Integral condition applicable for modeling environmental process polluting

Kamen Nikolov, Rositsa Velichkova

***Abstract**: Flows which polluting the environment with distribution of hazards were fitted with non-isothermal two-phase stream flowing in space. The conclusion is made for integral conditions apply to vertical non-isothermal two-phase stream considering the influence of lift force.*

Key words: *integral condition, two-phase vertical flow, boundary condition*

INTRODUCTION

Environmental pollution with harmful and creating methods for their control and eventually capture is an important task for modern ecology. Contaminants are often fires in space, leakage of harmful emissions into the atmosphere by human activities and etc.

BASIC OUTPUT EQUIATIONS

The distribution of pollutants in the environment is approximated with a vertical non-isothermal two-phase flow with positive lift force lift. Output equations are given in [1], which have the form:

$$\frac{\partial}{\partial x}\left[y^j U_g \rho_g\right]+\frac{\partial}{\partial y}\left[y^j V_g \rho_g\right]=0 \tag{1}$$

$$\frac{\partial}{\partial x}\left[y^j U_p \rho_p\right]+\frac{\partial}{\partial y}\left[y^j V_p \rho_p\right]=0 \tag{2}$$

$$\left(y^j \rho_g U_g\right)\frac{\partial U_g}{\partial x}+\left(y^j \rho_g V_g\right)\frac{\partial U_g}{\partial y}=\frac{\partial}{\partial y}\left(y^j \rho_g \upsilon_{tg}\frac{\partial U_g}{\partial y}\right)-F_x y^j-\left(\rho_2-\rho_g\right)\pi g y^{2j} \tag{3}$$

$$\left(y^j \rho_p U_p\right)\frac{\partial U_p}{\partial x}+\left(y^j \rho_p V_p\right)\frac{\partial U_p}{\partial y}=\left(y^j \rho_g \frac{\upsilon_{tp}}{Sc_t}\frac{\partial x}{\partial y}\right)\frac{\partial U_p}{\partial y}+\frac{\partial}{\partial y}\left(y^j \rho_p \upsilon_{tp}\frac{\partial U_p}{\partial y}\right)+F_x y^j \tag{4}$$

$$\left(y^j U_p\right)\frac{\partial \chi}{\partial x}+\left(y^j V_p\right)\frac{\partial \chi}{\partial x}=\frac{\partial}{\partial y}\left(y^j \frac{\upsilon_{tp}}{Sc_t}\frac{\partial \chi}{\partial y}\right) \tag{5}$$

$$\left(y^j \rho_g U_g c_{pg}\right)\frac{\partial T_g}{\partial x}+\left(y^j \rho_g V_g c_{pg}\right)\frac{\partial T_g}{\partial y}=\left(y^j \rho_g c_{pg}\frac{\upsilon_{tg}}{\mathrm{Pr}_t}\frac{\partial T_g}{\partial y}\right)+Qy^j+F_x y^j\left(U_g-U_p\right) \tag{6}$$

$$\left(y^j \rho_p U_p c_{pp}\right)\frac{\partial T_p}{\partial x}+\left(y^j \rho_p V_p c_{pp}\right)\frac{\partial T_p}{\partial y}=\left(y^j \rho_p c_{pp}\frac{\upsilon_{tp}}{Sc_t}\frac{\partial x}{\partial y}\right)\frac{\partial T_p}{\partial y}+\frac{\partial}{\partial y}\left(y^j \rho_p c_{pp}\frac{\upsilon_{tp}}{\mathrm{Pr}_t}\frac{\partial T_p}{\partial y}\right)+Qy^j \tag{7}$$

BOUNDARY CONDITION

In order to solve the system of differential equations it is necessary to be set boundary and initial conditions. In the present system of two-phase non-isothermal vertical jet which leakage into a stationary fluid environment these conditions are:

- ***for axis of symmetry*** $(y=0)$**:**

$$\frac{\partial U_g}{\partial y}=\frac{\partial U_p}{\partial y}=0 \qquad \frac{\partial \mathrm{T_g}}{\partial y}=\frac{\partial T_p}{\partial y}=0 \qquad \frac{\partial \rho_p}{\partial y}=0$$

$$\overline{U_g^{'}V_g^{'}}=\frac{\partial \overline{U_g^{'}V_g^{'}}}{\partial y}=0 \qquad \overline{U_p^{'}V_p^{'}}=\frac{\partial \overline{U_p^{'}V_p^{'}}}{\partial y}=0$$

$$\overline{V_p^{'}\rho_p^{'}} = \frac{\partial \overline{V_p^{'}\rho_p^{'}}}{\partial y} = 0 \quad \mathrm{V_g} = V_p = 0$$

- ***for outer boundary of the stream*** $(y \to \infty)$

$$\overline{U_g^{'}V_g^{'}} = \frac{\partial \overline{U_g^{'}V_g^{'}}}{\partial y} = 0 \quad \overline{\mathrm{U_p^{'}}V_p^{'}} = \frac{\partial \overline{U_p^{'}V_p^{'}}}{\partial y} = 0$$

$$\overline{V_p^{'}\rho_p^{'}} = \frac{\partial \overline{V_p^{'}\rho_p^{'}}}{\partial y} = 0 \quad \frac{\partial \mathrm{T_g}}{\partial \mathrm{y}} = \frac{\partial T_p}{\partial y} = 0$$

$$V_g = V_p = 0; U_p = U_2; \rho_p = 0; T_g = T_p = T_2; \rho_g = \rho_2$$

In the system (1)÷(7) are include binary correlations $\overline{U_g^{'}V_g^{'}}, \overline{U_p^{'}V_p^{'}}, \overline{V_p^{'}\rho_p^{'}}$. These correlations take the following form:

$$\overline{U_g^{'}V_g^{'}} = -\nu_{tg}\frac{\partial U_g}{\partial y}; \quad \overline{\mathrm{U_p^{'}}V_p^{'}} = -\nu_{tp}\frac{\partial U_p}{\partial y}; \overline{V_p^{'}\rho_p^{'}} = -\frac{\nu_{tp}}{Sc_t}\frac{\partial \rho_p}{\partial y}$$

$$\overline{\mathrm{T_g^{'}}V_g^{'}} = -\frac{\nu_{tg}}{\mathrm{Pr}_t}\frac{\partial T_g}{\partial y} \quad \overline{\mathrm{T_p^{'}}V_p^{'}} = -\frac{\nu_{tp}}{\mathrm{Pr}_t}\frac{\partial T_p}{\partial y}$$

$$\mathrm{V_p^{'2}} = \overline{U_p^{'}V_p^{'}} = -\nu_{tp}\frac{\partial U_p}{\partial y} \quad \mathrm{V_g^{'2}} = -\nu_{tg}\frac{\partial U_g}{\partial y}$$

where: ν_{tg}, ν_{tp} - coefficient of kinematic viscosity of gases and impurities phases; Sc_t - turbulent number of Scmid; Pr_t - turbulent number of Prandtl.

CONCLUSION OF BASIC BOUNDARY INTEGRAL CONDITIONS

1. Condition for conservation the content of impurities at two phase turbulent vertical jets

Equations (1) and (2) are considering:

$$+\begin{cases} \dfrac{\partial}{\partial x}\left(y^j \rho_p U_p\right) + \dfrac{\partial}{\partial y}\left(y^j \rho_p V_p\right) = 0 \\ \left(y^j U_p\right)\dfrac{\partial \rho_p}{\partial x} + \left(y^j V_p\right)\dfrac{\partial \rho_p}{\partial y} = \dfrac{\partial}{\partial y}\left(y^j \dfrac{\nu_{tg}}{Sc_t}\dfrac{\partial \rho_g}{\partial y}\right) \end{cases}$$

After integrating the above equation in the range $y = 0 \div \infty$ we have:

$$\frac{\partial}{\partial x}\left(\int_0^\infty \rho_p U_p y^j dy\right) = 0 \qquad (8)$$

Equation (8) give the condition for conversation of mass transfer in jet.

2. Condition for movement quality of two-phase non-isothermal vertical jet

This conditions are received using equations (1) and (3) for gas phase and (1) and(4) for the phase of impurities:

$$+\begin{cases} \dfrac{\partial}{\partial x}\left(y^j \rho_g U_g\right) + \left(U_g - U_2\right)\dfrac{\partial}{\partial y}\left(y^j \rho_g V_g\right) = 0 \\ \left(y^j \rho_g U_g\right)\dfrac{\partial U_g}{\partial x} + \left(y^j \rho_g V_g\right)\dfrac{\partial U_g}{\partial y} = \dfrac{\partial}{\partial y}\left(y^j \rho_g \nu_{tg}\dfrac{\partial U_g}{\partial y}\right) - F_x y^j - g\pi\left(\rho_2 - \rho_g\right)y^{2j} \end{cases}$$

Then for the gas phase we obtain:

$$\frac{\partial}{\partial x}\left[y^j \rho_g U_g \left(U_g - U_2\right)\right] + \frac{\partial}{\partial y}\left[y^j \rho_g V_g \left(U_g - U_2\right)\right] = \frac{\partial}{\partial y}\left(y^j \rho_g \nu_{tg} \frac{\partial U_g}{\partial y}\right) - F_x y^j - g\pi\left(\rho_2 - \rho_g\right) y^{2j} \quad (9)$$

- phase of impurities

$$+\begin{cases} U_p \frac{\partial}{\partial x}\left(y^j \rho_p U_p\right) + U_p \frac{\partial}{\partial y}\left(y^j \rho_p V_p\right) = 0 \\ y^j \rho_p \left(U_p \frac{\partial U_p}{\partial x} + V_p \frac{\partial U_p}{\partial y}\right) = \left(y^j \rho_g \frac{\nu_{tp}}{Sc_t}\right) \frac{\partial U_p}{\partial y} + \\ + \frac{\partial}{\partial y}\left(y^j \rho_p \nu_{tp} \frac{\partial U_p}{\partial y}\right) + F_x y^j \end{cases}$$

$$\frac{\partial}{\partial x}\left(y^j \rho_p U_p^2\right) + \frac{\partial}{\partial y}\left(y^j \rho_p V_p U_p\right) = \frac{\partial}{\partial y}\left(y^j \rho_g U_p \frac{\nu_{tp}}{Sc_t} \frac{\partial \chi}{\partial y}\right) + + \frac{\partial}{\partial y}\left(y^j \rho_g \nu_{tp} \frac{\partial U_p}{\partial y}\right) + F_x y^j \quad (10)$$

Equation (9) and (10) are integrated in range $y = 0 \div \infty$ and it is obtain:

$$\frac{\partial}{\partial x}\int_0^\infty \rho_g U_g \left(U_g - U_2\right) y^j dy = -\int_0^\infty F_x y^j dy - g\pi \int_0^\infty \left(\rho_2 - \rho_g\right) y^{2j} dy \quad (11)$$

$$\frac{\partial}{\partial x}\int_0^\infty \rho_p U_p^2 y^j dy = \int_0^\infty F_x y^j dy \quad (12)$$

Equation (11) and (12) are integral condition for quality of movement for vertical non-isothermal turbulent jet.

3. *Conditions for transfer of kinetic energy of motion for gas phase and the phase of impurities*

This conditions are obtain when sum eq.(1) with eq.(3). Eq.(1) is multiplied with $\left(U_g - U_2\right)^2$, and eq. 3 with $2\left(U_g - U_2\right)$. For the phase of impurities sre used eq. (2) and (4). Equation (2) is multiplied with U_p^2 and (4) with $2U_p$

$$+\begin{cases} \left(U_g - U_2\right)^2 \frac{\partial}{\partial x}\left(y^j \rho_g U_g\right) + \left(U_g - U_2\right)^2 \frac{\partial}{\partial y}\left(y^j \rho_g V_g\right) = 0 \\ 2\left(U_g - U_2\right)\left(y^j \rho_g V_g\right)\frac{\partial U_g}{\partial x} + 2\left(U_g - U_2\right)\left(y^j \rho_g V_g\right)\frac{\partial U_g}{\partial y} = \\ = 2\left(U_g - U_2\right)\frac{\partial}{\partial y}\left(y^j \rho_g \nu_{tg} \frac{\partial U_g}{\partial y}\right) - 2\left(U_g - U_2\right) F_x y^j \\ -2g\pi\left(U_g - U_2\right)\left(\rho_2 - \rho_g\right) y^{2j} \end{cases}$$

$$\frac{\partial}{\partial x}\left[y^j\rho_g U_g\left(U_g-U_2\right)^2\right]+\frac{\partial}{\partial y}\left[y^j\rho_g V_g\left(U_g-U_2\right)^2\right]=$$
$$=2\left\{\frac{\partial}{\partial x}\left[y^j\rho_g\nu_{tg}\frac{\partial U_g}{\partial y}\left(U_g-U_2\right)\right]-y^j\rho_g\nu_{tg}\frac{\partial U_g}{\partial y}\frac{\partial U_g}{\partial y}\right\}- \tag{13}$$
$$-2\left(U_g-U_2\right)F_x y^j-g\pi\left(U_g-U_2\right)\left(\rho_2-\rho_g\right)y^{2j}$$

Equation 13 is integrated in range $y=0\div\infty$:

$$\frac{\partial}{\partial x}\left[\int_0^\infty\rho_g U_g\left(U_g-U_2\right)^2 y^j dy\right]=-2\int_0^\infty\rho_g\nu_{tg}\left(\frac{\partial U_g}{\partial y}\right)^2 y^j dy-2\int_0^\infty\left(U_g-U_2\right)F_x y^j dy$$
$$-2g\pi\int_0^\infty\left(U_g-U_2\right)\left(\rho_2-\rho_g\right)y^{2j}dy$$

For the phase of impurities:

$$+\begin{cases}U_p^2\frac{\partial}{\partial x}\left(y^j\rho_p U_p\right)+U_p^2\frac{\partial}{\partial y}\left(y^j\rho_p V_p\right)=0\\ 2U_p\left(y^j\rho_p U_p\right)\frac{\partial U_p}{\partial x}+2U_p\left(y^j\rho_p U_p\right)\frac{\partial U_p}{\partial y}=\\ =2U_p\left(y^j\rho_g\frac{\nu_{tp}}{Sc_t}\frac{\partial\chi}{\partial y}\right)\frac{\partial U_p}{\partial x}+2U_p\left(y^j\rho_p\nu_{tp}\frac{\partial U_p}{\partial y}\right)+2U_pF_x y^j\end{cases}$$

$$\frac{\partial}{\partial x}\left(y^j\rho_p U_p^3\right)+\frac{\partial}{\partial y}\left(y^j\rho_p U_p^2\right)=2\frac{\partial}{\partial y}\left\{y^j\rho_p\nu_{tp}\left[U_p\frac{\partial U_p}{\partial y}-\left(\frac{\partial U_p}{\partial y}\right)^2\right]\right\}+ \tag{14}$$
$$+\frac{\partial}{\partial y}\left(y^j\rho_g U_p^2\frac{\nu_{tp}}{Sc_t}\frac{\partial\chi}{\partial y}\right)+2U_pF_x y^j$$

Equation 13 is integrated in range $y=0\div\infty$:

$$\frac{\partial}{\partial x}\int_0^\infty\rho_p U_p^3 y^j dy=-2\int_0^\infty\rho_p\nu_{tp}\left(\frac{\partial U_p}{\partial y}\right)^2 y^j dy+2\int_0^\infty U_pF_x y^j dy \tag{15}$$

By analogous conversions can be obtained an equation of higher order relating to the concentration of the impurities. Equation (5) is multiplied by 2χ and then follows:

$$2\chi\frac{\partial}{\partial x}\left(\chi y^j U_p\right)+2\chi\frac{\partial}{\partial y}\left(\chi y^j V_p\right)=2\chi\frac{\partial}{\partial y}\left(y^j\frac{\nu_{tp}}{Sc_t}\frac{\partial\chi}{\partial y}\right)$$
$$\frac{\partial}{\partial x}\left(\chi y^j U_p\right)+\frac{\partial}{\partial y}\left(\chi y^j V_p\right)=2\frac{\partial}{\partial y}\left(y^j\frac{\nu_{tp}}{Sc_t}\frac{\partial\chi}{\partial y}\chi\right)-2y^j\frac{\nu_{tp}}{Sc_t}\left(\frac{\partial\chi}{\partial y}\right)^2$$

This equation is integrated in range $y=0\div\infty$:

$$\frac{\partial}{\partial x}\left[\int_0^\infty\left(\chi^2U_p y^j\right)dy\right]=-2\int_0^\infty\frac{\nu_{tp}}{Sc_t}\left[\frac{\partial\chi}{\partial y}\right]^2 y^j dy \tag{16}$$

4. Condition for conservation of heat transfer for gas phase and the phase of impurity

- it is using equation (6) and (1) which is multiplied with $c_{pg}\left(T_g - T_2\right)$:

$$
+\begin{cases}
c_{pg}\left(T_g - T_2\right)\frac{\partial}{\partial x}\left(\rho_g y^j U_g\right) + c_{pg}\left(T_g - T_2\right)\frac{\partial}{\partial y}\left(\rho_g y^j V_g\right) = 0 \\
c_{pg}\rho_g y^j\left(U_g\frac{\partial T_g}{\partial x} + V_g\frac{\partial T_g}{\partial y}\right) = \frac{\partial}{\partial y}\left(c_{pg}\rho_g y^j\frac{\nu_{tg}}{\Pr_t}\frac{\partial T_g}{\partial y}\right) + \\
F_x y^j\left(U_g - U_p\right) - Qy^j
\end{cases}
$$

$$
\frac{\partial}{\partial x}\left[\int_0^\infty c_{pg}\rho_g U_g\left(T_g - T_2\right)y^j dy\right] = \int_0^\infty F_x\left(U_g - U_p\right)y^j dy - \int_0^\infty Qy^j dy \qquad (17)
$$

For the phase of impurity are used equation (7) and (2), which is multiplied with $c_{pp}T_p$

$$
+\begin{cases}
c_{pp}T_p\frac{\partial}{\partial x}\left(\rho_p U_p y^j\right) + c_{pp}T_p\frac{\partial}{\partial y}\left(\rho_p y^j V_p\right) = 0 \\
c_{pp}\rho_p y^j\left(U_p\frac{\partial T_p}{\partial x} + V_p\frac{\partial T_p}{\partial y}\right) = \left(c_{pp}y^j\rho_g\frac{\nu_{tp}}{Sc_t}\frac{\partial \chi}{\partial y}\right)\frac{\partial T_p}{\partial y} + \\
+\frac{\partial}{\partial y}\left(c_{pp}y^j\rho_p\frac{\nu_{tp}}{\Pr_t}\frac{\partial T_g}{\partial y}\right) + Qy^j
\end{cases}
$$

or

$$
\frac{\partial}{\partial x}\left(c_{pp}T_p\rho_p y^j U_p\right) + \frac{\partial}{\partial y}\left(c_{pp}T_p\rho_p y^j V_p\right) = \frac{\partial}{\partial y}\left(c_{pp}y^j\rho_g\frac{\nu_{tp}}{Sc_t}\frac{\partial \chi}{\partial y}T_p\right) +
$$

$$
+\frac{\partial}{\partial y}\left(c_{pp}y^j\rho_p\frac{\nu_{tp}}{\Pr_t}\frac{\partial T_p}{\partial y}\right) + Qy^j
$$

$$
\frac{\partial}{\partial x}\left(\int_0^\infty c_{pp}T_p\rho_p y^j U_p dy\right) = \int_0^\infty Qy^j dy \qquad (18)
$$

For the system equations for two-phase turbulent vertical jets is obtained:

$$
\frac{\partial}{\partial x}\left(\int_0^\infty \rho_p U_p y^j dy\right) = 0 \qquad (19)
$$

$$
\frac{\partial}{\partial x}\int_0^\infty \rho_g U_g\left(U_g - U_2\right)y^j dy = -\int_0^\infty F_x y^j dy - \int_0^\infty\left(\rho_2 - \rho_g\right)\pi g y^{2j} dy \qquad (20)
$$

$$
\frac{\partial}{\partial x}\int_0^\infty \rho_p U_p^2 y^j dy = \int_0^\infty F_x y^j dy \qquad (21)
$$

$$
\frac{\partial}{\partial x}\left[\int_0^\infty \rho_g U_g\left(U_g - U_2\right)^2 y^j dy\right] = -2\int_0^\infty \rho_g \nu_{tg}\left(\frac{\partial U_g}{\partial y}\right)^2 y^j dy - 2\int_0^\infty\left(U_g - U_2\right)F_x y^j dy -
$$

$$
-2\int_0^\infty\left(U_g - U_2\right)\left(\rho_2 - \rho_g\right)\pi g y^{2j} dy \qquad (22)
$$

$$\frac{\partial}{\partial x}\int_0^\infty \rho_p U_p^3 y^j dy = -2\int_0^\infty \rho_p \nu_{tp}\left(\frac{\partial U_p}{\partial y}\right)^2 y^j dy + 2\int_0^\infty U_p F_x y^j dy \tag{23}$$

$$\frac{\partial}{\partial x}\left[\int_0^\infty \left(\chi^2 U_p y^j\right) dy\right] = -2\int_0^\infty \frac{\nu_{tp}}{Sc_t}\left[\frac{\partial \chi}{\partial y}\right]^2 y^j dy \tag{24}$$

$$\frac{\partial}{\partial x}\left[\int_0^\infty c_{pg}\rho_g U_g\left(T_g - T_2\right) y^j dy\right] = \int_0^\infty F_x\left(U_g - U_p\right) y^j dy - \int_0^\infty Q y^j dy \tag{25}$$

$$\frac{\partial}{\partial x}\left(\int_0^\infty c_{pp} T_p \rho_p y^j U_p dy\right) = \int_0^\infty Q y^j dy \tag{26}$$

CONCLUSION

In current work is given numerical model of two-phase non-isothermal vertical jet using the integral method. It was reported the influence of buoyancy and the influence of the interfacial heat and mass transfer. This model will be used to solve at specific problem from practice.

Literature

[1]. Antonov I., Applied fluid mechanics, Sofia 2010
[2]. Lien H.D., Two-phase non isothermal jets, PhD thesis, Sofia 1996
[3]. Nikolov K., R.Velichkova, H.D. Lien, I.Antonov, Modeling of movement and heat exchange at gaseous harmful in environment,Proceeding of 20 National scientific conference with int ernational participation FPEPM 2015, Sozopol, vol. 2 pp 33-38

The paper is financed under the contract N152ПД055-02 „Modeling of movement of harmful in environment”

Contacts:

PhD student Kamen Nikolov, department ”Hydroaerodynamics and hydraulic machines” Technical university of Sofia phone 02-965-34-43, kamen.nikolaev.nikolov@gmail.com

Assoc. prof. PhD Rositsa Velichkova department ”Hydroaerodynamics and hydraulic machines” Technical university of Sofia phone 02-965-34-43,rositsavelichkova@abv.bg

This paper has been reviewed.