Design and Stability Analysis of Supervisor-based Adaptive Fuzzy Logic Control System for Temperature

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ABSTRACT

The application of fuzzy logic (FL) supervisors (FLS) for on-line nonlinear auto-tuning of the basic FL controllers (FLCs) gains popularity as a simple adaptive technique for improvement of the performance of control systems for plants with nonlinearities, inertia, time-delay, model uncertainty and variable parameters. Various approaches for the FLS design are developed for different types of FLCs and performance measures considered. However, structure simplification techniques and closed loop system stability analysis are needed to promote the FLC -FLS industrial applications which constitute the aim of the present work. A parallel distributed compensation (PDC) equivalent in operation to the FLC-FLS is suggested that consists of a Sugeno model for fuzzy blending of the outputs of several linear PID controllers depending on the operation point. The PID parameters are optimized using genetic algorithms, simulations, experimental data and random inputs. The simpler in structure PDC-PID control system is validated as equivalent in operation to the supervisory control system and also its stability is proven by deriving Lyapunov conditions and solving them using linear matrix inequalities (LMI) numerical technique. All investigations are performed on the example of the control of the air temperature in a laboratory dryer using MATLAB™ and its real time facilities.

Keywords
Fuzzy Logic Supervisor, Genetic Algorithms, Lyapunov Stability, Parallel Distributed Compensation, Temperature Real Time Control, TSK Models.

1. INTRODUCTION AND STATE-OF-THE-ARTS

Processes are difficult to control for their inertia, time delays, model uncertainties, nonlinearities and parameter variations due to time-variance, aging, different operation modes and conditions that move the operation point along nonlinear characteristics, etc. The recent development of the process fuzzy logic controllers (FLCs) addresses essential problems related to their industrial applications - adaptation facilities to plant parameter changes which ensures also system stability in industrial environment, and a completion of the adaptive FLC (AFLC) by standard PLC and embedded techniques. This implies general application-independent adaptation fuzzy algorithms which are simple and fast to compute in real time and also easy to design and program. The AFLCs follow basically the main principles of adaptive control [1]. A fuzzy model reference adaptive/learning control (FMRLC) as a specific type of performance-based learning controller with performance specifications given by a reference model is developed in [2-6]. The main FLC is a model-based parallel distributed compensation (PDC) with local state feedback to ensure system asymptotic stability in [4], or a PI FLC for level with adjusted gains in [6], or a PD FLC in [5]. The learning mechanism built on fuzzy inverse plant model (FIM) and knowledge-based modifier (KBM) ensures on-line automatic synthesis and adaptation that improves system performance by interacting with environment and memorizing past experience. It tunes the centres of the output membership functions (MFs) in the active rules at the current time considering the delay between plant input and output in [5]. Different modifications are also suggested – in [2] the adaptation/learning of the MFs of the main PD FLC ensures minimal integral absolute error (IAE) in a servo-system tracking of the reference model in real time. In [3] the learning mechanism is subjected to adaptation instead of the main FLC in order to reduce system sensitivity with respect to initial conditions.

A fuzzy self-organising control (FSOC) which implies fuzzy rules modifications, for unknown dynamic nonlinear plant is developed in [7-9]. The controller's parameters in the rules conclusions are tuned to minimize both squared tracking error and control effort by gradient search algorithm in [9]. The weighted constants and the learning rate are determined from convergence and stability analysis using Lyapunov function. In [7] FSOC for water level and air pressure is suggested. The learning mechanism modifies and memorizes on-line the rule-bases of the main FLCs, starting from no rules and no information about the plant and accumulating the knowledge gained with the control. In experimental testing FSOC outperforms linear PI/PID control in accurate reference tracking at changes of the plant dynamics. No off-line pre-training is required and high control performance is achieved for the designed in [8] FSOC through a three-stage algorithm – a coarse fuzzy tuning of the output scaling factor (ScF), a correction of the fuzzy rule consequents (singletons) from zero initial position as function of the degree of activation of the rule and the system error at a previous time accounting for the plant time delay, and finally a bounded correction of the position of the peaks of the triangular inputs MFs from requirement of homogeneous distribution of the integral squared error (ISE) throughout the MFs defined operating regions – the ISE computation, however, requires several runs.

Common drawbacks of the FMRLC and FSOC are: 1) Trial-and-error guesses and intuitive decisions about the data needed – thresholds against oscillations, initial values, plant model structure and inertia, universes of discourse, bounds to tuning parameters (MFs centres, etc.), time window for computing performance, etc.; 2) Unproved stability and convergence, possible conflict in combining several adaptation approaches – good results can be achieved by using a simple approach at more intensive adaptation; 3) Insufficient on-line testing in industrial environment - noisy data and varying plant response to reference changes and disturbances may decrease convergence rate, simulation tests are not always consistent as they use repeated step changes - impossible in on-line operation; 4) Complexity in design and in algorithm – many MFs, several runs required "to learn", high computational resources needed (memory for storage of previous experience, processing time for many calculations, etc.) which hinders the PLCs embeding of the algorithm for real time on-line operation. In the industrial practice the reliable control is of crucial importance – this means computationally and structurally simple algorithms with easy and transparent design and ensuring system stability and adaptation convergence in industrial environment.

Great popularity for their simplicity and good performance gain the self-tuning FLCs for auto- tuning of the ScFs of the main FLCs [10-16]. They resemble in structure the FL supervisors (FLS) but have the same inputs of the main FLC - the system error and its derivative. In [11] the master FLC has the system error and a function of its first and second derivatives as inputs and tunes the ScFs of the derivative and integral component of the main PID-FLC. The algorithm is implemented on industrial PLC and the improvements are proven in real time control. A microcontroller completion of a two-level FLC with 2-input master FLC with error and control signals as inputs is suggested in [10], where a real world plant is controlled in real time at reference and load step changes. In [12] two Sugeno neuro-fuzzy PID main controllers with output singletons are off-line adapted using stochastic optimization algorithms for minimization of ISE. Generalization in tracking is proven via simulations for TITO plants.

The FLS in supervising systems auto-tunes on-line in a nonlinear manner the main FLCs gains, MFs, weights, etc., thus introducing adaptation and using simple structure and control algorithm of the fuzzy adaptive system [17-19]. It supervises and improves the operation of the main controller from upper control level using available system data different from the used by the main controller from extra sources (operator or other systems). The two-level FLC-FLS is viewed upon as a modification of the direct AFLC since the FLS's inputs are most often performance measures computed on the base of on-line available previous and current time measurements (control effort, over-/undershoot estimates, etc.). The ScFs tuning of the main FLC is a preferred approach for performance tuning as it can be easily implemented via industrial PLCs and has the effect of adaptation of MFs, fuzzy rules and signal resolution [18]. At the same time this facilitates the design and the tuning, allowing the use of standard empirical MFs and rules. FL gain scheduling control of resolution, based on two rule tables for coarse and fine ScFs tuning is suggested in [18].

In [19] a two-input FLS is designed for real time auto-tuning of the output ScF of a PI PDC-FLC with application to temperature control. The FLS inputs are y/y_r (relative with respect to reference y_r plant output y) and magnitude $|e|$ of system error $e(t)= y_r-y(t)$ - performance measures easily estimated from on-line measurements.

The FLS can be designed as a gain scheduler or a fuzzy switcher between fixed rule-bases (i.e., course and fine), or adjusting mechanism of fuzzy set representation. It can also tune the plant model or the adaptation mechanism of an AFLC. The system performance is improved significantly at relatively great, random and unknown plant changes by simple means – mainly by adjusting the ScFs or other FLC's gains which is equivalent to tuning of rules, MFs which demand more complicated techniques. Thus the FLS introduces the necessary local changes in the control surface on-line continuously. The simple FLS's design and structure need a small amount of knowledge about the plant and determine a low computational effort which facilitates the feasibility for industrial applications in noisy environment and real time.

Genetic algorithms (GA) or other optimization techniques are used to tune both the main FLCs and also their auto-tuning FLS in order to improve different system performance and energy efficiency indices [20-22]. The GA is a random derivative-free search technique for multi-objective global optimization [23] of a non-analytically defined multimodal function of many variables (parameters). The GA optimization is difficult to apply on-line as it interferes the plant operation, is slow - a great number of experiments are required, is inaccurate because of the many disturbances from the industrial environment, and is restricted by the system stability and parameter and signal constraints. The off-line GA optimization is based on an accurate plant model, an accepted fitness function and a sample of experiments/simulations used in its evaluation. In [20] a systematical determination of a multi-objective optimization as a weighted sum of criteria is suggested for a FLS PID auto-tuning with various fitness functions.

Main problems of the system with the two-level supervisor-based AFLC are the stability analysis and the complexity of the FLC-FLS structure when on-line improvement of several performance measures is considered. Then in order to preserve the FLS design simplicity separate fuzzy units (FUs) for each performance-tuning parameter channel are used which increases the number of the FUs in the FLS configuration. This makes the PLC programming difficult. To solve these problems in [24] a neuro-fuzzy Sugeno model representing a TSK-PDC is trained as an equivalent substitute of the two-level FLC-FLS structure of many FUs. The proper input-output training data is collected during the on-line operation of the complex FLC-FLS system. The simple Sugeno substitute can be easily programmed in a PLC and for it a corresponding TSK plant model derived which enables the fuzzy system stability analysis.

The aim of the present paper is to develop a simple fuzzy PDC equivalent to a designed supervisor-based process AFLC which is suitable for easy embedding in industrial PLCs and proving of closed loop system stability on the example of temperature control. The simplification procedure is based on a derived TSK plant model and the GA optimization techniques. The stability of the fuzzy closed loop system of a PDC-FLC and a TSK plant model is studied by deriving Lyapunov stability conditions and solving them using the Linear Matrix Inequalities (LMI) numerical technique.

The investigation is based on MATLAB^{TM}, simulation and real time experimentation [25,26] and considers the restrictions which the future completion on industrial PLC imposes on the simplified two-level FLC-FLS substitute [27,28].

The rest of the paper is organized as follows. In Chapter 2 the preliminary results on TSK plant modeling and design of a FLS-based AFLC for on-line ScFs auto-tuning in order to keep selected performance measures to desired Norms are presented and applied for the control of the air temperature in a dryer. Chapter 3 discusses the development of a PI-PDC functional equivalent of the FLS-based AFLC with structure based on the TSK plant model and GA optimized PI local controllers' parameters. In Chapter 4 the PI-PDC system stability is proven by derivation and solution of the Lyapunov stability conditions. The analysis of the results, the conclusion and the future work are outlined in Chapter 5.
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2. PRELIMINARY INVESTIGATIONS AND PROBLEM FORMULATION

The investigation stems from previous results related to TSK plant modeling and PI-FLC-FLS design [19, 22,24]. The plant is the air-temperature *y* in a laboratory dryer controlled by changing the voltage *u* to a Pulse-Width Modulator (PWM) thus varying the duty ratio of switching of an electrical heater and a fan. The dryer is equipped with industrial sensors, transmitters, solid state relays, a fan, an electrical heater and an interfacing board to a computer supplied with analog-to-digital converters and digital outputs. The controller, the PWM, the graph recorders and the noise filters are accomplished as software in a Simulink model.

The TSK plant model is determined on the basis of experimental study of the plant step responses in different operation points. Three overlapping linearization zones are distinguished with the increase of the plant output *y* on the base of similarity of adjacent step responses. Average Ziegler-Nichols (ZN) plant models are obtained for each zone $P_i(s) = K_i . e^{-\tau_i s} . (T_i s + 1)^{-1}$, i=1÷3, $\mathbf{q}_{ZN}$=[K_1=11.5,

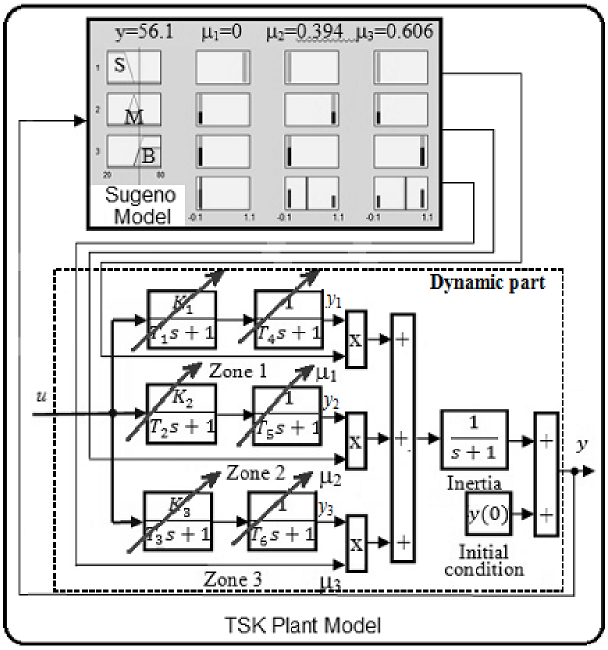

Fig. 1 TSK plant model for the air temperature $\mathbf{q}_{TSK}$ =[K_1=13.72, K_2=12.2, K_3=8.3, T_1=57.3, T_2=96.5, T_3=49.1, T_4=11.6, T_5=15.4, T_6=2.5]

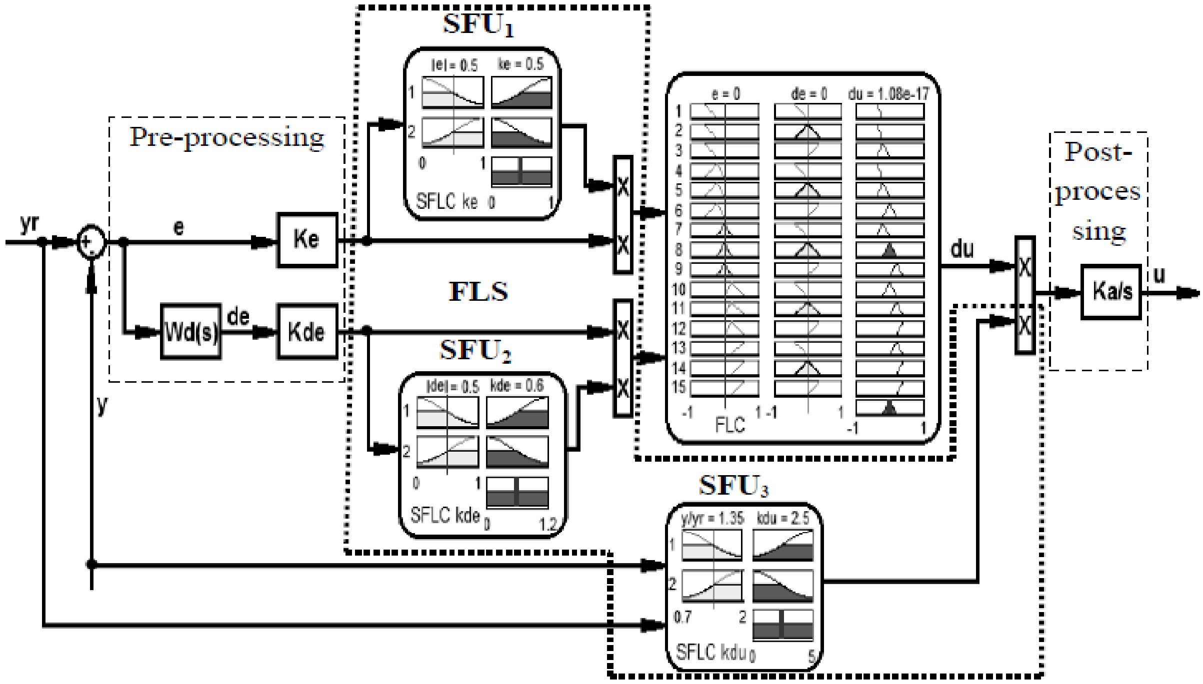

Fig. 2 Block Diagram of the Designed PI-FLC-FLS

T_1=95, τ_1=4; K_2=6.5, T_2=86, τ_2=3.7; K_3=4.5, T_3=50, τ_3=6.3]. The different ZN model parameters in the linearization zones prove plant nonlinearity and justify the choice of FLC for its control.

The TSK plant model is shown in Fig.1. It is built of a dynamic part and a Sugeno Model - fuzzy blender for the outputs y_i of the dynamic part. The Sugeno Model with input y in the range [20, 80] ^{o}C, which is partitioned by 3 MFs [S M B] for Small, Medium and Big, allocated according to the definition of the three overlapping zones is used to identify the linearization zones and to yield three outputs of singletons for the degrees of belonging μ_i of the current measured y to each zone. The dynamic part is comprised of three parallel channels with input – the plant input u. Each channel consists of two time-lags in series - the first with the physical meaning of the time lag in the ZN model, and the second – the linear term of the Taylor's series expansion of the time delay element in the ZN model - $e^{-\tau_i s} \approx (\tau_i s+1)^{-1}$. The inertia of the plant is increased by the common time lag - (K=1, T=1) at the plant model output, where the initial condition (initial or ambient air temperature y(0)) in the dryer is added.
The TSK plant model parameters – gains K_i and time constants T_j, j=1÷6, are GA optimized to minimize the integral squared relative error between experimentally recorded real plant and simulation model outputs with respect to real plant output.

The designed FLC-FLS is shown in Fig. 2. It consists of a main Mamdani PI-FLC and a FLS of three FUs – $SFLC_{ke}$, $SFLC_{kde}$, $SFLC_{kdu}$ with subscripts denoting the scale to the corresponding ScF. Inputs are the normalized magnitude of error |e|, of derivative of error |de| and of the relative with respect to reference plant output y_r/y respectively. The PI-FLC has pre-processing of a differentiator with transfer function $W_d(s)=K_dT_ds(T_ds+1)^{-1}$ and an integrating post-processing 1/s. The ScFs K_e and K_{de} normalize the FLC inputs - the system error e and the derivative of error de respectively, while K_a denormalizes the FLC output - the derivative of control du. The design is based on the approach, developed in [19]. First the main PI-FLC is determined – FU with standard both MFs in normalized universes of discourse and rule base, necessary pre- and post-processing and parameter tuning – T_d=3, K_d=50, K_a=0.025, K_e=K_{de}=0.1 [24]. Then the FLS is specified – number of FUs, their input and output connections and ranges (necessary measurements, performance indicators computations, PI-FLC parameters to be tuned), MFs in absolute universes of discourse and rule bases.

The problem is to reduce the number of FUs in the designed PI-FLC-FLS in order to ease its implementation in an industrial PLC. The equivalent in behaviour simplified FLC should also allow to study the closed loop system stability. Proper type for the simple FLC is the PDC [29,30] since the available TSK plant model facilitates its design determining the PDC structure – input y, the same simple Sugeno Model of r rules with singletons as outputs –available as a fuzzy block in most PLCs [27,28], for identification of linearization zones and for fuzzy blending the outputs u_i of three PI linear controllers – one for each local plant.

This structure of the PI-PDC is easy to program for PLCs and to compute in real time as it consists of one simple FU and linear PI controllers. Besides, for a system of a TSK plant model and a PI-PDC Lyapunov stability conditions can be derived and solved in order to study the closed loop system stability. Therefore, the problem is to tune the parameters of the PI linear controllers from requirement for functional equivalence between the system with the simple PI-PDC and the system with the easily designed FLS-based AFLC. One possible tuning approach is the GA optimization.

3. DEVELOPMENT OF PI-PDC FUNCTIONAL EQUIVALENT OF THE FLS-BASED AFLC

The structure of the suggested PI-PDC functional equivalent of the FLS-based AFLC is shown in Fig. 3. It is determined by the TSK plant model. To each local plant in a linearization zone a linear PI controller with input the system error e is assigned. The PI-PDC output u is computed as weighted by μ_i sum of the individual PID controllers' outputs u_i. This corresponds to fuzzy blending and defuzzification, which are performed by the same Sugeno Model like the used in the TSK plant model for identification of the degree of belonging of the current plant output to each defined zone of linear range of operation. The PI controllers' parameters $\mathbf{q}_{PI}=[K_{p1}, T_{i1}; K_{p2}, T_{i2}; K_{p3}, T_{i3}]$ are tuned from criterion for minimization of the relative difference between the outputs of the designed PI-FLC-FLS (FLS-based AFLC) system $y_{AFLC}(t)$ (target) and the simplified PI-PDC system $y_{PDC}(t)$ using GA with fitness function:

$$\mathbf{F}=\int[E(t)/y_{AFLC}(t)]^2.dt, \qquad (1)$$

where $E(t)=y_{AFLC}(t)-y_{PDC}(t)$.

The GA minimization of $\mathbf{F}$ is carried out in simulations using the TSK plant model in Fig. 1. The outputs of the two systems – the PI-FLC-FLS and the simplified PI-PDC, are recorded as responses to one and the same pattern of generated random in magnitude and duration step references changes $y_r(t)$. Thus independent of the specific references and rich in magnitudes and frequencies signals are ensured in the whole range of the input signal, necessary for the PI-PDC to learn the AFLC system nonlinearity and ability to adapt. The GA performs selection, crossover and mutation to build a new generation of offspring and provide a new set of controller's parameters. After a new simulation the fitness (1) is computed. This cycle continues till the end condition is met – the final number of generations of 100 is reached. The parameters of the GA are: population size=20; number of generations=100; elite=2; crossover rate = 0.8 and method – single point; mutation operator –adapt feasible; fitness scaling – rank based; selection – roulette wheel; binary coding. The bounds for the tuning parameters $\mathbf{q}_{PI}$ are computed on the base of approximate tuning of the local PI-controllers considering a simple empirical engineering method [24] and the parameters of the average ZN local plants models.

The simulation is fast, more realistic, with full control on the experiment, without unplanned noise or disturbances, safe for the plant which ensures the best solution with respect to system stability and gradual parameter changes without restrictions on parameters and signals. It allows various inputs – reference changes and disturbances with different magnitude and frequency that cover the realistic industrial environment impact range. However, a reliable plant model is required for the whole range of operation conditions.

For the tuned PI controllers' parameters it is computed $\mathbf{q}^{opt}_{PI}=[K_{p1}=0.09,\ T_{i1}=59.2;\ K_{p2}=0.11,\ T_{i2}=70.6;\ K_{p3}=0.07,\ T_{i3}=38.3]$.

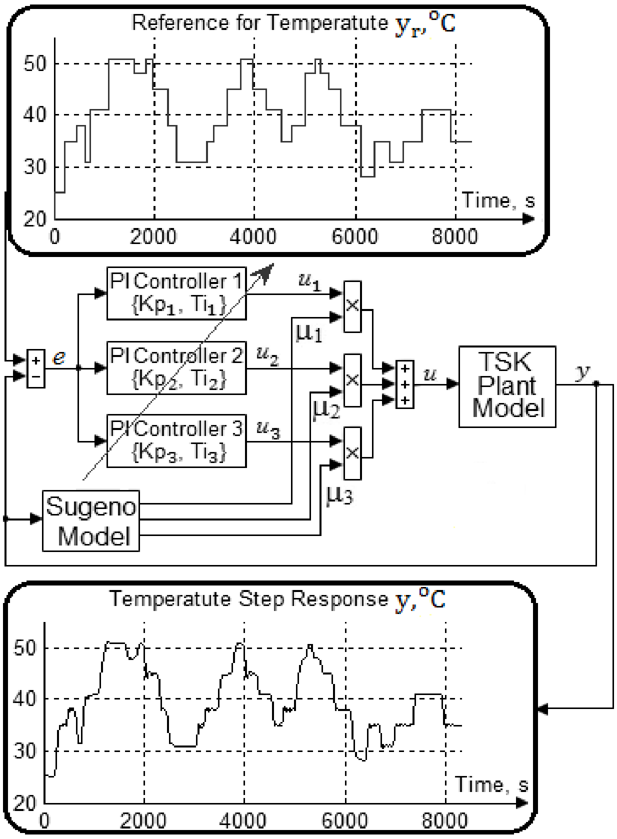

Fig. 3 PI-PDC equivalent system with random reference input y_r and target output y_{AFLC} from FLS-based AFLC for GA tuning of the linear PI controllers' parameters

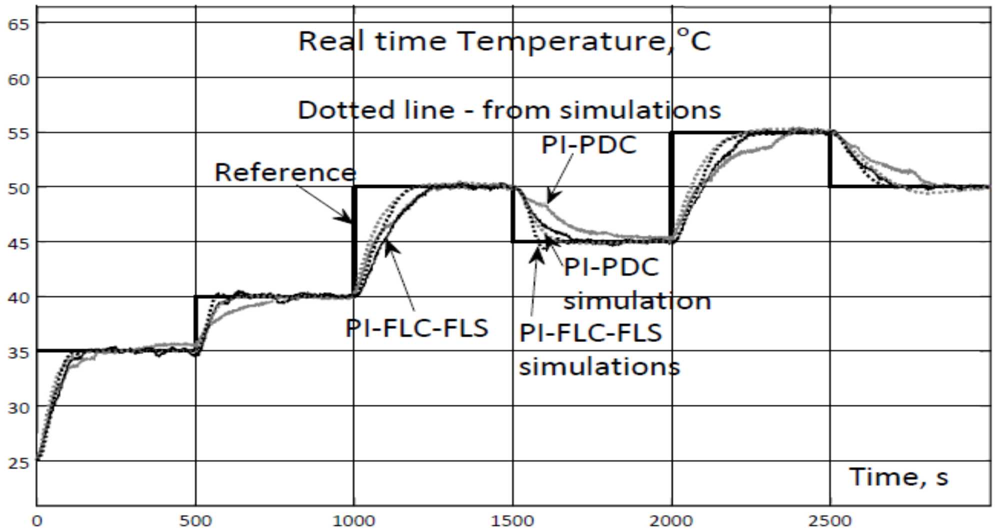

Fig. 4 PI-PDC and PI-FLC-FLS systems real time temperature step responses

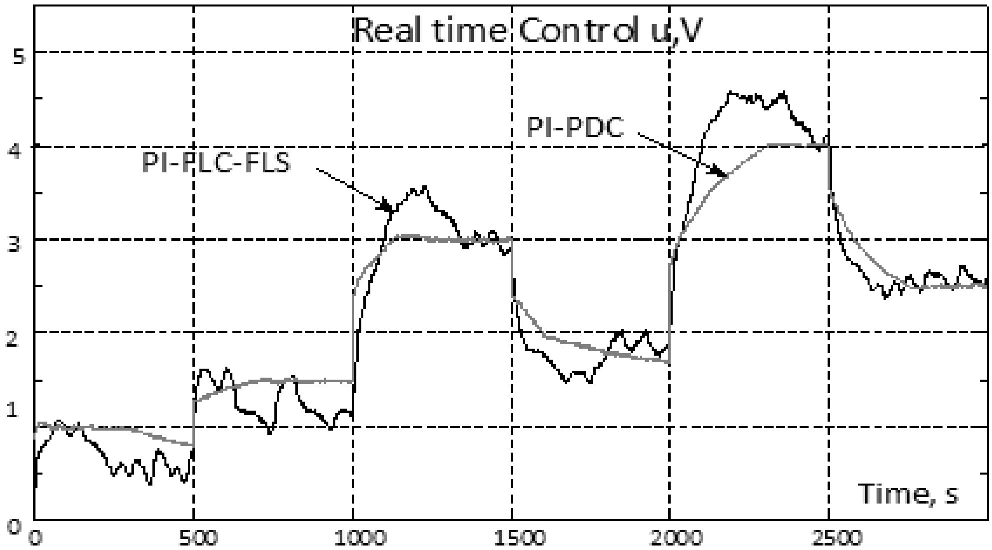

Fig. 5 PI-PDC and PI-FLC-FLS systems real time control

The validation of the tuned PI-PDC is based on the laboratory dryer real time control and experimental investigation of the temperature step responses in the systems with PI-PDC and with PI-FLC-FLS to input references, different from the used in modeling. The temperature step responses in the two systems are shown in Fig. 4. The temperature responses of the system with the equivalent in functioning but simple in structure (with 1 FU) PI-PDC are close to the temperature responses of the system with the easy to design but more complex in structure (with 4 FUs) PI-FLC-FLS. In Fig. 4 are given also for comparison the simulated temperature step responses in dotted lines. The simulation results are close to the obtained from the real time control of the dryer air temperature.

In Fig. 5 is given the control. The grey graph corresponds to the PI-PDC system while the black - to the PI-FLC-FLS. The PI-PDC uses less and more smooth control with no peaks and oscillations, which is good for the actuator and a prerequisite for energy efficiency – peaks and falls of control means overheating followed by cooling, high values for the control means longer duration of connecting of heater and fan to nets, so a greater energy consumption for the same system performance, expressed in fast and robust temperature step responses with no overshoot in all operation points despite the changed plant parameters.

In Fig. 6 are shown the temperature responses from simulations of systems with designed various controllers [19,22,24] – linear PI (dashed line), ordinary PI-FLC (black) and ordinary GA optimized PI- PDC (light grey),

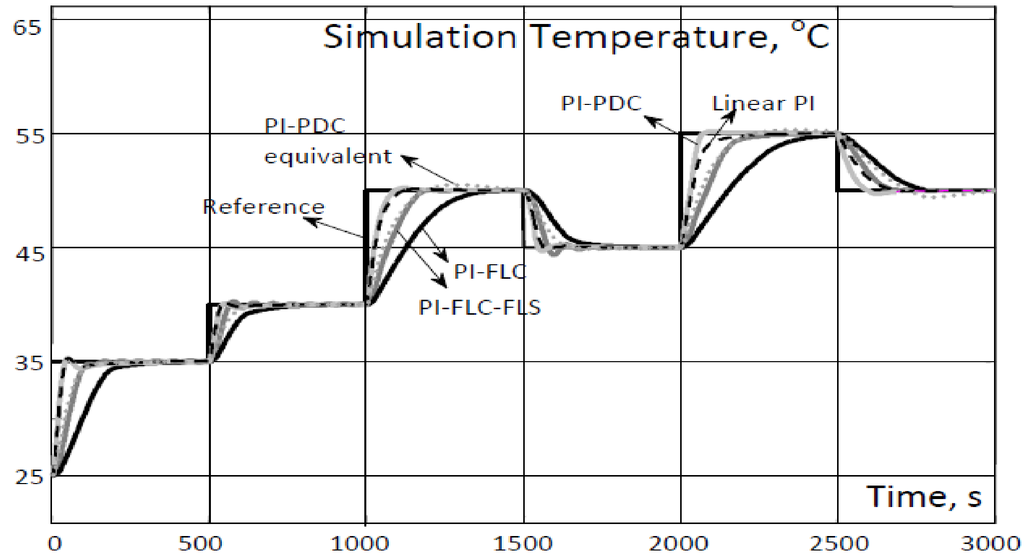

Fig. 6 Temperature step responses from simulation of systems with various fuzzy controllers

PI-PDC equivalent (grey dotted) and PI-FLC-FLS (dark grey). This investigation proves the PI-FLC control is better than the linear PI as it ensures no overshoot and similar in performance step responses in all operation areas, so it is robust. However, it is slower. The PI-PDC is almost like linear PI for low references - fast but with small overshoot. For high references it becomes faster. Further improvement is achieved by the PI-FLC-FLS two-level structure – a little less fast but without overshoot robust responses. Its simplified PI-PDC equivalent has close step responses – so by simple means the ordinary PI-FLC is made faster and the overshoot of the PI-PDC and linear PI eliminated. Since simulation results are closed to the obtained in real time control for PI-PDC equivalent and PI-FLC-FLS as seen in Fig. 4, the relationship among all types of control from simulations in Fig. 6 can be considered trustful and is confirmed in real time control in previous investigations [19,22,24].

4. STABILITY ANALYSIS OF PI-PDC SYSTEM

The design of an ordinary PI-PDC is based on tuning of the local linear PI controllers considering the average local plant models in the conclusions of the TSK plant model and the selected method from the linear control theory that ensures local linear system stability, desired performance and robustness [24,29]. The global nonlinear fuzzy system stability is studied mainly by the help of Lyapunov methods and linear matrix inequalities (LMIs) numerical technique [24,29-35]. The model-based stability conditions and the corresponding LMIs have been already derived for a great number of TSK-PDC systems in which various system peculiarities such as plant uncertainties, time-delays, signal constraints, constraints on system performance, etc. and conservatism reduction techniques are considered. In the present research the PI-PDC is designed and GA tuned to be functionally equivalent to a FLS-based AFLC. The local linear systems tuning is not based on stability requirements. Therefore, the Lyapunov stability analysis is of crucial importance especially considering the PI-PDC system adaptive properties. The stability analysis steps on derived TSK plant model and designed TSK-PDC which are described with the following fuzzy rules:

IF z_1 is M_{i1} **AND**…**AND** z_n is M_{in} **THEN**

$$\left| \begin{aligned} \dot{x}_i(t) &= \mathbf{A}_i x_i(t) + \mathbf{B}_i u_i(t) \\ y_i(t) &= \mathbf{C}_i x_i(t) + \mathbf{D}_i u_i(t) \end{aligned} \right. \tag{2}$$

IF z_1 is Lz_1 **AND**…**AND** z_n is Lz_n **THEN** $u_i=-\mathbf{F}_i x_i$,

where:

- $i=1\div r$, the number of the rules r is equal to the number of the linearization zones;
- the premise variables z with linguistic values M recognize the zone for the corresponding local linear plant model;
- the local linear plant models in the conclusions of the fuzzy rules are described by the state vector x_{nx1}, the output vector y_{mx1}, the input control vector u_{dx1} and the corresponding matrices $\mathbf{A}_{nxn}$, $\mathbf{B}_{nxd}$, $\mathbf{C}_{mxn}$ and $\mathbf{D}_{mxn}$;
- the local linear controllers for each local plant are presented in the state space as a state feedback - $u=-\mathbf{F}x$, dynamic compensator, PID, etc.

The overall nonlinear plant output is computed as a fuzzy blending of individual rules (local plants) outputs via the fuzzy inference and defuzzification mechanisms. The final nonlinear control is a fuzzy blending of the individual rules control actions.

The used in the present investigation ZN linear plant models and PI linear local controllers lead to modification of the fuzzy rules (2) [24,30]:

$$\textbf{IF } y(t) \text{ is } \mathrm{M_i} \textbf{ THEN } \left| \begin{array}{l} \dot{x}_i(t) = \mathbf{A}_{io} x_i(t) + \mathbf{B}_{id} u_i(t - \tau_i) \\ y_i(t) = \mathbf{C}_i x_i(t) \end{array} \right. \tag{3}$$

$$\textbf{IF } y(t) \text{ is } \mathrm{M_i} \textbf{ THEN } \left| \begin{array}{l} \dot{u}_i(t) = -\mathbf{F}_i x_i(t) + \mathbf{G}_i x_r \\ \text{or } \dot{u}_i(t) = K_{pi}\dot{e}(t) + (K_{pi}/T_{ii}) e(t) \end{array} \right. ,$$

where:

$$x_i(t) = \begin{bmatrix} x_{i1}(t) = y(t) \\ x_{i2}(t) = \dot{x}_{i1}(t) \end{bmatrix}, \ \mathbf{A}_{io} = \begin{bmatrix} 0 & 1 \\ 0 & -1/T_i \end{bmatrix}, \ \mathbf{B}_{id} = \begin{bmatrix} 0 \\ K_i / T_i \end{bmatrix},$$

$$\mathbf{C}_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \text{ and } x_r = \begin{bmatrix} x_{r1} = y_r \\ x_{r2} = 0 \end{bmatrix};$$

$$\mathbf{F}_i = \begin{bmatrix} K_{pi}/T_{ii} & K_{pi} \end{bmatrix}, \mathbf{G}_i = \begin{bmatrix} K_{pi}/T_{ii} & 0 \end{bmatrix}.$$

The Lyapunov sufficient condition for quadratic stability of the closed loop system (3) is the existence of matrices **P**>0, and **Q**>0 such that the following matrix inequalities are satisfied for $i, j=1 \div r, j>i$ [29-30]:

$$\mathbf{PA}_{io} + \mathbf{A}_{io}^T\mathbf{P} + \mathbf{PB}_{id}\mathbf{F}_i\mathbf{Q}^{-1}\mathbf{F}_i^T\mathbf{B}_{id}^T\mathbf{P} + \mathbf{Q} < 0 \tag{4}$$

$$\begin{array}{l} 0.5\mathbf{P}(\mathbf{A}_{io} + \mathbf{A}_{jo}) + [0.5(\mathbf{A}_{io} + \mathbf{A}_{jo})]^T\mathbf{P} + \\ + 0.5(\mathbf{B}_{id}\mathbf{F}_j\mathbf{Q}^{-1}\mathbf{F}_j^T\mathbf{B}_{id}^T + +\mathbf{B}_{jd}\mathbf{F}_i\mathbf{Q}^{-1}\mathbf{F}_i^T\mathbf{B}_{jd}^T) + \mathbf{Q} \le 0 \end{array} \tag{5}$$

The LMIs numerical computation technique for solving optimization problems of mathematical programming under convex restrictions is applied for the solution of (4)-(5) [29,33]. The nonlinear inequality (4) is first turned to linear by the help of the Shur decomposition:

$$\begin{bmatrix} \mathbf{PA}_{io} + \mathbf{A}_{io}^T\mathbf{P} + \mathbf{Q} & \mathbf{PB}_{id}\mathbf{F}_i \\ \mathbf{F}_i^T\mathbf{B}_{id}^T\mathbf{P} & -\mathbf{Q} \end{bmatrix} < 0 \tag{6}$$

Then the LMIs of the problem are obtained. They consist of (6), (5) and

$$-\mathbf{P} < -\mathbf{O} \text{ and } -\mathbf{Q} < -\mathbf{O}. \tag{7}$$

Here the novelty is in the introduction of a small positively determined matrix **O** (accepted $\mathbf{O}=10^{-3}\mathbf{I}$) instead of a zero matrix. This aims at ensuring of positively determined solutions for the matrices **P** and **Q** in the case of PI-PDC which contains an integrator that makes the open loop system critically stable. The solutions **P** and **Q** should also guarantee a low sensitivity to parameter variations, inaccurate data and computational errors, so they should have small condition numbers.

The computed matrices are:

$$\mathbf{P} = \begin{bmatrix} 0.0041 > 0 & -0.8141 \\ -0.8141 & 177.9804 \end{bmatrix}, \ \mathbf{Q} = \begin{bmatrix} 0.0004 > 0 & -0.0020 \\ -0.0020 & 2.5347 \end{bmatrix}$$

$$\det(\mathbf{P}) = 0.0740 > 0 \qquad \det(\mathbf{Q}) = 0.0011 > 0$$

These matrices stay positively determined for rounded data as well.

5. ANALYSIS OF RESULTS, CONCLUSION AND FUTURE WORK

The main results of the present research conclude in the following.

An approach is suggested for development of a simple fuzzy PDC functional equivalent to a designed FLC-FLS adaptive controller. The FLC-FLS controller needs several fuzzy blocks to be easily designed. The PDC equivalent is built of a single fuzzy block and several linear controllers – an economic structure with respect to the needed computational resources (memory and computation time) which enables easy embedding in any PLC for wide industrial applications in real time control of process variables.

The approach requires a TSK plant model which is possible to derive when the plants dynamics can be partitioned into a finite number of linear models, yielding a simple TSK model with only few fuzzy rules The TSK plant model determines the structure of the PDC. The parameters of the PDC are GA optimized from requirement of functional equivalence between the systems with PDC and with the initial FLC-FLS adaptive controller. The GA optimization is based on systems simulations using the TSK plant model and random in magnitude and duration step references to enable the PDC system to learn the original FLC-FLS system nonlinearity and ability to adapt.

The approach is applied to the control of the air temperature in a laboratory dryer. The simplified PDC equivalent is validated in real time temperature control for references different from the used in its design. The outputs of the two systems – with PI-PDC and FLC-FLS controllers are close; the PDC system has a smoother and more economic control.

The TSK plant model with linear local models with time delay (processes are inertial) and the PI-PDC allow to study the global fuzzy closed loop system stability – essential problem since stability requirements are not included in the initial and simplified adaptive FLC system design. Existing Lyapunov conditions are applied and solved by LMI techniques to prove TSK-PDC system stability. A modification is introduced as a harder restriction to the possible solution to guarantee insensitivity to inaccurate data and to ensure stability in case of critically stable open loop system.

The future work will focus on completion of the PI-PDC on a Siemens PLC and its implementation for the control of other industrial plants.

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REFERENCES

[1] Astrom K.J and Wittenmark B (1989) *Adaptive Control*. New York: Addison-Wesley.
[2] Aydogdu O and Alkan O (2013) Adaptive control of a time-varying rotary servo system using a fuzzy model reference learning controller with variable adaptation gain. *Turkish Journal of Electrical Engineering & Computer Sciences.* Epub ahead of print 25 Nov 2013. DOI:10.3906/elk-1201-115.
[3] Cerman O and Hussek P (2011) Fuzzy model reference learning control with modified adaptation mechanism. In: *7th international conference on natural computation*, Shanghai, Chinese, 26-28 July 2011, pp. 123-128. Shanghai: Institute of Electrical and Electronics Engineers.
[4] Cho Y-W, Seo K-S and Lee H-J (2007) A direct adaptive fuzzy control of nonlinear systems with application to robot manipulator tracking control. *International Journal of Control, Automation, and Systems* 5(6): 630-642.
[5] Passino KM and Yurkovich S (1998) *Fuzzy Control*. California: Addison-Wesley Longman.
[6] Ram A and Lincoln S (2013) A model reference-based fuzzy adaptive PI controller for non-linear level process system. *International Journal of Research and Reviews in Applied Sciences* 14(2): 477-486.
[7] Kaminskas V and Liutkevicius R (2002) Adaptive fuzzy control of nonlinear plant with changing dynamics. *Informatica* 13(3): 287–298.
[8] Rojas I, Pomares H, Gonzalez J, Herrera L, Guillen A, Rojas F and Valenzuela O (2006) Adaptive fuzzy controller: Application to the control of the temperature of a dynamic room in real time. *Fuzzy Sets and Systems* 157: 2241-2258.
[9] Treesatayapun C (2014) Balancing control energy and tracking error for fuzzy rule emulated adaptive controller. *Applied Inteligencel.*Epub ahead of print 21 December 2013. DOI: 10.1007/s10489-013-0493-x.
[10] Kanagaraj N, Sivashanmugam P and Paramasivam S (2009) A Fuzzy logic based supervisory hierarchical control scheme for real time pressure control. *International Journal of Automation and Computing* 6(1): 88-96.
[11] Karasakal O, Yesil E, Guzelkaya M and Eksin I (2005) Implementation of a new self-tuning fuzzy PID controller on PLC. *Turkish Journal Electrical Engineering* 13(2): 277-286.
[12] Lutfy O and Khduir E (2013) A PID-like ANFIS Controller Trained by PSO Technique to Control Nonlinear MIMO Systems. *Australian Journal of Basic and Applied Sciences* 7(4): 626-640.
[13] Sharma K, Ayyub M, Saroha S and Faras A (2014) Advanced controllers using fuzzy logic controller for performance improvement. *International Electrical Engineering Journal* 5(6): 1452-1458.
[14] Soyguder S, Karakose M and Alli H (2009) Design and simulation of self-tuning PID-type fuzzy adaptive control for an expert HVAC system. *Expert Systems with Applications* 36(3): 4566–4573.
[15] Tsai M-T, Tung P-C and Chen K-Y (2011) Experimental evaluations of proportional–integral–derivative type fuzzy controllers with parameter adaptive methods for an active magnetic bearing system. *Expert Systems* 28(1): 5-18.
[16] Zulfatman and Rahmat M (2009) Application of self-tuning fuzzy PID controller on industrial hydraulic actuator using system identification approach. *International Journal on Smart Sensing and Intelligent Systems* 2(2): 246-261.
[17] Driankov D, Hellendoorn H and Reinfrank M (1993) *An Introduction to Fuzzy Control*. New York: Springer-Verlag.
[18] Li H.-X. and Tso S (1999) A Fuzzy PLC with Gain-Scheduling Control Resolution for a Thermal Process – A Case Study. *Journal Control Engineering Practice* 7: 523-529.
[19] Yordanova S and Yankov V (2014) Fuzzy supervisor for nonlinear self-tuning of controllers in real time operation. *Proceedings of the Technical University of Sofia.* Sozopol: Bulgaria, 131-140.
[20] Wu C-J, Ko C-N, Fu Y-Y and Tseng C-H (2009) A Genetic-Based Design of Auto-Tuning fuzzy PID Controllers, *International Journal of Fuzzy Systems* 11(1): 49-58.
[21] Yordanova S and Georgieva A (2013) Genetic algorithm based optimization of fuzzy controllers tuning in level control. *Journal Electrotechnica and Electronica E+E* 48(9-10): 45-51.
[22] Yordanova S and Sivchev Y (2014) Design and tuning of parallel distributed compensation-based fuzzy logic controller for temperature. *Journal of Automation and Control* 2(3): 79-85.
[23] Haupt RL and Haupt S (1998) *Practical Genetic Algorithms*. New York: John Wiley & Sons.

[24] Yordanova S (2011) *Methods for Design of Fuzzy Logic Controllers for Robust Process Control*. Sofia: KING. (in Bulgarian)

[25] *Fuzzy logic toolbox. User's guide for use with MATLAB*, MathWorks Inc., 1998.

[26] *MATLAB – Genetic algorithm and direct search toolbox. User's guide*, MathWorks, Inc., 2004.

[27] *SIMATIC S7. Fuzzy control. User's manual*, SIEMENS AG, 2002.

[28] *IEC 61131–Programmable controllers*. 1999.

[29] Tanaka K and Wang HO (2001) *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. New York: John Wiley & Sons.

[30] Yordanova S (2009) Lyapunov stability and robustness of fuzzy process control system with parallel distributed compensation. *Information Technologies and Control* 4:38-48. (Bulgarian Union of A&I)

[31] Fang C-H, Liu Y-S, Kau S-W, Hong L and Lee C-H (2006) A new LMI-based approach to relaxed quadratic stabilization of T-S fuzzy control systems. *Institute of Electrical and Electronics Engineers Transactions on Fuzzy Systems* 14(3): 386-397.

[32] Gu Y, Wang H.O. and Tanaka K (2001) Fuzzy control of nonlinear time-delay systems: stability and design issues. *Proceedings of the 2001 American Control Conference*. Arlington: VA, 25-27.

[33] Lam H and Leung F (2007) Fuzzy controller with stability and performance rules for nonlinear systems. *Journal Fuzzy Sets and Systems* 158(2): 147-163.

[34] Li L and Liu X (2007) New approach on robust stability for uncertain T–S fuzzy systems with state and input delays. *Chaos, Solitons & Fractals* 40(5): 2329-2339.

[35] Yoneyama J (2007) New robust stability conditions and design of robust stabilizing controllers for Takagi–Sugeno fuzzy time-delay systems. *Institute of Electrical and Electronics Engineers Transactions on Fuzzy Systems* 15(5): 828-839.