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MODAL PROPERTIES OF DRIVE TRAIN IN HORIZONTAL-AXIS WIND TURBINE

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Abstract: The staring point in the research of the vibrations is the finding of natural frequencies and mode shapes, which indicate the frequency range of interest. A dynamic multibody model for determination of the torsional vibrations of a wind turbine drive train is presented in this paper. The model of a wind turbine consists of a rotor with rigid blades, elastic shafts, a drive train and a generator. The drive train has a gearbox with three gear stages. The gear stages include two high-speed stages (helical gear pairs) and a low-speed planetary gear stage (three identical planets with spur teeth, sun and a fixed ring wheel). The model consists of 10 bodies and has 11 degrees of freedom. The model takes into account the stiffness of the engaged tooth pairs and shafts. In this model the aerodynamic and generator torques are applied as external loads. Computer simulation is performed by MATLAB. The natural frequencies and vibration modes are obtained for an industrial wind turbine.

Keywords - Wind turbine, drive train, torsional vibrations, modal analysis.

1. Introduction

The wind energy application has been growing rapidly for the last few years. In the last ten years the global installed capacity of wind energy has increased 20 times. This trend is expected to continue in Europe. The increase in the rotor and hence size of the turbine leads to a complicated design of the drive train in the wind turbine beside higher requirements of turbine reliability.

Design calculations for a wind turbine base on simulation of mechanical loads on the turbine components caused by external forces. The external forces are the wind, the electricity grid and sea waves for offshore applications.

The multi-body simulation techniques are used to analyze the loads on internal components of drive trains. The simplest model with one degree of freedom (DOF) for each drive train component is used to investigate only torsional vibrations in the drive train. In this model all bodies have one DOF, i.e. the rotation around their axis of symmetry. Therefore, the coupling of two bodies involves 2 DOF's. Gear contact forces between two wheels are modelled with a linear spring acting in the plane of action along the contact line (normal to the tooth surface), [17, 21]. More complex model with 6 DOF's for each drive train component is used for investigation of the influence of bearing stiffness on the internal dynamics of the drive train. All drive train components are treated as rigid bodies. The linkages in the multi-body model, representing the bearing and tooth flexibilities, have 12 DOF's, [38]. Finally, it is used a flexible model in which the drive train components are modelled as finite element models instead of rigid bodies, [2, 4]. This model adds a possibility of calculating stress and deformation in the drive train components in some time. Any addition to the model leads to additional information about dynamics of the drive train but makes the modelling and the simulation more complicated.

The modern wind turbines have a planetary gearbox. Studies on the vibrations in a planetary gear system have been done in [2, 10, 18, 19, 20, 23, 34]. The tooth meshes are modelled as a linear spring with stiffness which is a time function. For this reason the vibration equations of a planetary gear system are differential equations with periodic coefficients, [2, 5, 18, 19 20, 23]. References [6, 7, 11, 12, 14, 19, 23] investigate the vibrations of compound planetary gears.

The applications of these modelling techniques on different drive trains of wind turbines are presented in [13, 15, 16, 24-27, 28, 29, 31-33]. References [35, 36] present the numerical investigations for the given wind turbine in this paper, where the meshes stiffness are modelled as constant springs. In this case the differential equations, which describe the torsional vibrations of the wind turbine, have

constant coefficients. Reference [37] presents a dynamical model of an wind turbine, but the meshes stiffness are modelled as a time function.
This paper is based on the work [37]. The Lagrange's equations are used to obtain the equations of the torsional vibrations of the wind turbine, [3, 8, 22, 30, 40]. Computational modal analysis of an example wind turbine is presented.

2. Dynamic Model Of Wind Turbine

The wind turbine consists of a rotor, a drive train and a generator (Fig.1). The drive train has a gearbox with three stages. The gear stages include two high-speed parallel gear stages (helical gear pairs) and a low-speed planetary gear stage (three identical planets with spur teeth, sun and fixed ring wheel) (Fig.2).

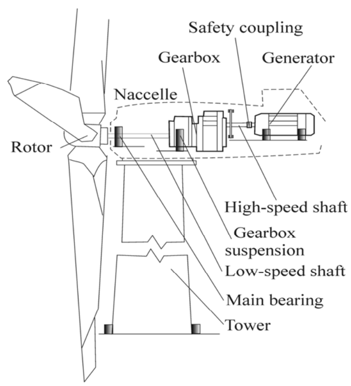

Figure 1: Schematic sketch of wind turbine

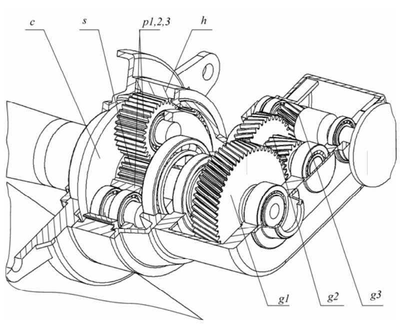

Figure 2: Sketch of gearbox: h-hull, c-carrier, p1,2,3-planets, s-sun, g1,2,3-gears

The dynamic multi-body model is shown in Fig.3. It consists of a rotor with 3 rigid blades, a low-speed elastic shaft, a gearbox with 3 gear stages, a high-speed elastic shaft and a generator rotor. Thus, the model consists of 10 bodies and 11 DOF's

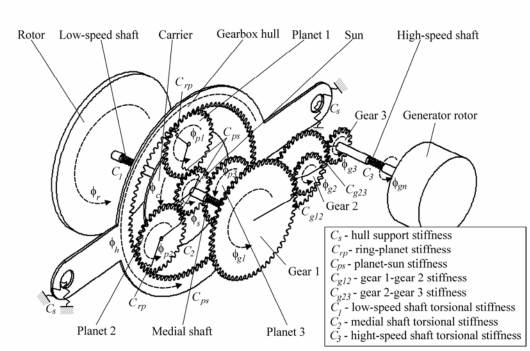

Figure 3. Dynamical model of wind turbine

The gear contact forces between wheels are modeled by linear spring acting in the plane of action along the contact line (normal to the tooth surface), [21, 41]. The stiffness gear is defined as a normal distributed tooth force in a normal plane causing the deformation of one or more engaging tooth pairs, over a distance of 1 μm, normal to a envolvent profile in a normal plane, [9]. This deformation results from the bending of the teeth in contact between the two gear wheels, the one of which is fixed and the other is loaded. The stiffness varies in the time and can be expressed in a time Fourier series form, [1, 18-20, 34, 37].
Damping and friction forces are not included. These assumptions are valid for heavily to moderately loaded gears that are correctly for a large wind turbine, [17, 21].
It is also accepted that the masses of the planets are identical.
The vector of the generalized co-ordinates is

$$\{q\} = \begin{bmatrix}\phi_h & \phi_c & \phi_r & \phi_{p1} & \phi_{p2} & \phi_{p3} & \phi_s & \phi_{g1} & \phi_{g2} & \phi_{g3} & \phi_{gn}\end{bmatrix}^T \quad (1)$$

where ϕ_i (i=*h,c,r,p1,p2,p3,s,g1,g2,g3,gn*) are the rotational angles of the ring (gearbox hull), carrier, rotor (hub), planet 1, planet 2, planet 3, sun, gear 1, gear 2, gear 3 and the generator rotor (Fig. 3).

The differential equations, describing the torsional vibrations of the wind turbine, are

$$[M]\{\ddot{q}\} + [C(t) - \omega^2 C_\omega]\{q\} = \{T\} \quad (2)$$

where *M* is the inertia matrix and *C* is the stiffness matrix. The matrix C_ω results from the carrier rotation.

The vector of the external forces, caused by the wind and the electricity grid, is

$$\{T\} = \begin{bmatrix}0 & 0 & T_{aero} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_{gen}\end{bmatrix}^T \quad (3)$$

where T_{aero} and T_{gen} are the aerodynamic and electromagnetic torques.

The non-zero numbers of inertia matrix M are

$$m_{1,1} = J_h + J_c + m_c l_{cr}^2 + J_r + m_r l_{cr}^2 + 3J_p + 3m_p l_{cr}^2 + 3m_p r_c^2 + J_s + m_s l_{cr}^2 + I_{g1} + m_{g1} l_{g12}^2 + $$
$$+ I_{g2} + m_{g2} l_{g22}^2 + I_{g3} + m_{g3} l_{g23}^2;$$

$m_{1,2} = m_{2,1} = J_c + J_r + 3J_p + 3m_p r_c^2$; $m_{1,3} = m_{3,1} = m_{2,3} = m_{3,2} = m_{3,3} = J_r$;

$m_{1,4} = m_{4,1} = m_{1,5} = m_{1,6} = m_{6,1} = m_{5,1} = m_{2,4} = m_{4,2} = m_{2,5} = m_{5,2} = m_{2,6} = m_{6,2} = m_{4,4} = m_{5,5} = m_{6,6} = I_p$;

$m_{1,7} = m_{7,1} = m_{7,7} = I_s$; $m_{1,8} = m_{8,1} = m_{8,8} = I_{g1}$; $m_{1,9} = m_{9,1} = m_{9,9} = I_{g2}$; $m_{1,10} = m_{10,1} = m_{10,10} = I_{g3}$;

$m_{2,2} = J_c + J_r + 3J_p + 3m_p r_c^2$; $m_{11,11} = I_{gn}$.

The non-zero numbers of stiffness matrix C are

$$c_{1,1} = C_{s1}\left(l_{s1}^2 + l_{s2}^2\right) + C_{rp}(t)\left(3r_R^2 + 3r_c^2 + 3r_p^2 - 6r_R r_c\right) + C_{ps}(t)\left(3r_c^2 + 3r_p^2 + 3r_s^2 - 6r_c r_s \cos\alpha\right) +$$
$$+ C_{g12}(t)\left(l_{g12} - l_{g22}\right)^2 + C_{g23}(t)\left(l_{g22} - l_{g23}\right)^2;$$

$$c_{1,2} = c_{2,1} = C_{rp}(t)\left(3r_c^2 + 3r_p^2 - 3r_R r_c\right) + C_{ps}(t)\left(3r_c^2 + 3r_p^2 - 3r_c r_s \cos\alpha\right);$$

$$c_{1,4} = c_{4,1} = C_{rp}(t)\left[r_p^2 - r_R r_p \cos(\omega t + \alpha) + r_c r_p \cos(\omega t + \alpha)\right] + C_{ps}(t)\left[r_p^2 - r_c r_p \cos(\omega t - \alpha) + r_s r_p \cos\omega t\right];$$

$$c_{1,5} = c_{5,1} = C_{rp}(t)\left[r_p^2 + r_R r_p \cos(60 - \omega t - \alpha) - r_c r_p \cos(60 - \omega t - \alpha)\right] +$$
$$+ C_{ps}(t)\left[r_p^2 + r_c r_p \cos(60 - \omega t + \alpha) - r_s r_p \cos(60 - \omega t)\right];$$

$$c_{1,6} = c_{6,1} = C_{rp}(t)\left[r_p^2 + r_R r_p \cos(60 + \omega t + \alpha) - r_c r_p \cos(60 + \omega t + \alpha)\right] +$$
$$+ C_{ps}(t)\left[r_p^2 + r_c r_p \cos(60 + \omega t - \alpha) - r_s r_p \cos(60 + \omega t)\right];$$

$c_{1,7} = c_{7,1} = C_{ps}(t)\left(3r_s^2 - r_c r_s \cos\alpha\right)$; $c_{1,8} = c_{8,1} = C_{g12}(t) r_{g1}\left(l_{g12} - l_{g22}\right)\cos\alpha\cos\beta$;

$c_{1,9} = c_{9,1} = C_{g12}(t) r_{g21}\left(l_{g12} - l_{g22}\right)\cos\alpha\cos\beta + C_{g23}(t) r_{g22}\left(l_{g22} - l_{g23}\right)\cos\alpha\cos\beta$;

$c_{1,10} = c_{10,1} = C_{g23}(t) r_{g3}\left(l_{g22} - l_{g23}\right)\cos\alpha\cos\beta$; $c_{2,2} = C_{rr}(t)\left(3r_c^2 + 3r_p^2\right) + C_{ps}(t)\left(3r_c^2 + 3r_p^2\right)$;

$c_{2,3} = -c_{3,3} = -C_1$; $c_{2,4} = c_{4,2} = C_{rp}(t)\left[r_p^2 + r_c r_p \cos(\omega t + \alpha)\right] + C_{ps}(t)\left[r_p^2 - r_c r_p \cos(\omega t - \alpha)\right]$;

$c_{2,5} = c_{5,2} = C_{rp}(t)\left[r_p^2 - r_c r_p \cos(60 - \omega t - \alpha)\right] + C_{ps}(t)\left[r_p^2 + r_c r_p \cos(60 - \omega t + \alpha)\right]$;

$c_{2,6} = c_{6,2} = C_{rp}(t)\left[r_p^2 - r_c r_p \cos(60 + \omega t + \alpha)\right] + C_{ps}(t)\left[r_p^2 + r_c r_p \cos(60 + \omega t - \alpha)\right]$;

$c_{2,7} = c_{7,2} = -3C_{ps}(t) r_c r_s \cos\alpha$; $c_{4,4} = c_{5,5} = c_{6,6} = C_{rp}(t) r_p^2 + C_{ps}(t) r_p^2$; $c_{4,7} = c_{7,4} = C_{ps}(t) r_s r_p \cos\omega t$;

$c_{5,7} = c_{7,5} = -C_{ps}(t) r_s r_p \cos(60 - \omega t)$; $c_{6,7} = c_{7,6} = -C_{ps}(t) r_s r_p \cos(60 + \omega t)$; $c_{7,7} = C_2 + 3C_{ps}(t) r_s^2$;

$c_{7,8} = -C_2$; $c_{8,8} = C_2 + C_{g12}(t) r_{g1}^2\left(\cos^2\alpha\cos^2\beta + \sin^2\alpha\right)$; $c_{8,9} = C_{g12}(t) r_{g21}^2\left(\cos^2\alpha\cos^2\beta + \sin^2\alpha\right)$;

$c_{9,9} = \left(C_{g12}(t) r_{g21}^2 + C_{g23}(t) r_{g22}^2\right)\left(\cos^2\alpha\cos^2\beta + \sin^2\alpha\right)$; $c_{9,10} = C_{g23}(t) r_{g22} r_{g3}\left(\cos^2\alpha\cos^2\beta + \sin^2\alpha\right)$;

$c_{10,10} = C_3 + C_{g23}(t) r_{g3}^2\left(\cos^2\alpha\cos^2\beta + \sin^2\alpha\right)$; $c_{10,11} = -c_{11,11} = -C_3$.

The non-zero number of C_ω is

$$c_{\omega_{2,2}} = 3m_p r_c^2.$$

3. Numerical Results

The rotor, drive train and generator characteristics of an example turbine can be seen in Ref. [37]. To determinate the natural frequencies and mode shapes the time invariant system is considered. All mesh stiffnesses are considered to be constant and equal to their average stiffness over one mesh cycle. All externally apllied forces are assumed to be zero.

The natural frequencies and modes shapes are obtained by Equation (2)

$$\left([M]^{-1}[C] - \lambda[E]\right)\{q\} = 0 \tag{4}$$

All calculations are accomplished using the codes of MATLAB. The natural frequencies in Hz are **728 387 288 327 324 318 219 178 62 0 2.5**

The mode shapes are shown in Fig.4

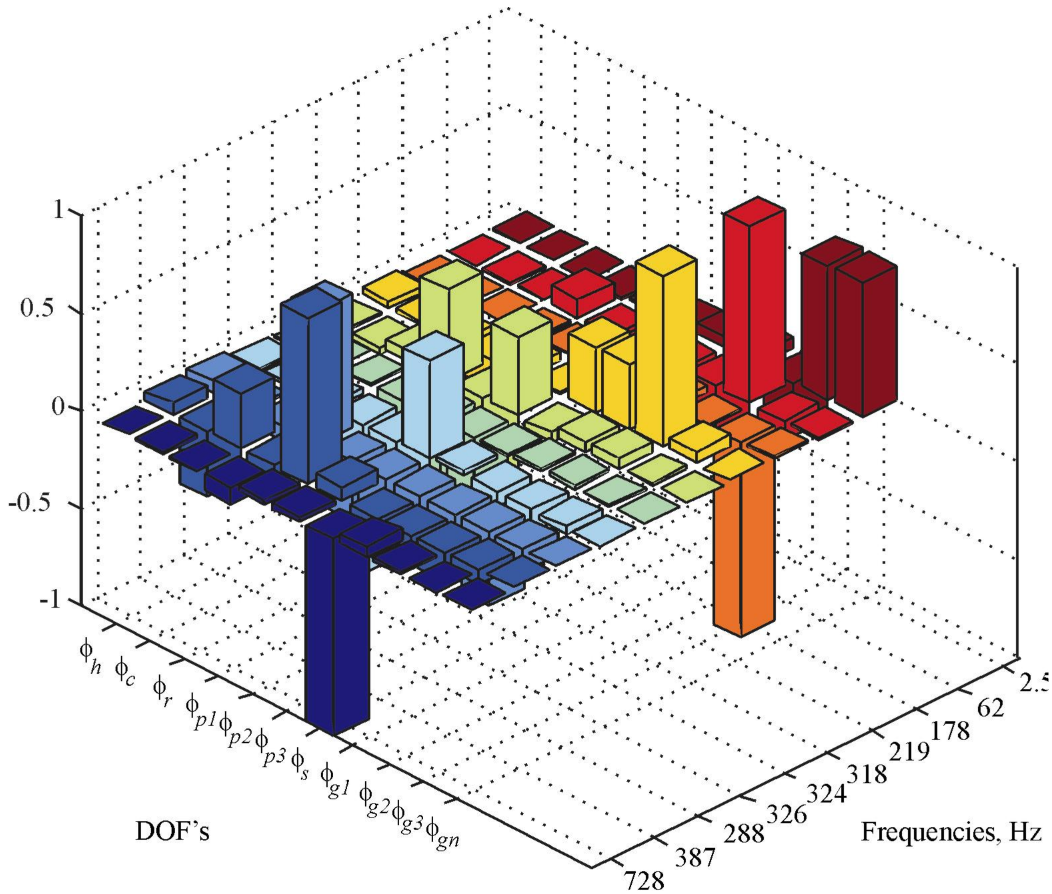

Figure 4: Mode shapes of wind turbine
For beter ilustration the mode shapes are also presented in Fig.5-14.

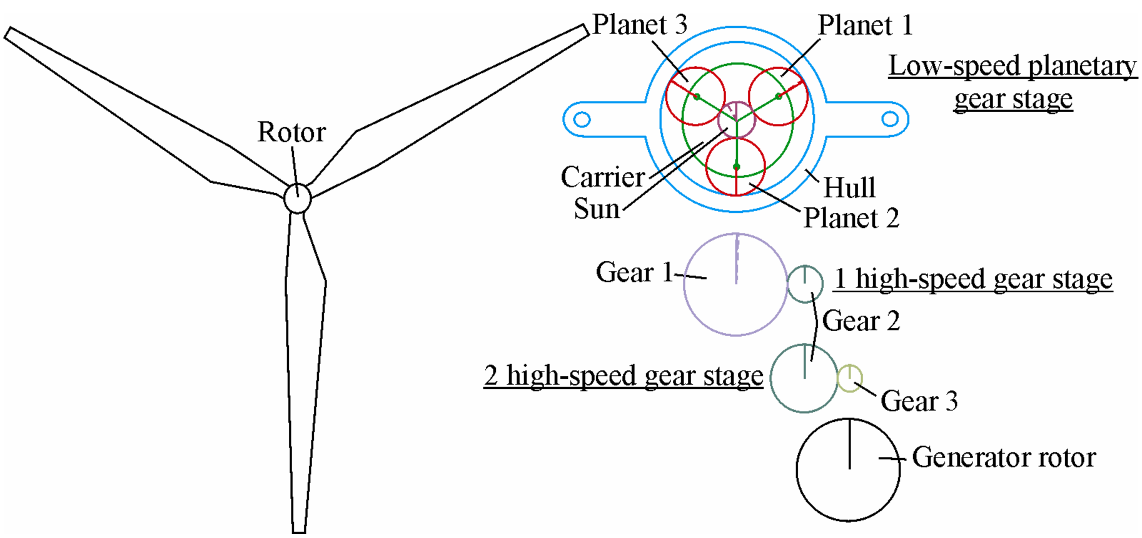

Fig.5. Mode shapes for natural frequency 782 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

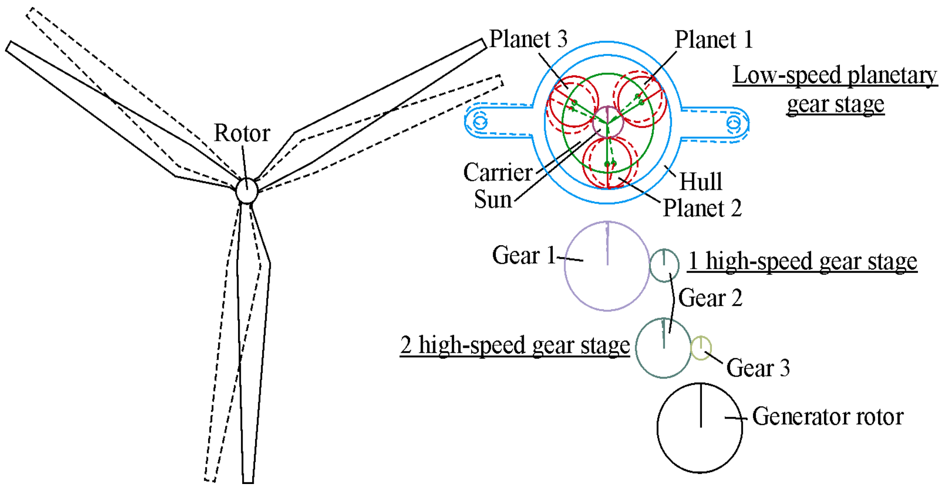

Fig.6. Mode shapes for natural frequency 387 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

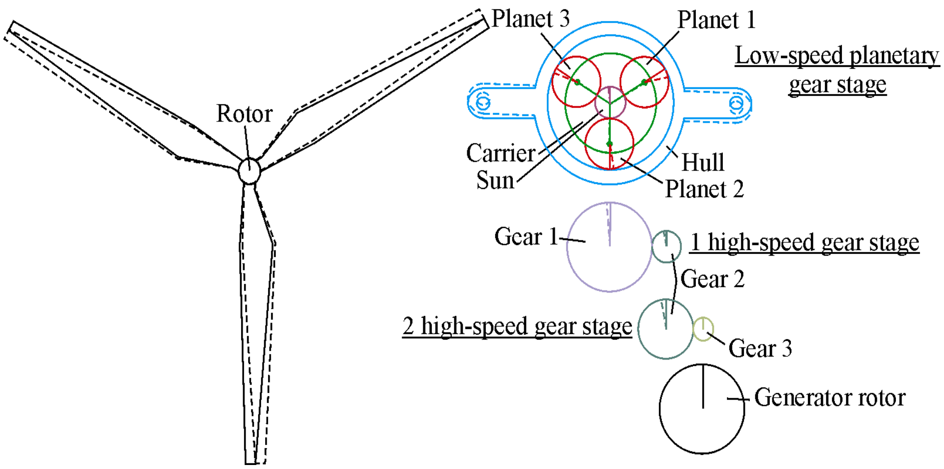

Fig.7. Mode shapes for natural frequency 288 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

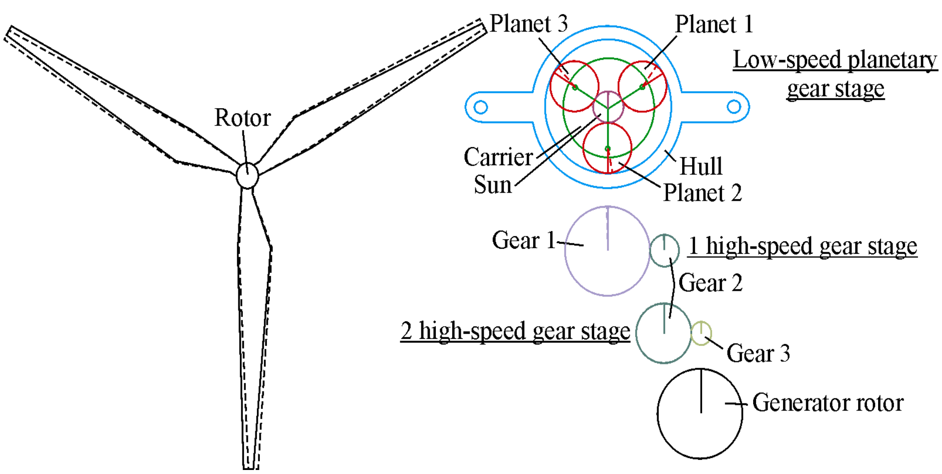

Fig.8. Mode shapes for natural frequency 327 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

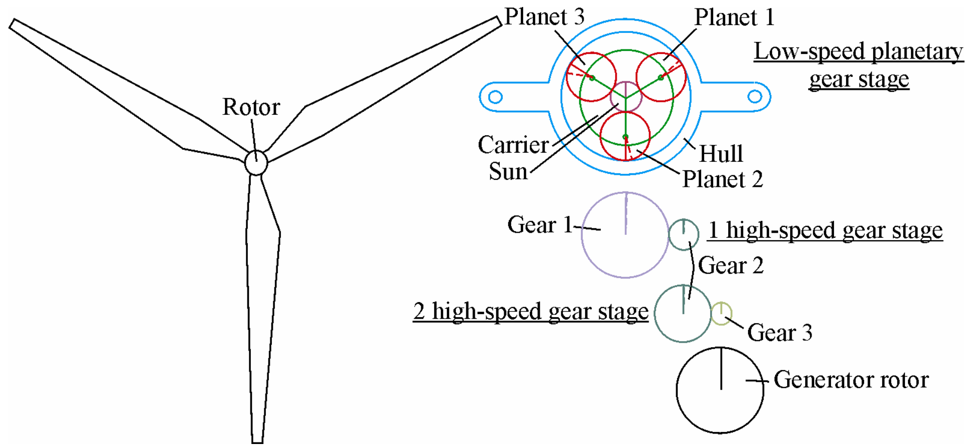

Fig.9. Mode shapes for natural frequency 324 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

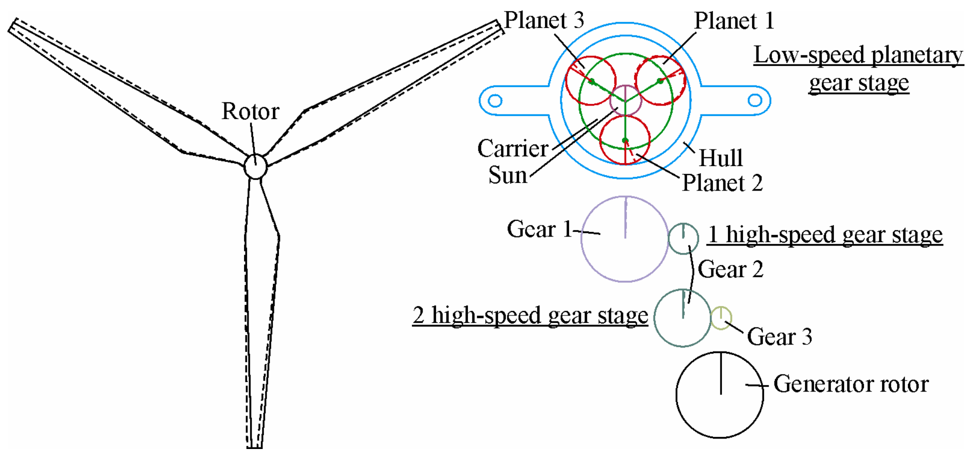

Fig.10. Mode shapes for natural frequency 318 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

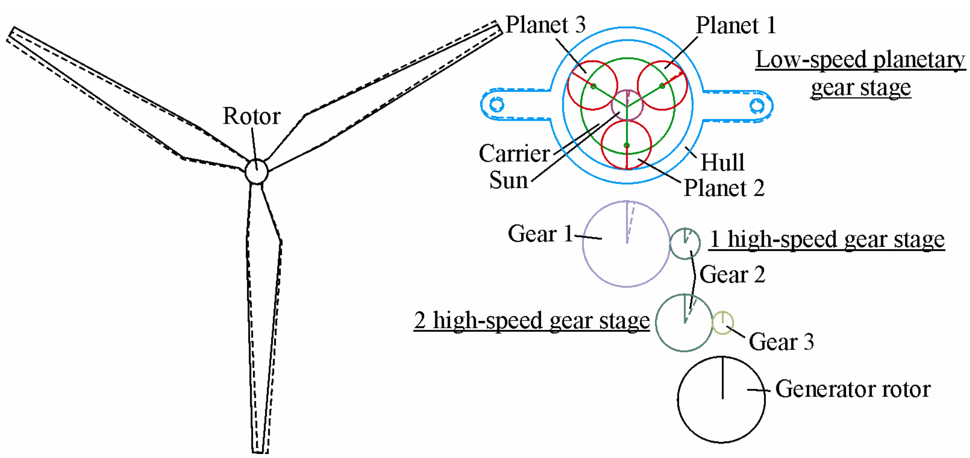

Fig.11. Mode shapes for natural frequency 219 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

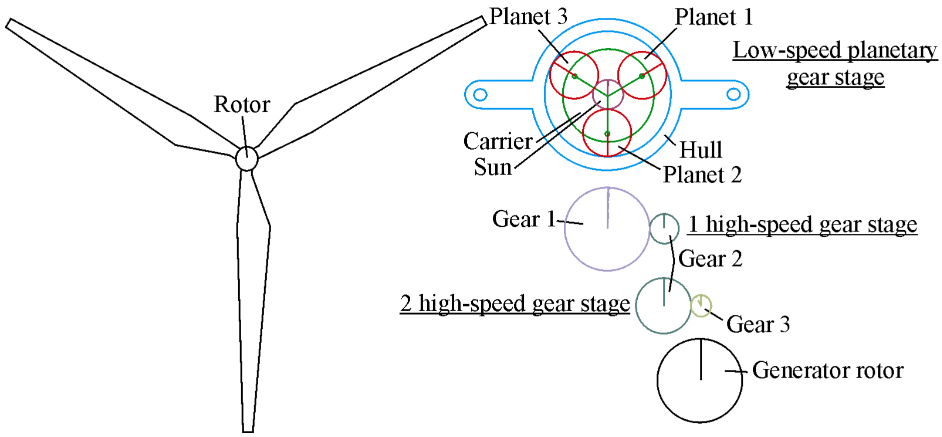

Fig.12. Mode shapes for natural frequency 178 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

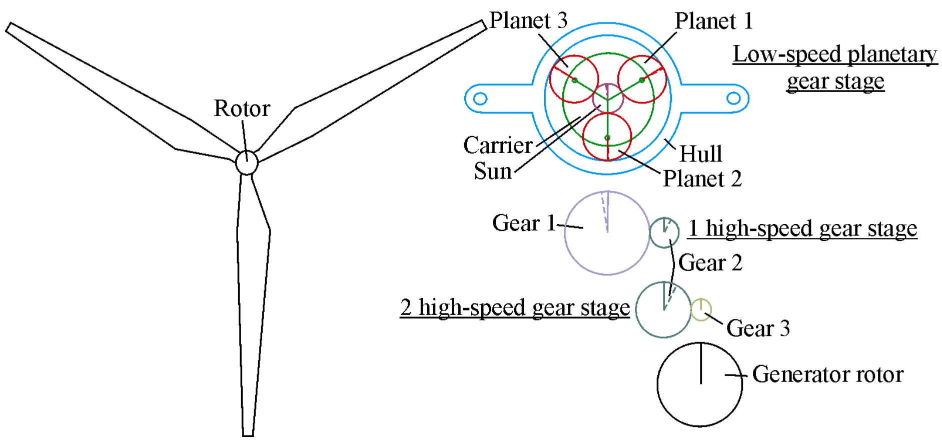

Fig.13. Mode shapes for natural frequency 62 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

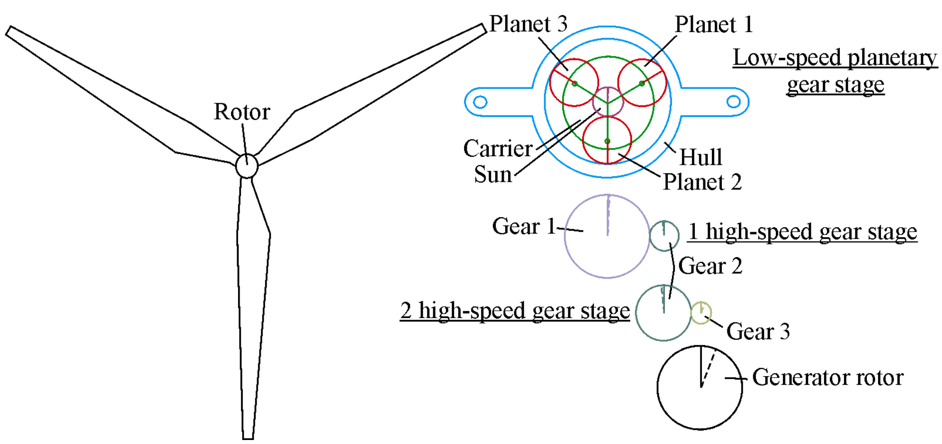

Fig.14. Mode shapes for natural frequency 2.5 Hz. Solid lines are the equilibrium positions and dashed lines are the deflected positions.

4. Conclusions

This work identifies the properties of the natural frequency spectra and vibration modes of a wind turbine with a complex drive train. The developed model and results, obtained by its help, are useful for the gearbox designing and scientific researches. The prediction of natural frequencies allows taking actions for avoiding of resonance regimes in the gearbox and wind turbine designing. This model gives an opportunity for investigation of torsional vibrations excited by wind and electromagnetic loads. The results can be used for wind turbine vibrodiagnostics.

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