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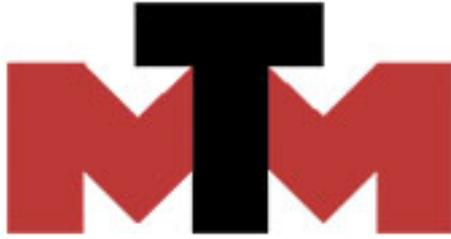


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**MATERIALS**

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# DYNAMIC ANALYSIS OF A DRIVE TRAIN OF A WIND TURBINE WITH FAULT CAUSED BY TOOTH PITTING

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**Abstract:** Planetary gear systems are widely used in wind power systems because of the advantages of compact design, large carrying capacity, and high transmission efficiency. Despite these advantages, the several conditions under which such gears are typically used may lead to failure. Tooth pitting is one of the frequently encountered failure modes. The damage of teeth (as tooth pitting) is modeled for a wind turbine drive train. The drive train has got a three-stage gearbox that contains two high-speed parallel gear stages and a low-speed planetary gear stage. The proposed dynamic model of a wind turbine also includes a rotor and an electric generator. The model consists of 10 bodies and has got 11 degrees of freedom. The aerodynamic and generator torques are applied as external loads. The influence of tooth pitting on the gear mesh stiffness is scrutinized. Healthy drive train dynamic response and response of drive train containing tooth pitting are compared. The obtained results give an useful information about health monitoring of wind turbine drive train.

**Keywords:** PITTING, GEAR MESH STIFFNESS, DRIVE TRAN, WIND TURBINE, DYNAMIC ANALYSIS

## 1. Introduction

The wind energy application has been growing rapidly for the last few years. In the last ten years the global installed capacity of wind energy has increased 20 times. This trend is expected to continue in Europe. However, the increase of the wind turbine sizes leads to component failures and an increase of operation and maintenance costs and subsequently, the cost of energy. Therefore, there is a need for the industry to reduce the wind turbine downtime and to increase its reliability. An investigation of dynamic behavior of wind turbine can potentially help for detecting incipient failures early, thereby reducing the reasons for catastrophic failures.

Figures 1 and 2 illustrate the failure statistics published by Wissenschaftliches Mess- und Evaluierungs- programm (WMEP) database from 1993 to 2006 [24, 25, 36].

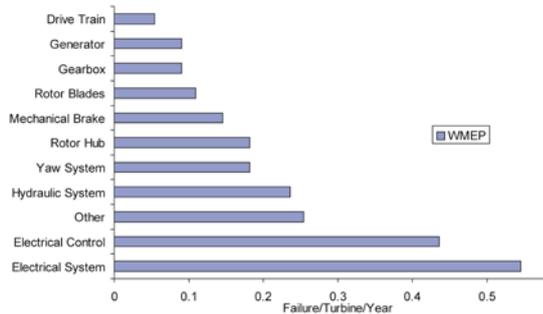


Fig. 1 Reliability of turbine subsystems [24]

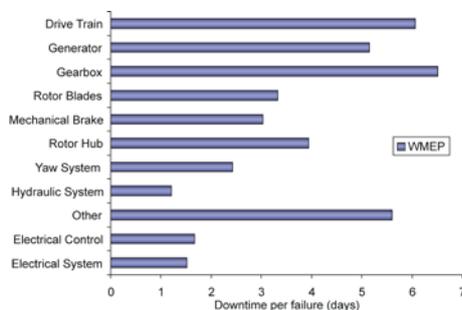


Fig. 2 Downtime of turbine subsystems [24]

Figure 3 illustrates the gearbox damage distribution published by NREL database [24, 25, 36].

From the figures it can be seen that the electrical systems had highest failure rate, but the gearboxes caused longest downtime per failure. The gearbox damage distributions show that both bearing and gear faults are concentrated in the parallel section.

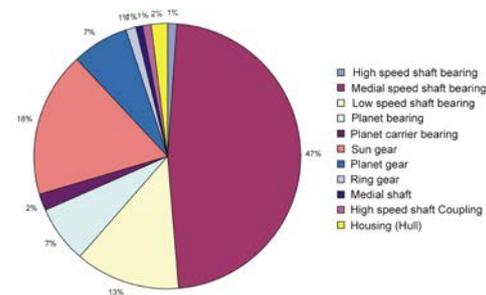


Fig. 3 Gearbox damage distribution [25]

Design calculations for a wind turbine are based on simulation of mechanical loads on the turbine components caused by external forces. The external forces are the wind, the electricity grid and sea waves for offshore applications.

The multi-body simulation techniques are used to analyse the loads on internal components of drive trains. All drive train components are treated as rigid bodies. The linkages in the multi-body model, representing the bearing and tooth flexibilities, are modelled with a springs acting in the plane of action [7, 10, 13, 14, 26-28].

It is also used a flexible model in which the drive train components are modelled as finite element models instead of rigid bodies. This model adds a possibility of calculating stress and deformation in the drive train components at the same time. Any addition to the model leads to additional information about dynamics of the drive train but makes the modelling and the simulation more complicated [1, 12, 20].

The modern wind turbines have a planetary gearbox. Studies on the vibrations in a planetary gear system have been done in [1, 3, 5, 16, 17]. The tooth meshes are modelled as a linear spring with stiffness that is a time function. For this reason the vibration equations of a planetary gear system are differential equations with periodic coefficients, [2-5, 16, 17].

The applications of these modelling techniques on different drive trains of wind turbines are presented in [1, 10-13, 20-23, 26-28]. References [29, 30] present the numerical investigations for the given wind turbine in this paper, where the meshes stiffness are modelled as constant springs. In this case the differential equations, which describe the torsional vibrations of the wind turbine, have constant coefficients. In [31, 32, 34], a dynamic model of wind turbine is proposed, where the mesh stiffness is modelled as a time function, and the aerodynamic and electromagnetic torques are constants. In Ref. [33], the same dynamical model is proposed, but the aerodynamic and electromagnetic torques are modelled as time functions.

References [2, 4, 35, 37, 38] present the effects of tooth damages and the wearing on the gear dynamics. Typical gear failures are: broken tooth, cracked tooth, worn tooth, pitting, spalling and chipping. Several approaches are proposed in these references to understand the influence of local damages on the dynamic behavior of gearbox.

Pitting occurs when fatigue cracks are initiated on the tooth surface or just below the surface. Usually pits are the result of surface cracks caused by metal-to-metal contact of asperities or defects due to low lubricant film thickness. High-speed gears with smooth surfaces and good film thickness may experience pitting due to subsurface cracks. These cracks may start at inclusions in the gear materials, which act as stress concentrators, and propagate below and parallel to the tooth surface. Pits are formed when these cracks break through the tooth surface and cause material separation. When several pits join, a larger pit (or spall) is formed. Pitting can also be caused by foreign particle contamination of lubricant. These particles create surface stress concentration points that reduce lubricant film thickness and promote pitting [15].

## 2. Dynamic model of wind turbine

The wind turbine consists of a rotor, a drive train and a generator (Fig.4).

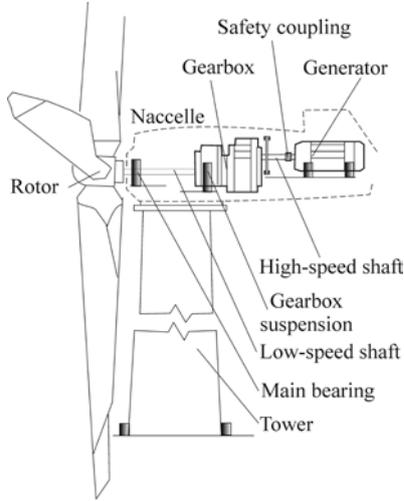


Fig.4 Schematic sketch of wind turbine

The drive train has a gearbox with three stages. The gear stages include two high-speed parallel gear stages and a low-speed planetary gear stage (three identical planets with spur teeth, sun and fixed ring wheel) (Fig.5).

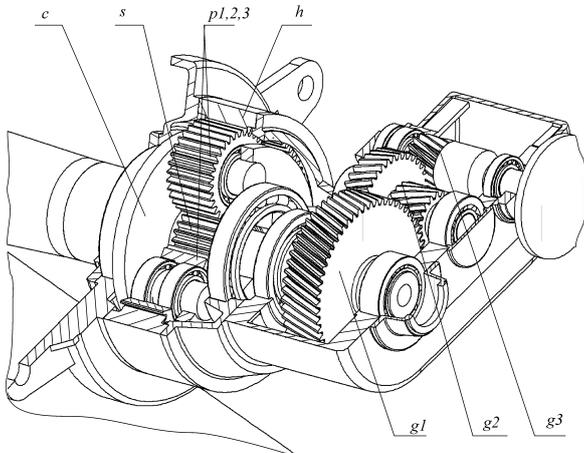


Fig. 5 Sketch of gearbox: h-hull, c-carrier, p1,2,3-planets, s-sun, g1,2,3-gears

The dynamic multi-body model is shown in Fig.6. It consists of a rotor with 3 rigid blades, a low-speed elastic shaft, a gearbox with

3 gear stages, a high-speed elastic shaft and a generator rotor. Thus, the model consists of 10 bodies and 11 DOF's.

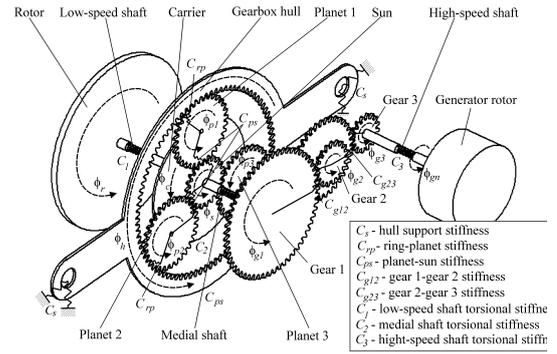


Fig. 6 Dynamical model of wind turbine

The differential equations, describing the torsional vibrations of the wind turbine, are

$$\mathbf{M}\ddot{\mathbf{q}} + [\mathbf{C} - \omega^2 \mathbf{C}_\omega] \mathbf{q} = \mathbf{T}$$

where  $\mathbf{q}$  is the degrees of freedom vector

$$\mathbf{q} = [\phi_h \ \phi_c \ \phi_r \ \phi_{p1} \ \phi_{p2} \ \phi_{p3} \ \phi_s \ \phi_{g1} \ \phi_{g2} \ \phi_{g3} \ \phi_{gn}]^T$$

and  $\mathbf{M}$  is the inertia matrix,  $\mathbf{C}$  is the stiffness matrix, and  $\mathbf{C}_\omega$  is the centripetal stiffness matrix. The angles  $\phi_i$  ( $i=h,c,r,p1,p2,p3,s,g1,g2,g3,gn$ ) are the rotational angles of the ring (gearbox hull), carrier, rotor (hub), planet 1, planet 2, planet 3, sun, gear 1, gear 2, gear 3 and the generator rotor (Fig. 6). The vector of the external forces,  $\mathbf{T}$ , caused by the wind and the electricity grid, is

$$\mathbf{T} = [0 \ 0 \ T_{aero} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ T_{gen}]^T$$

The non-zero numbers of inertia matrix  $\mathbf{M}$ , stiffness matrix  $\mathbf{C}$ , and  $\mathbf{C}_\omega$  can be seen in [31, 32].

### Gear Mesh Stiffness

- Healthy (perfect) gears

The gear contact forces between wheels are modelled by linear spring acting in the plane of action along the contact line (normal to the tooth surface), [3-5, 17, 18]. The stiffness gear is defined as a normal distributed tooth force in a normal plane causing the deformation of one or more engaging tooth pairs, over a distance of 1  $\mu\text{m}$ , normal to an involute profile in a normal plane, [6]. This deformation is a result from the bending of the teeth in contact between the two gear wheels, the first of which is fixed and the other is loaded. The stiffness varies in the time and can be expressed in a time Fourier series form, [17, 18]. Each mesh stiffness is presented by

$$C_{g_i}(t) = C_{g_i} + C_{g_{iv}}(t)$$

where  $C_{g_i}$  and  $C_{g_{iv}}$  are mean and time-varying components of the stiffness. The variation part is periodic with frequency  $\Omega_i = z_i \omega_i$  ( $z_i$  is the number of teeth on the gears,  $\omega_i$  is mean angular velocity of the gear shafts) and it is expressed in Fourier series as

$$C_{g_{iv}} = 2C_{g_{vs}} \sum_{s=1}^{\infty} (a_s \sin s\Omega_i t + b_s \cos s\Omega_i t)$$

where

$$a_s = -\frac{2}{s\pi} \sin[s\pi(\varepsilon - 2p)] \sin(s\pi\varepsilon)$$

$$b_s = -\frac{2}{s\pi} \cos[s\pi(\varepsilon - 2p)] \sin(s\pi\varepsilon)$$

Without loss of generality, it can be accepted that  $p = 0$  ( $p$  is the phasing between planets) [18]. In practice, three or four Fourier terms reasonably approximate the stiffness variation.

The rectangular waves are often used to approximate the mesh stiffness between 2 pairs of teeth in contact [3, 5, 17]. In this study, the  $C_{g_{iv}}$  are specified as rectangular waves with variational amplitudes and periods.

The rotating frequency of the carrier, sun, planets, and mesh frequency for the planetary gear stage can be calculated by

$$\begin{aligned}\Omega_c &= \omega_c / 2\pi \\ \Omega_s &= \omega_c (z_s + z_r) / z_s \\ \Omega_p &= (\omega_s z_s - \omega_c (z_s + z_p)) / z_p \\ \Omega_{mesh} &= \Omega_c z_r\end{aligned}$$

#### • Tooth Crack Defect Modelling

It has been established [4] that gear tooth failure will induce amplitude and phase changes in vibration, which in turn can be represented by magnitude and phase changes in gear mesh stiffness. The tooth-pitting-induced variations of mesh stiffness used for this simulation is shown in Fig.7.

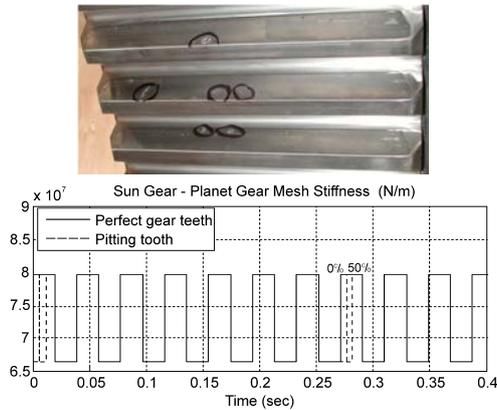


Fig.7 Photo of tooth pitting and effect on mesh stiffness

An amplitude modulation of the gear mesh signal is expected from this pitting-induced change. The new mesh signal, resulting from crack defect modelling can be expressed by

$$C_{g_{id}}(t) = C_{g_i}(t)d(t)$$

where  $d(t)$  is the modulating function.

As a result of this amplitude modulation, an exciting force is appeared, and the frequency content of the response is also affected [4, 5].

Assuming a localized tooth pitting on only one sun tooth, the defect frequency is written as

$$\Omega_d = 3(\Omega_s - \Omega_c)$$

### 3. Results

All calculations are accomplished by using the codes of MATLAB.

The drive train data can be seen in [30, 31]. It is assumed that the aerodynamic torque and electromagnetic torque are  $T_{aero} = -T_{gen} = 15000$  Nm. The rotor is turned with angular velocity  $\omega = 18$  tr/min. It is also assumed that there is a pitting tooth of the sun of the planetary gear stage. The rate of degradation of stiffness is 25%. Figure 8 presents the time series and frequency domain of torsional vibrations of the wind drive train hull. The vibration signal of the gearbox hull can be easily measured, and thus to be made some conclusions about the state of the gearbox.

In Fig. 8, the influence of pitting tooth of the sun of the planetary gear stage on the vibration signals is shown for presented degradation value. It is seen that the obvious periodical impulses caused by the pitting tooth are appeared. This carries diagnostic information that is important for extracting features of tooth defects.

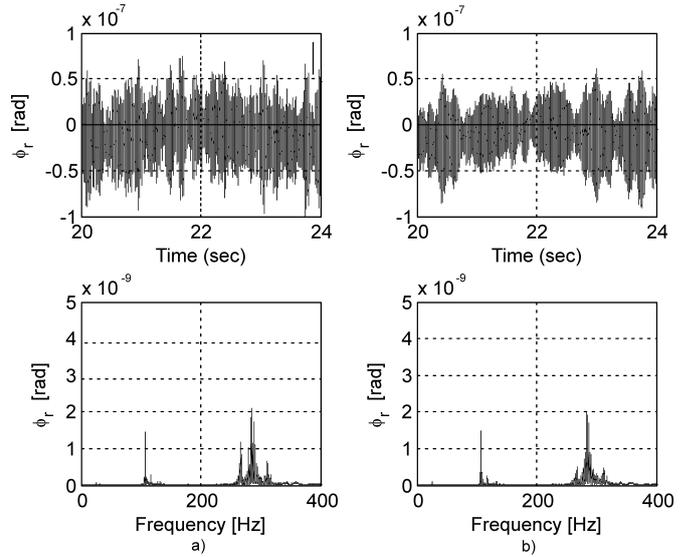


Fig.8 Hull torsional vibration: a) health gears case and b) pitting sun case

### 4. Conclusion

1. A detailed multi-body model of the wind turbine with a complex drive train was developed to examining the gearbox dynamics in the presence of defect such tooth pitting.
2. This effect of pitting tooth breakage on the sun-planet gear mesh stiffness was modeled by an amplitude reduction.
3. The time series and frequency domains of rotation angle of the drive train hull were presented. The teeth pitting effect on the gear mesh stiffness leads to an increase of impulses corresponding to the mesh of defected tooth. This information is very useful in a condition-monitoring system and can detect defect during early stage of failure in wind turbine gearbox.

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