DETERMINATION OF THE PARTICLE SHAPE FACTOR USING CAUCHY'S THEOREM AND IMAGE ANALYSIS

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Abstract

The basis of this study for determination of particle shape factor is Cauchy's theorem, according to which the average projected area of randomly oriented convex particles is just one quarter of the average surface area of these particles. A large number of particles are photographed with digital camera on a flat surface and in ethanol-water mixture. By the means of experimental measurements and image analysis of different images, which are obtained through these two ways of photographing, are calculated two values of particle shape factor. All captured images are processed with commercial software ImageJ. The main aim is to achievement a random orientation of the particles and this is implemented through the photographing of the particles in a ethanol-water mixture.

Keywords: *particle shape factor, Cauchy's theorem, digital camera, image analysis.*

INTRODUCTION

Particulate materials are widely used in many industries. The determination of particle shape is accepted as very important when Computational Fluid Dynamics (CFD) tools are used in industrial sector for the simulation and prediction of particle behavior in fluid flows. The particle shape has a significant effect on the modeling of solid phase by the means of the Euler-Lagrange method, which is implemented into the commercial code ANSYS Fluent as a Discrete Phase Model (DPM). The drag laws in DPM are spherical and non-spherical [3]. The real particles are non-spherical and for this the shape factor is needed to calculate the drag coefficient. A number of definitions through the years of the shape factor have been proposed in the literature. Typically particle shape is quantified using how close is the particle geometry or 2D projection to a perfect sphere or circle respectively. Wadell (1932) defined the shape factor as the ratio of the surface area of a sphere with the same volume as the particle S_{sp} to the actual surface area of the particle S_p :

$$\Psi = \frac{S_{sp}}{S_p} \quad , \quad (1)$$

It should be noted that this equation is used to the determination of the drag coefficient in ANSYS Fluent .

Also in 1935 Wadell used the term "degree of circularity" as the ratio of the perimeter of a circle of the same area as the particle to the actual particle perimeter. The shape of particle can also be quantified by the relations between the lengths of the three orthogonal long, intermediate and short axes of the particle (Folk, 1974; Bates and Jackson, 1980) [10].

Many techniques have been used to obtain the particle shape factor: Fourier analysis, fractal dimension, tomography, laser scanning and digital image analysis. In the recent years due to the advent of digital cameras and commercial software packages for image analysis, we can more easily to calculate the shape factor through 2D captured images of the particles. Using the digital image analysis the total number of particles and geometry of two dimensional projections of their form were obtained from the authors [6] [7].The advantages of this method are more, but an important disadvantage is error due to the overlapping or hiding of the particles during the capture process. Also image analysis is limited for the direct determination of particle shape factor because used two dimensional projections of the three dimensional particles. The first aim of this paper is the showing of a relationship between the two-dimensional measure of particle geometry using an image analysis software named ImageJ and the three-dimensional determination of shape factor as defined by Waddell (1932). The second and main aim is detection of way of photographing, in which the condition of Cauchy's theorem of randomly oriented convex particles is performed.

DETERMINATION OF THE SHAPE PARAMETERS

Image analysis does not measure directly the parameters of the shape factor from equation (1). For this purpose the surface area of a sphere with the same volume as the particle S_{sp} was obtained experimentally as follows below. The overall mass of the 400 non-spherical polypropylene particles M=11.852g and density of the particle ρ_p=0.876 g/cm^3 were measured by precision laboratory scale and picnometer. Then the average mass m_p=0.0218 g and the average volume $V_p=m_p/\rho_p$ =0.0248 m^3 of a particle were obtained. From other equation for the volume of a sphere was calculated the diameter of a sphere with the same volume as particle d_p=3.6 mm. Finally the surface area of a sphere S_{sp}=40.6 mm^2 was determined. In order that to calculate the actual surface area of the particle S_p is adopted Cauchy's theorem [9]. Cauchy's theorem says, that the average projected area A_p of randomly oriented convex particles is just one quarter of the average surface area of these particles:

$$A_p=\frac{S_p}{4}. \quad (2)$$

The applications of the theorem are various: in astrophysics, military and computer graphics [1] [4] [11]. But in many cases it is a starting point for determining the 3D shape of the particles using their projected areas. Houston 1957 and Nussinovitch 2010 were used theorem as a step in estimating the sphericity of fruits and of polymer beads in gels [5] [8].
In this study by means of image analysis is possible to determine the 2D parameters of the particles including average projected area of these particles and distances or angles of user defined objects [2]. After the determination of average projected and actual surface area of the particle it will be possible to calculate the particle shape factor.

PHOTOGRAPHING OF THE PARTICLES ON A FLAT SURFACE.

In order to be performed the Cauchy's theorem (2), randomly oriented particles are needed. In this first method that condition has not been satisfied due to the fact that particles are placed on a flat surface and this eliminates the possibility to be processed incorrectly during the image analysis. The 400 polypropylene particles were placed on a dark background without touching or overlapping each other. A scale was placed at the right of the dark background to obtain dimensions in mm and this was used during image analysis process. A high quality image of the particles is needed before any image processing can be performed. The image was captured from the top view of the particles with a digital camera with an effective resolution 18 MP. Also during the capture process a top light was applied. The output of camera is a JPEG image with 4272x2848 pixels, 24-bits of RGB color. The aim of the image analysis is determination of average projected area of particles A_p and first step is the conversion of pixel values into mm using a scale factor. The calibration of the scale factor was done with scale placed on image as shown in Fig. 1. Then unnecessary parts of the image were cut and original image was converted into binary. The binary image has only black or white pixels to clearly identify particles from their background. After that the tool *Threshold B&W* was used and the particles were displayed in black and background in white as shown in Fig. 2. The value of the average projected area was measured 12.03 mm^2. The output image of the ImageJ is shown in Fig. 3.

Using equation (2) actual surface area of the particle S_p= 48.12 mm^2 was obtained. Then shape factor defined by Waddell (1) was calculated 0.84.

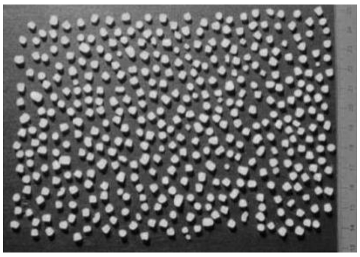

Fig. 1. Image of the particles

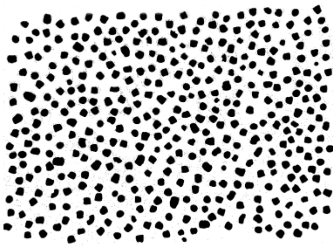

Fig. 2. Binary image

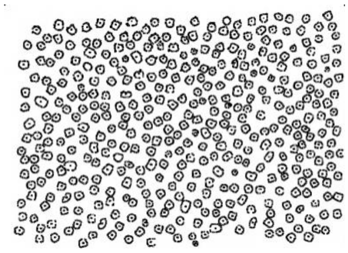

Fig.3. Output of the ImageJ

PHOTOGRAPHING OF THE PARTICLES IN ETHANOL-WATER MIXTURE.

When the first way is used the force of the gravity of an particle affects its stability. The convex particles are in stable equilibrium, when they have minimum potential energy. Then stable equilibrium of particles placed on a flat surface is achieved, when the center of gravity of a body is below the point of suspension or support. As a result the requirement of Cauchy's theorem, about random orientation, isn't satisfied. In order that to avoid this disadvantage in second method during the capture process the particles are situated in ethanol-water mixture with the same density as the particle, instead on a flat surface. If the particle has the same density as the fluid, then its buoyancy equals its weight. The particles will remain submerged in fluid and they will neither sink nor float. By this way we achieved completely random position of the all of them. The ethanol-water mixture has a density in the range 0.789÷0.998 g/cm^3 and from 65% ethanol, 35% water, the density of fluid was achieved 0.877 g/cm^3. Also the ethanol-glycerin mixture includes particle density ρ_p in the range 0.789÷1.26 g/cm^3.

The general experimental arrangement of method is shown in Fig. 4. The 207 polypropylene particles were placed in a rectangular transparent container filled with ethanol-water mixture with the same density as the solid phase. A scale and black background were placed behind the transparent container. The main light was scattered light from the room, because back and top lights were caused shadows on the captured image.

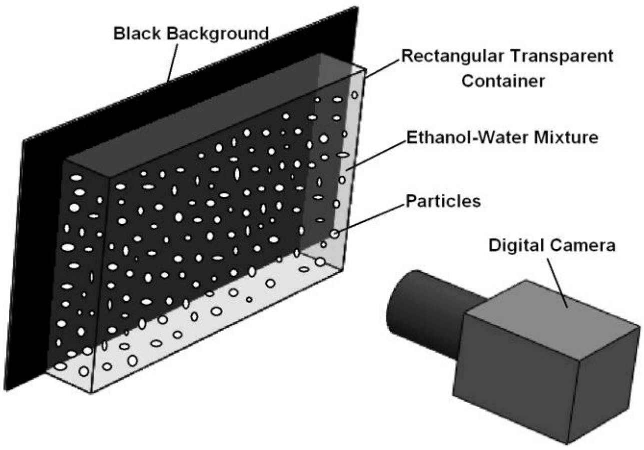

Fig. 4. Experimental arrangement

The output of the camera is shown in Fig. 5. The same steps of the image analysis were repeated as in first method. The unnecessary parts and overlapping particles were cut of the image and output of the ImageJ is shown in Fig. 6. The value of average projected area was measured 13.12 mm^2 and using Cauchy's theorem actual surface area of the particle was obtained S_p= 52.49 mm^2. Then shape factor was obtained 0.77.

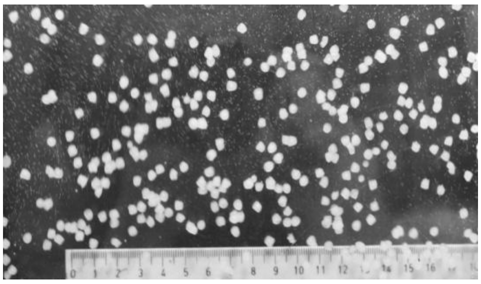

Fig. 5. Output of digital camera

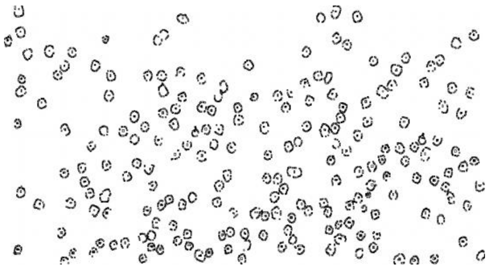

Fig. 6. Output of image analysis

CONCLUSION

This study has been presented particle shape factor determination of the large convex particles by means of the experimental measurements, Cauchy's theorem and software ImageJ for image analysis. The first way of the photographing of the particles includes main steps of the image analysis for determination of the particle shape factor. In regard to accuracy in the implementation of Cauchy's theorem, the second way was developed. The difference is about 10% between the obtained results, regarding shape factor and it is considered that the second way is more accurate. By this study, image analysis is considered to be a significant technological advancement towards particle shape determination with images obtained from digital cameras and processed with commercial software ImageJ.

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