Modeling of 3D Impedance Tube with a Complex Termination Impedance using Finite Element Method

Tsvetan Nedkov[1]

Abstract – **Impedance tubes are used to measure the acoustic impedance of a sound absorbing material and are essentially one-dimensional wave guides, with a source at one end and the acoustic load – typically the test sample of the absorptive material – placed at the other end. There are two common methods by which the impedance of the material is measured in an impedance tube. The first involves a moveable microphone that traverses the length of the tube. This method is the older and arguably simpler of the two methods but is slow. The second method is known as the "two-microphone" or "transfer function" method and will be the focus of the method employed in this section. In this study will be modeled a 3D impedance tube with a complex termination using finite element method software and the results will be compared with experimental data.**

Keywords – **Impedance tube, 3D modelling, Finite element method, Complex impedance termination.**

I. Introduction

Characterizing the absorptive properties of acoustic materials is critical for understanding their behavior when deployed in engineering applications. Mainly for building and industrial acoustic or high frequency noise control absorptive materials are often used. This engineering solution is known as passive noise control and is applied in room acoustics, enclosures, boxes, silencers and barriers. Different types of absorptive materials are available and mainly used are produced from stone or glass wool, technical polyurethane foams and wood based fiber plates.

The most important acoustic parameter for porous materials is absorption coefficient. According to it, these materials are classified as absorptive or reflective. The absorption coefficient can be defined as the relationship between the acoustic energy that is absorbed by a material and the total incident impinging upon it. This coefficient is limited between 0 for non absorbent materials and 1 for totally absorbent. Measurement methods have been developed and standardized in order to enable the study of the acoustic properties of different materials in order to use these data in acoustic projects [1]. There are two types of measurement of absorptive materials: in reverberation room described in EN ISO 354:2003 [2] and measurement in impedance tube described in EN ISO 10534:1 [3] and 10534:2 [4].

Using impedance tube method accurate acoustic absorbing measurements can be provided for normal incident sounds waves only [5]. So with the methods described by [2] or [3] it is possible to determine the normal incidence absorption coefficient and the specific acoustic surface impedance. At low frequencies the impedance tube method will not give accurate results because the sample must be attached air tightly and at the same time the sample must be able to vibrate freely [6].

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II. Theory

With the application of the two-microphone method the impedance, complex reflection coefficient, and the normal-incidence sound absorption coefficient can be determined. Regarding the one-dimensional wave guide shown in Fig. 1, it is excited by plane wave source on the left side ($x = -L$) while it is terminated with a complex impedance, Z, at the opposite end ($x = 0$). Two pressure sensors (microphones) are located at $x = -X_1$ and $x = -X_2$ and are used to determine the magnitude of the forward and backward traveling waves, from which the impedance, flow resistivity, and sound absorption coefficient may be determined [7]. It is assumed that there are no losses along the length of the tube and that only plane waves propagate. The origin of the system is at the right end of the tube and the wave incident on the specimen will be traveling in a positive x direction.

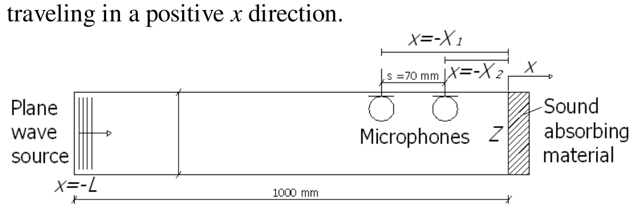

Fig. 1. Impedance tube with source at one end ($x = -L$) and impedance, Z, at the opposite ($x = 0$). The two microphones are used to estimate the termination impedance using the two-microphone technique.

The sound absorption coefficient of a material, α, is defined as the ratio of sound power absorbed by a surface to the incident sound power. For a plane wave at normal incidence, the sound absorption coefficient is given by [8]:

$$\alpha = 1 - |r|^2 \tag{1}$$

r is the complex sound reflection coefficient and is defined as follow equation:

$$r = \frac{p_r}{p_i} \tag{2}$$

p_r is reflected pressure, p_i is the incident pressure.

The incident and reflected waves can be written as [7]:

$$p_i = p_0 e^{j(\omega t - kx)} \text{ And } p_r = r p_0 e^{j(\omega t + kx)} \quad (3)$$

where p_0 is the amplitude of the incident wave, rp_0 is the amplitude of the reflected wave, $\omega = 2\pi f$ is the angular frequency, and k is the wave number. The total sound pressure, p_t , at a location x in the tube is the sum of the forward and backward traveling waves:

$$p_t = p_0 e^{j(\omega t - kx)} + r p_0 e^{j(\omega t + kx)} \quad (4)$$

and the total particle velocity, u_t ,is given by:

$$u_t = \frac{p_0}{\rho_0 c_0}\left(e^{j(\omega t - kx)} - r e^{j(\omega t + kx)}\right) \quad (5)$$

The complex transfer function between the two microphones located at $x = -X_1$ and $x = -X_2$ is:

$$H_{12} = \frac{p_1}{p_2} = \frac{p_0 e^{j(\omega t + kX_1)} + r p_0 e^{j(\omega t - kX_1)}}{p_0 e^{j(\omega t + kX_2)} + r p_0 e^{j(\omega t - kX_2)}}, \quad (6)$$

which can be solved for ***r*** giving the reflection coefficient as a function of microphone positions and the transfer function between the two as [9] :

$$r = e^{2jkX_2} \frac{H_{12} - e^{-jk\Delta X}}{e^{jk\Delta X} - H_{12}}, \quad (7)$$

where $\Delta X = X_2 - X_1$ .Hence, knowing the locations of the microphones , the complex sound reflection coefficient, and thus sound absorption coefficient, can be determined directly from the pressure transfer function between two microphones. At the same time, the reflection coefficient can be represented as:

$$r = \frac{p_r}{p_i} = \frac{Z - Z_0}{Z + Z_0}, \quad (8)$$

where $Z_0 = \rho_0 c_0$ is the characteristic impedance of the fluid and Z is the specific acoustic impedance of the absorbing surface.

The specific acoustic impedance at any point, x, is defined as the ratio of the total pressure and total particle velocity and is given by [10]:

$$Z(x) = \frac{p_t(x)}{u_t(x)} = \rho_0 c_0 \frac{1 + r e^{2jkx}}{1 - r e^{2jkx}} \quad (9)$$

The specific acoustic impedance ratio (or normalized acoustic impedance) is defined as:

$$\frac{Z}{Z_0} = \frac{p_t}{p_0 c_0 u_t} = \frac{1 + r e^{2jkx}}{1 - r e^{2jkx}} \quad (10)$$

The impedance ratio as $x = 0$ is therefore

$$\frac{Z}{Z_0} = \frac{1 + r}{1 - r} \quad (11)$$

And the complex reflection in terms of the impedance ratio is:

$$r = \frac{Z/Z_0 - 1}{Z/Z_0 + 1} \quad (12)$$

In terms of a real (resistive) part and an imaginary (reactive) part, the specific acoustic impedance ratio (also known as the normalized specific acoustic impedance) is:

$$\frac{Z}{Z_0} = R + jX, \quad (13)$$

where R is resistance and X is reactance. The equivalent admittance is:

$$\frac{Z}{Z_0} = \frac{1}{R + jX} = \left[\frac{R}{R^2 + X^2}\right] + j\left[\frac{-X}{R^2 + X^2}\right] \quad (14)$$

It can be shown that the sound absorption coefficient in terms of the real and imaginary parts of the impedance ratio is given by:

$$\alpha = \frac{4R}{(R^2 + X^2) + 2R + 1} = 1 - |r|^2 \quad (15)$$

III. Modeling

A 3D configuration of the test set-up as described in10534:2 [4] was modeled using FEM software Ansys. It was used a linear 3D brick acoustic fluid element FLUID 30 defined by 8 nodes [11]. The acoustic body of the tube is presented with two parameters: speed of sound in air and volume density. At the end of the tube is inserted Harmonic Mass Source (Loudspeaker) generating plane wave and on the opposite side Acoustic attenuation surface (the tested specimen from PU elastic foam). At two different positions (Fig. 2) are placed microphones. It was provided three different models of the mesh size showed in Table I. As the observed frequency range is between 244 Hz to 2 kHz, so the mesh element size of Model 1 is 0.01 m to ensure that there are at least 16 elements per wavelength at the highest frequency of interest. A mesh sensitivity analysis was carried out and the maximum length must satisfy the equation:

$$E_l = \frac{c}{8f}, \tag{16}$$

where E_l is the maximum element length, c is the speed of sound in air and f is the highest frequency of excitation.

For Model 2 was applied equation (16) for every one-third-octave band in the software simulation. For Model 3 was applied equal mesh size of 0.014 m for frequency region 250 – 400 Hz in reason of save computation time and for comparison with Model 2.

TABLE I
ELEMENT SIZE DISTRIBUTION WITH RESPECT TO FREQUENCY AT ONE-THIRD-OCTAVE BANDS

Frequency	Model 1	Model 2	Model 3
250	0,01	0,17	0,014
315	0,01	0,14	0,014
400	0,01	0,11	0,014
500	0,01	0,086	0,086
630	0,01	0,068	0,068
800	0,01	0,054	0,054
1000	0,01	0,043	0,043
1250	0,01	0,034	0,034
1600	0,01	0,027	0,027
2000	0,01	0,021	0,021

The upper limiting frequency f_u is calculated from the follow condition [9]:

$$d < 0.58\lambda_u \tag{17}$$

The lower limiting frequency f_l is defined using the follow conditions:

$$s_0 > 0.05\lambda_l . \tag{18}$$

And the follow condition must be satisfied:

$$f_u . s_0 < 0.45 c_0 . \tag{19}$$

Taking into account described conditions for distance *s =70 mm* the following limiting frequencies can by calculated [11]:

f_u < 2193 Hz and f_l > 244 Hz ;

The parameters used in the analysis are presented in Table II.

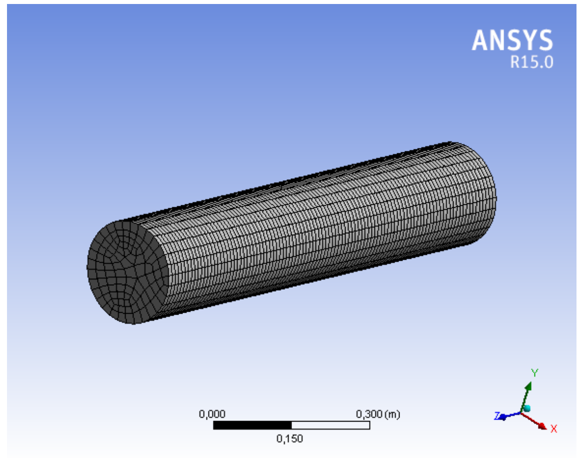

Fig. 2. Element size distribution of Model 1

TABLE II
PARAMETERS USED IN THE SIMULATIONS

Description	Unit	Value
Impeadance tube length	m	1
Impedance tube width	m	0,1
Speed of sound c_0	m/s	343,24
Density ρ	kg/m^3	1,2041
Acoustic flow	kg/s^2	1
Porosity φ	-	0,99
Tortuosity α_{ϖ}	-	1,02
Viscous charact. length Λ	m	0,000078
Thermal charact. length Λ	m	0,000192
Thickness of PU elastic foam	m	0,05
Flow resistivity σ	$Nm^{-4}s$	12569
Density of frame ρ_1	$kg\ m^{-3}$	8,85

IV. COMPARISON BETWEEN CALCULATED AND MEASURED RESULTS

To verify the correctness of the model are used measurements provided in the book of Allard and Atalla [7]. Three different mesh densities were applied in order to study the mesh sensitivity in calculating the absorption coefficient with respect to frequency. Comparing α coefficient values shown in Fig. 3, it can be seen that the mesh size has effect on the obtained results. The highest differences for all the frequencies were exhibited for a mesh size of 0.01 m (Model 1) followed by Model 2 even though comply with equation (16) for a frequency range of 244 to 400 Hz, the values obtained at these frequencies were poor compared to Model 3 mesh size tested.

When the absorption coefficient from a mesh size of 0.01 m for a frequency of 244 to 400 Hz were compared to higher mesh sizes of 0.017 m, 0.14 m, 0.11 m and 0.014 m, an average difference of α = 0.08 was observed with a lowest difference of α = 0.03 at 400 Hz and highest of α = 0.1 at 250 Hz. Therefore for a frequency range of 244 to 400 Hz a mesh size from Model 3 can yield satisfactory results compared to a mesh size of Models 1 and 2.

It was found that the best results for a frequency range of 250 – 2000 Hz were observed at mesh sizes of Model 3.

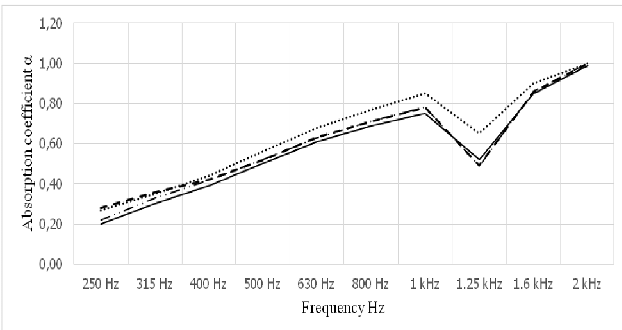

Fig. 3. Absorption coefficient: a) —— measured results; b) .... mesh size Model 1; c) - - - mesh size Model 2; d) — · · mesh size Model 3

V. Conclusion

From the provided simulations it's obvious that the usage of FEM models is applicable when measurement equipment like impedance tube is not available. Difficulty may occur in the possibility of collect correct parameters like viscous and thermal characteristic length of the tested absorbing specimen. For future work it can be observed frequency range extended from 63 to 244 Hz and between 2000 to 4000 Hz and the optimal mesh sizing for related frequencies.

Acknowledgement

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