said to be fulfilled in a medium with no reflection at a distance where the scattered sound pressure level decreases with slope of 6 dB per doubling the distance. In case of single plane diffuser in two dimensional space the scattering surface can be regarded as infinite in z direction, that is the third dimension, and thus acting as ideal line source instead of point source; the decrease in sound pressure is then 3 dB per doubling the distance in the far field. If an observation point is closer than this distance, interference effects will cause severe constructive and destructive interference. If the source is considered far away from the diffuser, than the receivers are in the far field if, [5]:

- the distance from receiver to diffuser is large compared to wavelength;
- the distance between path lengths from points on the surface to the receiver is small compared to the wavelength.

The later one is problematic in real world applications and frequencies down to 100 Hz.

The fine structure of a reflection polar plot is not of practical concern, because of the details are reduced at the end to a single number coefficient. Additionally, as also ISO standard requires, the results shall be presented in ⅓ octave frequency bands averages; this will decrease the frequency resolution. Both arguments serve in favour of a practical approach for determining the far field distance. ISO standard defines so called specular zone; this concept is shown in the next Figure 2. It is defined as the lines crossing the receivers' positions arc and the reflections from both ends of the sample. Practical recommendation set by the same standard, but also by [7], is that the distance from source to diffuser and from diffuser to receivers, which is defined by receiver arc radius, should be such as 80% of the receiver positions should be out of the specular zone.

Figure 2 Specular zone definition for calculating freefield sound diffusion coefficient; □ – sources positions; ○ – receiver positions



III. The results from Matlab simulation algorithm for the proposed method of sound diffusion coefficients calculation

The proposed method for calculation of sound diffusion coefficients is subject of Matlab simulations in this article developing a suitable algorithm for FDTD. Simulations have been performed in a discrete domain with 570 discretisation points in each direction. Sound diffuser sample has been located on the bottom end of discrete domain. A sound wave is emitted from a point source with pressure according to the description found in [3]. In order to calculate random incidence sound diffusion coefficients, the sound source location is varied on a semi-arc with angle increment of 10 degree. Pressure is recorded each time iteration in 180 points on a semicircle with radius corresponding to 1,5 metres. In order to ensure sufficient frequency resolution down to 100 Hz third-octave band, 4096 time step iterations has been performed for each source position; this results in 19.7 Hz frequency resolution. For each source angle calculation the pressure has been recorded without reflective surface present, with a diffuser and with a reference plane surface. Later the incident sound wave has been subtracted from the reflected and thus acquiring "clean" response with no artefacts from the incident wave.

Calculation of random incidence value of the coefficients has been made by the use of Paris's formula, described for example in [5]. For the calculation of the diffusion coefficient a normalization with respect to the reference flat surface is necessary.

Figure 3 shows the angle dependent and random incidence diffusion coefficient for the diffuser, and the next Figure 4 shows angle dependent and random incidence diffusion coefficient calculated for reference plane surface. It looks like the 45 degree angle is the dominant for calculating the random incidence value.

Figure 3 Calculated angle dependant and random incidence diffusion coefficient of QRD sample

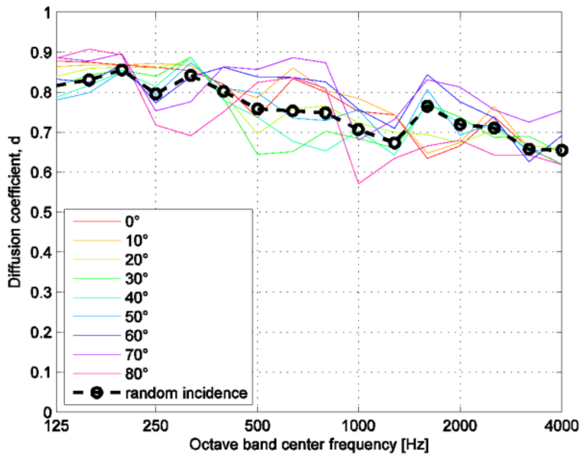


Figure 4 Calculated diffusion coefficients for QRD and reference surface, as well as diffusion coefficient after normalization

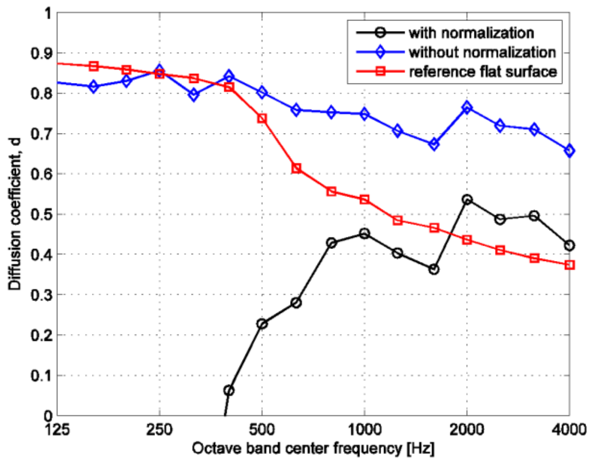
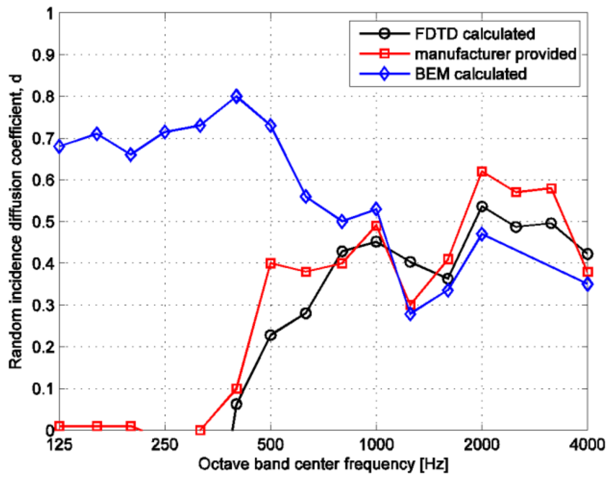


Figure 5 presents comparison between the values of diffusion coefficient calculated by FDTD, BEM and measured data for the commercial diffuser product. Data for the BEM calculated values has been taken from [8], and the graph is showing high values of the diffusion coefficient for low frequencies, most probably because no normalization has been applied after calculation. Also the data points on the region 2 kHz to 4 kHz are not calculated, leading to the loss of detail in this range.

Figure 5 Random incidence diffusion coefficient calculated for QRD sample by FDTD and BEM methods in comparison with commercial product measured data



IV. Conclusion

The presented simulation in Matlab results of sound scattering coefficients calculation with the proposed method and suitable algorithm for FDTD can be summarized as follow:
- the analysis and comparison of angle dependent and random incidence diffusion coefficient, calculated for the diffuser (Figure 3), and angle dependent and random incidence diffusion coefficient, calculated for reference plane surface (Figure 4) lead to conclusion, that it looks like the 45 degree angle is the dominant for calculating the random incidence value;
- the comparison in Figure 5, between the values of diffusion coefficient calculated by FDTD and data for the BEM calculated values taken from [8] shown high values of the diffusion coefficient for low frequencies, most probably because no normalization has been applied after calculation and also the data points on the region 2 kHz to 4 kHz are not calculated, leading to the loss of detail in this range.

Continuing the research from previous papers, it was possible to calculate the random incidence sound diffusion coefficient. In simulation with considering the sample as totally reflective, there is reasonable agreement between simulated and measured data.

This can be used for practical assessment of sound diffusers and further contribute to the sound quality in rooms with high demands for music and speech reproduction.

Future work will include simulation of room acoustic parameters with sound diffusive shapes and optimization of the room acoustic parameters. The use of parallel computation algorithms shall be also experimented with. This method could also be used for auralization of the simulated rooms.

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VI. Acknowledgments

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