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СЪДЪРЖАНИЕ том 67, книга 2

АВТОМАТИКА

1.	Дочо Цанков, Евтим Йончев, Тодор Йонков.....	15
	<i>Реализация и експериментално изследване на система: рекуперативен импулсен преобразувател-тягов постоянен ток електродвигател</i>	
2.	Христо Стоянов, Тодор Йонков.....	25
	<i>Управление на климатична камера за поддържане на температура и качество на въздуха в конферентни зали</i>	
3.	Владимир Христов.....	33
	<i>Концепции и аспекти в развитието на електромобилите</i>	
4.	Камен Христов.....	43
	<i>Методи за повишаване на енергийната ефективност при изграждане на асинхронни електрозадвижвания</i>	
5.	Иван Аврамов, Людмил Спиrow, Никола Ценов, Владислав Викторoв, Стефан Ангелoв, Димитър Георгиев.....	51
	<i>Развитие и усъвършенстване на функционалните възможности и локомоционните поведения на учебно - демонстрационни прототипи от малки хуманоидни роботи (част 1 - инженерно представяне на малките хуманоидни роботи и някои първи апробации)</i>	
6.	Иван Аврамов, Людмил Спиrow, Никола Ценов, Владислав Викторoв, Димитър Георгиев, Стефан Ангелoв.....	61
	<i>Развитие и усъвършенстване на функционалните възможности и локомоционните поведения на учебно - демонстрационни прототипи от малки хуманоидни роботи (част 2 - реални експерименти и виртуални 3D CAD моделни приложения, с цел, усъвършенстваня и нови функционалности)</i>	
7.	Найден Шиварoв, Денис Чикуртев, Ивайло Рангелoв, Александър Гигoв, Недко Шиварoв.....	71
	<i>Сензорна система за идентификация на препятствия при сервизни мобилни роботи</i>	
8.	Nikolay Bratovanov, Zlatko Sotirov, Vladimir Zamanov.....	81
	<i>Modeling and Simulation of Overconstrained Closed-Loop Mechanisms</i>	
9.	Емил Никoлoв.....	91
	<i>Функционална параметризация на динамични системи при априорна неопределеност</i>	
10.	Нина Никoлoвa.....	101
	<i>Репетитивни системи за елиминиране на периодична грешка по измерване</i>	
11.	Весела Карлoвa-Сергиевa, Александър Маринчeв.....	111
	<i>Робастно управление на обект с регулируема величина ниво</i>	
12.	Станислав Енeв.....	121
	<i>Субоптимално, по разход на гориво, управление на системи от втори ред с реални полюси</i>	

13.	Александър Маринчев	127
	<i>Решение на целочислена линейна оптимизационна задача посредством логическо програмиране</i>	
14.	Борис С. Грасиани	133
	<i>Фрактално управление на пневматично позициониращо устройство</i>	
15.	Борис С. Грасиани	143
	<i>Фрактално репетитивно управление на пневматично позициониращо устройство</i>	
16.	Кирил Борисов, Десислава Стоицева-Деличева, Васил Гълъбов	153
	<i>Интердисциплинарен подход за изследване на някои психологически характеристики</i>	
17.	Йорданка Генова Дунчева	161
	<i>Уейвлет анализ на хидрометеорологични времеви редове за седем български града</i>	
18.	Галина Чернева	171
	<i>Фрактални модели за апроксимация на случайни процеси</i>	
19.	Димитринка Владева	177
	<i>Проекциите на k-симплекс върху най-малките подсимплекси от произволен ред са диференцирания</i>	
20.	Димитринка Владева	187
	<i>Проекциите на k-симплекс върху най-големите подсимплекси от произволен ред са диференцирания</i>	
21.	Десислава Стоицева-Деличева, Борис Киров, Мартин Маринов	197
	<i>Разработване на програмно осигуряване за автоматизирано изграждане на виртуални метаболитни мрежи</i>	
22.	Борис Киров, Десислава Стоицева-Деличева, Мартин Маринов	205
	<i>Алгоритъм за анализ на баланса на потоците в метаболитни мрежи чрез рекурентни зависимости</i>	
23.	Емил Гарипов	213
	<i>Непрекъснат регулатор на Далин в схема на предиктор на Смит</i>	
24.	Цоньо Славов, Теофана Пулева	223
	<i>Линейно квадратично управление на мощността на ветрогенератор</i>	
25.	Цоньо Славов, Йордан Кралев, Петко Петков	233
	<i>Хардуерна симулация на ПИД регулатор за управление на система количка-махало</i>	
26.	Валентин Димитров, Алена Козакова, Йордан Кралев	243
	<i>Цифрово управление на температурата на почвата за стайни растения</i>	
27.	Камен Перев	253
	<i>Ортогонална апроксимация на нелинейни системи описани в ред на Волтера</i>	

28.	Божидар Раков, Георги Ружеков.	263
	<i>Програмна среда за проектиране на PLC ПИД управление на обект с два входа и два изхода</i>	
29.	Аспарух Марковски.	273
	<i>Учебна система "Hardware in the Loop"</i>	
30.	Йордан Кралев, Борислав Иванов, Иван Евг. Иванов, Андрей Йончев, Десислава Георгиева.	279
	<i>Един подход за управление на перфузионна помпа при нанопилтрация</i>	
31.	Димитър Юруков.	287
	<i>Методи за моделиране и управление на процесите в циментова пещ</i>	
32.	Христина Стойчева, Драгомир Чантов.	297
	<i>Проектиране на синхронизационни схеми на хаотични системи с изменящи се параметри на основата на втория метод за устойчивост на Ляпунов</i>	
33.	Стоян Димитров.	307
	<i>Последователни безквадратни числа от специален вид</i>	
34.	Стоян Димитров.	317
	<i>Диофантово неравенство с четири прости числа от специален вид</i>	
35.	Веселина Трашлиева.	327
	<i>Анализ на краткосрочна оптимизация на режима по активна мощност чрез множители на Лагранж</i>	
36.	Никола Георгиев, Василина Златанова, Снежана Терзиева.	337
	<i>Изследване на тактилни сензори с електропроводими еластомери захранвани с периодично несинусоидално напрежение</i>	
37.	Атанас Червенков, Тодорка Червенкова, Атанас Янев.	345
	<i>Симулиране на трифазен инвертор за квази-синусоидално напрежение</i>	
38.	Стоян Кирилов, Валери Младенов.	353
	<i>Изследване на зависимостите между ефективните стойности на входното и изходното напрежения и честотата на мемристорно-резисторна верига</i>	
39.	Zhivko Georgiev, Ivan Trushev, Atanas Chervenkov.	361
	<i>Transient and Steady-State Analysis of Electrical Circuits Supplied by Sequence of Rectangular Pulses by Z - Transform</i>	
40.	Симона Петракиева, Владимир Балабанов.	371
	<i>Web-базирано приложение за следене и управление на наличности в склад</i>	
41.	Илона Ячева, Малина Димитрова, Николина Петкова.	381
	<i>3D моделиране на разпределението на електромагнитно поле в близост до електропровод 400 kV</i>	
42.	Илона Ячева.	389
	<i>Определяне на параметри и зависимости в електромагнитни системи на базата на числено моделиране на електромагнитно поле</i>	

CONTENTS volume 67, Issue 2

AUTOMATICS

1.	Docho Tsankov, Evtim Yonchev, Todor Ionkov	15
	<i>The Realization and Experimental Study of System Regenerating Pulse Converter - Traction DC Motor</i>	
2.	Hristo Stoyanov, Todor Ionkov	25
	<i>Demand-Controlled Air Handling Unit for Conference Room Temperature and Indoor Air Quality Control</i>	
3.	Vladimir Hristov	33
	<i>Concepts and Aspects in the Development of Electric Vehicle</i>	
4.	Kamen Hristov	43
	<i>Method and Approaches of Increasing the Energy Efficiency at Developing of Asynchronous Drives</i>	
5.	Ivan Avramov, Lyudmil Spirov, Nikola Tsenov, Vladislav Viktorov, Stefan Angelov, Dimitar Georgiev	51
	<i>Development and Improvement of Functional Opportunities and Locomotion Behaviors of Study - Demonstration Prototypes of Small Humanoid Robots (Part One - Engineering Representation of Small Humanoid Robots and Some Initial Attempts)</i>	
6.	Ivan Avramov, Lyudmil Spirov, Nikola Tsenov, Vladislav Viktorov, Dimitar Georgiev, Stefan Angelov	61
	<i>Development and Improvement of Functional Opportunities and Locomotion Behaviors of Study - Demonstration Prototypes of Small Humanoid Robots (Part Two - Real Experiments and Virtual 3D CAD Model Applications for Improvement and New Functionalities)</i>	
7.	Nayden Chivarov, Denis Chikurtev, Ivaylo Rangelov, Aleksandar Gigov, Nedko Shivarov	71
	<i>Sensor System for Identification of Obstacles for Service Mobile Robots</i>	
8.	Nikolay Bratovanov, Zlatko Sotirov, Vladimir Zamanov	81
	<i>Modeling and Simulation of Overconstrained Closed-Loop Mechanisms</i>	
9.	Emil Nikolov	91
	<i>Functional Parameterization of Dynamic Systems in a Priority Uncertainty</i>	
10.	Nina Nikolova	101
	<i>Repetitive Systems for Eliminating Periodic Measurement Error</i>	
11.	Vessela Karlova-Sergieva, Alexander Marinchev	111
	<i>Robust Level Control of Tank System</i>	
12.	Stanislav Enev	121
	<i>Suboptimal Minimum-Fuel Control Policies for Second-Order Over Damped Systems</i>	
13.	Alexander Marinchev	127
	<i>Solution of Integer Linear Optimization Problem Through Logical Programming</i>	
14.	Boris S. Grasiani	133
	<i>Fractional Control of a Pneumatic Positioning Device</i>	

15.	Boris S. Grasiani	143
	<i>Fractional Repetitive Control of a Pneumatic Positioning Device</i>	
16.	Kiril Borisov, Desislava Stoitseva-Delicheva, Vasil Galabov	153
	<i>Interdisciplinary Approach for Evaluation of Some Psychological Aspects</i>	
17.	Iordanka Guenova Dountcheva	161
	<i>Wavelet Analysis of Hydrometeorological Time Series for Seven Bulgarian Locations</i>	
18.	Galina Cherneva	171
	<i>Fractal Models for Approximation of Random Processes</i>	
19.	Dimitrinka Vladeva	177
	<i>The projections of k-Simplex Onto the Smallest Subsimplices of Arbitrary Type Are Derivations</i>	
20.	Dimitrinka Vladeva	187
	<i>The Projections of k-Simplex Onto the Greatest Subsimplices of Arbitrary Type Are Derivations</i>	
21.	Desislava Stoitseva-Delicheva, Boris Kirov, Martin Marinov	197
	<i>Engineering of Software for Virtual Metabolic Networks Automated Construction</i>	
22.	Boris Kirov, Desislava Stoitseva-Delicheva, Martin Marinov	205
	<i>Metabolic Network Flux Balance Analysis Algorithm Based on Recurrence Relations</i>	
23.	Emil Garipov	213
	<i>Continuous Time Dahlin Controller in a Scheme of Smith Predictor</i>	
24.	Tsonyo Slavov, Teofana Puleva	223
	<i>LQR Power Control of Wind Generator</i>	
25.	Tsonyo Slavov, Jordan Kralev, Petko Petkov	233
	<i>HIL Simulation of PID Controller for Control of Inverted Pendulum-Cart</i>	
26.	Valentin Dimitrov, Alena Kozakova, Jordan Kralev	243
	<i>Digital Control of Soil Temperature for Growing Plants Indoors</i>	
27.	Kamen Perev	253
	<i>Orthogonal Approximation for the Volterra Series Model of Nonlinear Systems</i>	
28.	Bozhidar Rakov, Georgi Ruzhekov	263
	<i>Program environment for PLC PID Control Design of a Plant with Two Inputs and Two Outputs</i>	
29.	Asparuh Markovski	273
	<i>Laboratory System "Hardware in the Loop"</i>	
30.	Jordan Kralev, Borislav Ivanov, Ivan Evg. Ivanov, Andey Yonchev, Desislava Georgieva	279
	<i>An Approach for Perfusion Pump Control for Nanofiltering</i>	
31.	Dimitar Yurukov	287
	<i>Cement Dry Rotary Kiln Processes Modeling and Control Methods</i>	

32.	Hristina Stoycheva, Dragomir Chantov	297
	<i>Design of Synchronization Schemes of Chaotic Systems with Changing Parameters Based on Lyapunov Second Method of Stability</i>	
33.	Stoyan Dimitrov	307
	<i>Consecutive Square-Free Numbers of a Special Form</i>	
34.	Stoyan Dimitrov	317
	<i>A Quaternary Diophantine Inequality by Prime Numbers of a Special Type</i>	
35.	Vesselina Trashlieva	327
	<i>Lagrange Multipliers Application in a Short-Term Optimal Power Scheduling Post-Optimal Analysis</i>	
36.	Nikola Georgiev, Vasilina Zlatanova, Snejana Terzieva	337
	<i>Investigation of Tactile Sensors with Conductive Elastomers Powered by Periodic Non-Sinusoidal voltage</i>	
37.	Atanas Chervenkov, Todorka Chervenкова, Atanas Yanev	345
	<i>Simulation of Three-Phase Invertor for Quasi-Sinusoidal Voltage</i>	
38.	Stoyan Kirilov, Valeri Mladenov	353
	<i>Analysis of the Relationships Between the Effective Values of the Input and Output Voltages and the Frequency of a Memristor-Resistor Circuit</i>	
39.	Zhivko Georgiev, Ivan Trushev, Atanas Chervenkov	361
	<i>Transient and Steady-State Analysis of Electrical Circuits Supplied by Sequence of Rectangular Pulses by Z - Transform</i>	
40.	Simona Petrakieva, Vladimir Balabanov	371
	<i>Web-Based Application for Monitoring and Control the Stocks in Storage</i>	
41.	Ilona Iatcheva, Malina Dimitrova, Nikolina Petkova	381
	<i>3D Modelling of the Electromagnetic Field Distribution in the Vicinity of 400 kV Power Line</i>	
42.	Ilona Iatcheva	389
	<i>Determination of Parameters and Dependencies in Electromagnetic Systems, Based on Numerical Modeling of Electromagnetic Field Distribution</i>	

TRANSIENT AND STEADY-STATE ANALYSIS OF ELECTRICAL CIRCUITS SUPPLIED BY SEQUENCE OF RECTANGULAR PULSES BY Z - TRANSFORM

Zhivko Georgiev, Ivan Trushev, Atanas Chervenkov

Abstract: Transient and steady-state process in series RC-circuits when the supplied voltage is a sequence of periodic rectangular pulses are analyzed. Analytical formulas for the voltage on the capacitor in any period of the supplied voltage are derived. The analysis is performed using z - transformation. The approach applied may also be used in other circuits and other forms of the supplied quantity.

Key words: transient, steady-state process, sequence of pulses, z - transformation

1. INTRODUCTION

Very often in various electrical and electronic circuits, sources generate a sequence of identical pulses (eg. rectangular, triangular, saw-tooth, etc.). Both from theoretical and practical point of view it is interesting to analyze the transient and steady-state processes in such circuits. The most common approach in such cases is to use the techniques of Fourier series [1], [2]. Fourier series are valid only for periodic functions, thus it can be examined only the steady-state process. Moreover, unknown quantities are obtained in the form of series, which leads to cumbersome calculations and inconvenience in use.

This article offers an approach for analyzing both the transient and steady-state process, when the input signals are periodic non-sinusoidal functions. In this case the unknown quantities are described by analytical formulas valid both for the above mentioned two processes. This approach is based on the use of z - transformation. The approach will be applied for the specific electrical circuit and input signal, but the basic principles of the analysis can be applied in other circuits and signals.

A series RC-circuits is considered (Fig.1).

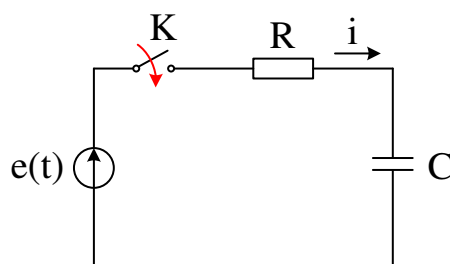


Fig.1. A series RC-circuit

Electromotive voltage $e(t)$ of the input source is a periodic sequence of rectangular pulses with an amplitude E , length t_H and repetition period T (Fig.2).

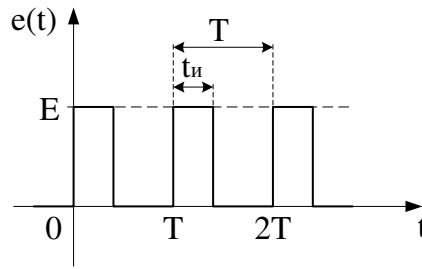


Fig.2. *Periodic series of rectangular pulses*

It can be described analytically by Heaviside function (continuous unit function) $u(t)$ as follows

$$e(t) = \sum_{n=0}^{\infty} E [u(t - nT) - u(t - nT - t_u)] , \quad (1)$$

and $u(t)$ is defined as follows

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad (2)$$

The instant of the closure of switch K coincides with the beginning of the first pulse. In general the capacitor voltage $u_c(t)$ satisfies the following differential equation [3], [4]

$$RC \frac{du_c}{dt} + u_c = e(t) , \quad t \geq 0 . \quad (3)$$

The time t , in the equation above is measured from the beginning of the first pulse.

2. THE MAIN IDEA FOR ANALYSIS

The main idea of the article is to determine the output response – capacitor voltage $u_c(t)$, of the electrical circuit (Fig.1) in discrete instants of time (samples) $t = 0, T, 2T, \dots, nT$, after sampling of input signal $e(t)$. This can be done by using of the discrete impulse function, z-transform and the transfer function of z-transform [5], [6], [7], [8]. In this case, the output variable within the interval of repetition of the pulses is obtained by using the samples as the initial conditions.

The input signal is sampled so that the samples are multiple of the period T of the pulse sequence $e(t)$. Using constant approximation of the input signal in every period, for the discrete input signal (Fig.3) the following equation is obtained

$$e[n] = e(nT) = Eu[n] , \quad (4)$$

where $u[n]$ is discrete unit function

$$u[n] = \begin{cases} 1 & n = 0, 1, 2, \dots \\ 0 & n = -1, -2, -3, \dots \end{cases}$$

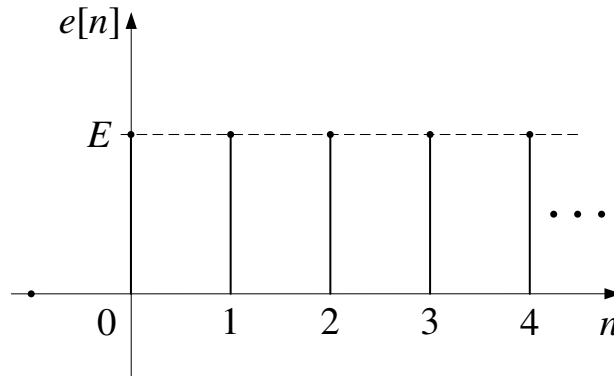


Fig.3. Sampled input signal

Thus the z-image of $e[n]$ is found

$$E(z) = Z\{e[n]\} = Z\{Eu[n]\} = E \frac{z}{z-1} \quad (5)$$

3. CONTINUOUS AND DISCRETE IMPULSE CHARACTERISTICS

Transient response of the output voltage $u_c(t)$ for the considered electrical circuit is determined when the input signal is continuous unite function $u(t)$.

When the input signal is $e(t) = E.u(t)$, $E = const$, and the continuous initial conditions are zero, the transient response is defined by the following equation

$$h(t) = \frac{u_c(t)}{E}. \quad (6)$$

Taking into account the expression for $u_c(t)$

$$u_c(t) = E \left(1 - e^{-\frac{t}{\tau}} \right), \quad \tau = RC, \quad (7)$$

for continuous transfer function the following equation is found

$$h(t) = \left(1 - e^{-\frac{t}{\tau}} \right) \quad (8)$$

The continuous impulse characteristic can be obtained as follows [9]

$$h_H(t) = h(t) - h(t - t_H), \quad t \geq 0 \quad (9)$$

For the instant of time $t = 0$, it is valid $h_H(0) = h(0) - 0 = 0$.

After substitution in (9) the following expression for $h_H(t)$ is found

$$h_H(t) = \left(1 - e^{-\frac{t}{\tau}} \right) - \left(1 - e^{-\frac{t-t_H}{\tau}} \right) = e^{-\frac{t-t_H}{\tau}} - e^{-\frac{t}{\tau}} = \left(e^{\frac{t_H}{\tau}} - 1 \right) e^{-\frac{t}{\tau}}, \quad t > 0 \quad (10)$$

$$h_H(t) = \begin{cases} \left(e^{\frac{t_H}{\tau}} - 1 \right) e^{-\frac{t}{\tau}}, & t > 0 \\ 0, & t = 0 \end{cases} \quad (11)$$

Discrete impulse characteristic is

$$\begin{aligned} h_H[n] &= h_H(nT), \quad \text{for } n=1,2,\dots \\ h_H[0] &= h_H(0) = 0 \end{aligned} \quad (12)$$

$$h_H[n] = \begin{cases} \left(e^{\frac{t_H}{\tau}} - 1 \right) e^{-\frac{nT}{\tau}}, & n=1,2,\dots \\ 0, & n=0 \end{cases} \quad (13)$$

4. DETERMINATION OF THE TRANSFER FUNCTION AT Z - TRANSFORMATION

It is well known that the transfer function $H_H(z)$ is the z – image of the discrete impulse characteristic

$$H_H(z) = Z \{ h_H[n] \} \quad (14)$$

An expression for $H_H(z)$ is obtained below

$$\begin{aligned} H_H(z) &= \sum_{n=0}^{\infty} h_H[n] z^{-n} = \sum_{n=1}^{\infty} h_H[n] z^{-n} = \left(e^{\frac{t_H}{\tau}} - 1 \right) \sum_{n=1}^{\infty} \left[e^{-\frac{nT}{\tau}} z^{-n} \right] = \\ &= \left(e^{\frac{t_H}{\tau}} - 1 \right) \sum_{n=1}^{\infty} \left(e^{-\frac{T}{\tau}} z^{-1} \right)^n = \left(e^{\frac{t_H}{\tau}} - 1 \right) \left[\sum_{n=0}^{\infty} \left(e^{-\frac{T}{\tau}} z^{-1} \right)^n - 1 \right] = \left(e^{\frac{t_H}{\tau}} - 1 \right) \left(\frac{1}{1 - e^{-\frac{T}{\tau}} z^{-1}} - 1 \right) = \\ &= \left(e^{\frac{t_H}{\tau}} - 1 \right) \left(\frac{z}{z - e^{-\frac{T}{\tau}}} - 1 \right) = \left(e^{\frac{t_H}{\tau}} - 1 \right) \left(\frac{z - z + e^{-\frac{T}{\tau}}}{z - e^{-\frac{T}{\tau}}} \right) \\ &H_H(z) = \left(e^{\frac{t_H}{\tau}} - 1 \right) e^{-\frac{T}{\tau}} \left(\frac{1}{z - e^{-\frac{T}{\tau}}} \right) \end{aligned} \quad (15)$$

5. DETERMINATION OF THE IMAGE OF THE OUTPUT SIGNAL

For the image of the output signal the following equality is valid (Fig.4)

$$u_C(z) = E(z)H_H(z) \quad (16)$$

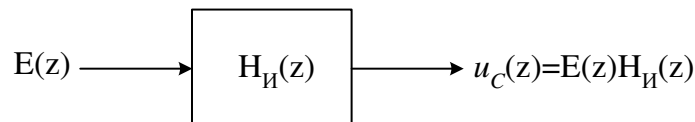


Fig.4. Block scheme for determination the image of the output signal

After substitution of (5) and (15) in (16) the following is obtained

$$u_C(z) = E \left(e^{\frac{t_H}{\tau}} - 1 \right) e^{-\frac{T}{\tau}} \frac{z}{(z-1) \left(z - e^{-\frac{T}{\tau}} \right)} \quad (17)$$

After introducing the notation

$$a = E \left(e^{\frac{t_H}{\tau}} - 1 \right) e^{-\frac{T}{\tau}} \quad (18)$$

the expression below is obtained

$$u_C(z) = \frac{az}{(z-1) \left(z - e^{-\frac{T}{\tau}} \right)} \quad (19)$$

6. DETERMINATION OF THE DISCRETE OUTPUT SIGNAL

To find the original $u_C[n]$, the expression (19) is decomposed into elementary terms

$$u_C(z) = \frac{az}{(z-1) \left(z - e^{-\frac{T}{\tau}} \right)} = \frac{c_1 z}{(z-1)} + \frac{c_2 z}{\left(z - e^{-\frac{T}{\tau}} \right)} \quad (20)$$

The aim is to find the unknown coefficients c_1 and c_2 . This can be done by using the undetermined coefficients method. The expression (20) is divided to z and the following is obtained

$$\frac{u_C(z)}{z} = \frac{a}{(z-1) \left(z - e^{-\frac{T}{\tau}} \right)} = \frac{c_1}{z-1} + \frac{c_2}{z - e^{-\frac{T}{\tau}}} \quad (21)$$

The expression (21) is multiplied to $(z-1)$ and the equality below is found

$$\frac{a(z-1)}{(z-1) \left(z - e^{-\frac{T}{\tau}} \right)} = \frac{c_1(z-1)}{z-1} + \frac{c_2(z-1)}{z - e^{-\frac{T}{\tau}}} \quad (22)$$

It is replaced $z = 1$ and the following is obtained

$$c_1 = \frac{a}{1 - e^{-\frac{T}{\tau}}} \quad (23)$$

For determination the expression for c_2 , (21) is multiplied to $\left(z - e^{-\frac{T}{\tau}} \right)$

$$\frac{a \left(z - e^{-\frac{T}{\tau}} \right)}{(z-1) \left(z - e^{-\frac{T}{\tau}} \right)} = \frac{c_1 \left(z - e^{-\frac{T}{\tau}} \right)}{z-1} + c_2 \quad (24)$$

After substituting $z = e^{-\frac{T}{\tau}}$, it is found

$$c_2 = \frac{a}{e^{-\frac{T}{\tau}} - 1}. \quad (25)$$

Expressions (23) and (25) are substituted in (20) and the following equation for $u_C(z)$ is obtained

$$u_C(z) = \frac{a}{\left(1 - e^{-\frac{T}{\tau}} \right)} \frac{z}{(z-1)} - \frac{a}{\left(1 - e^{-\frac{T}{\tau}} \right)} \frac{z}{\left(z - e^{-\frac{T}{\tau}} \right)}, \quad (26)$$

$$u_C(z) = \frac{a}{\left(1 - e^{-\frac{T}{\tau}} \right)} \left[\frac{z}{(z-1)} - \frac{z}{\left(z - e^{-\frac{T}{\tau}} \right)} \right]. \quad (27)$$

The inverse transformations are used

$$Z^{-1} \left\{ \frac{z}{(z-1)} \right\} = u[n] = 1 \quad \text{for } n \geq 0, \quad (28)$$

$$Z^{-1} \left\{ \frac{z}{\left(z - e^{-\frac{T}{\tau}} \right)} \right\} = e^{-\frac{nT}{\tau}}, \quad (29)$$

thus the following expressions for $u_C[n]$ are obtained

$$u_C[n] = \frac{a}{\left(1 - e^{-\frac{T}{\tau}} \right)} \left(1 - e^{-\frac{nT}{\tau}} \right) \quad (30)$$

$$u_C[n] = E \frac{\left(e^{\frac{nT}{\tau}} - 1 \right) e^{-\frac{nT}{\tau}}}{\left(1 - e^{-\frac{T}{\tau}} \right)} \left(1 - e^{-\frac{nT}{\tau}} \right), \quad n = 0, 1, 2, 3, \dots \quad (31)$$

The determined formulas give the value of the capacitor voltage at the discrete instants of time, which are multiple of the period of the input signal. The expression (31) coincides to the formula (24) from article [10], taking into account that τ is reciprocal value of α . Furthermore, after little transformations [10], for the capacitor voltage the following expressions are obtained:

1) For an arbitrary period with the number of $n, n = 1, 2, 3 \dots$ at $0 \leq t' \leq t_H$

$$u_C(t', n) = E + E \left\{ \frac{1 - e^{-\frac{t_H}{\tau}} e^{-\frac{nT}{\tau}} - \frac{1 - e^{-\frac{(T-t_H)}{\tau}}}{1 - e^{-\frac{T}{\tau}}} \right\} e^{-\frac{t'}{\tau}} ; \quad (32a)$$

2) For an arbitrary period with the number of $n, n = 1, 2, 3 \dots$ at $t_H \leq t' \leq T$

$$u_C(t', n) = E \left\{ \frac{1 - e^{-\frac{t_H}{\tau}} e^{-\frac{nT}{\tau}} - \frac{1 - e^{-\frac{t_H}{\tau}}}{1 - e^{-\frac{T}{\tau}}} \right\} e^{-\frac{t'}{\tau}} . \quad (32b)$$

The expressions obtained above are general. They are valid both for the transient and the steady-state process. Taking into account the relation $t' = t - (n - 1)T$ it is possible to go back to the initial time t in formulas (32).

As an application of the theory described above, the following example is considered.

Example: *The element parameters of the circuit on fig.1 are $E=200mV$, $T = 10ms$, $t_H = 5ms$, $R = 2k\Omega$, $C = 13\mu F$. To analyze the circuit by PSPICE and to show as a graph the capacitor voltage $u_C(t)$. To calculate the value of the capacitor voltage for the time instant $t = 47ms$ by PSPICE. To compare these results with results, obtained of analytical formulas.*

Solution: The graph of the capacitor voltage $u_C(t)$ obtained by PSPICE is shown on Fig.5. The value of the voltage for the time instant $t = 47ms$, calculated by PSPICE is $u_C(47ms) = 86.684mV$. The time instant $t = 47ms$ belongs to the fifth period of the input voltage, i.e. $n = 5$ and $t' = t - (n - 1)T = 47 - (5 - 1)10 = 7ms$.

The following values are calculated at first:

$$\tau = RC = 2 \cdot 10^3 \cdot 13 \cdot 10^{-6} = 0.026,$$

$$T / \tau \approx 0.38461538,$$

$$5T / \tau = 1.9230769,$$

$$t_H / \tau \approx 0.19230769,$$

$$t' / \tau \approx 0.269230769 .$$

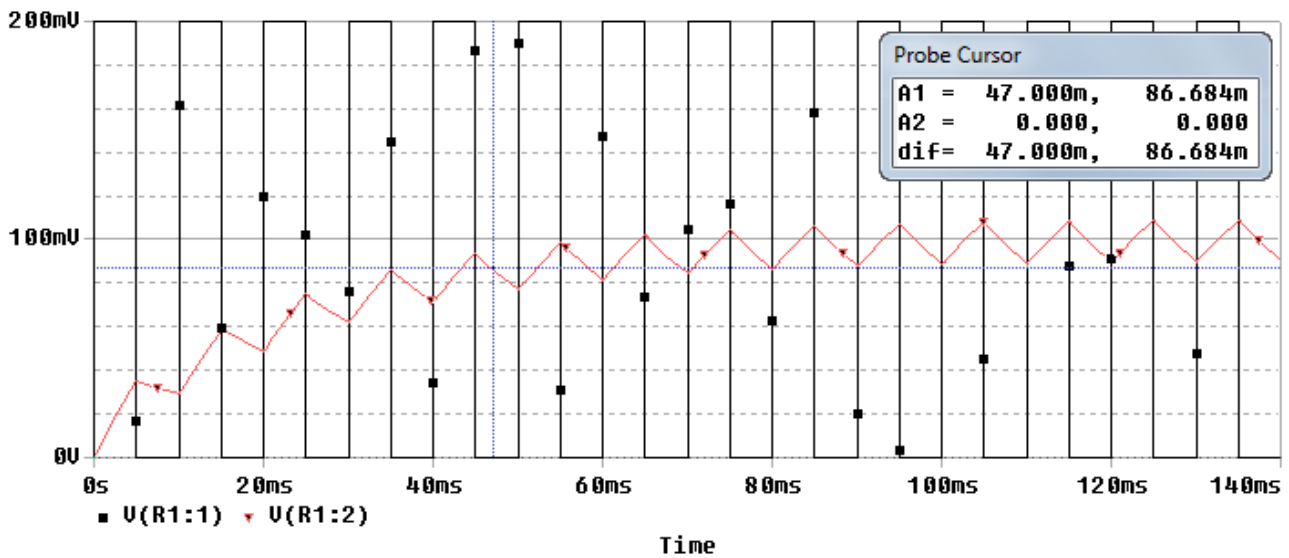


Fig.5. The capacitor voltage graph obtained by PSPICE

After that the obtained data are substituted in expression (32b):

$$\begin{aligned}
 u_C(0.007,5) &= 0.2 \left\{ \frac{1 - e^{0.19230769}}{1 - e^{-0.38461538}} e^{-1.9230769} - \frac{1 - e^{0.19230769}}{1 - e^{-0.38461538}} \right\} e^{-0.269230769} = \\
 &= 0.2 \left\{ \frac{-0.2120433932}{0.3192875985} 0.14615656 - \frac{-0.2120433932}{0.3192875985} \right\} 0.7639669354 = \\
 &= 0.2 \{ -0.09706463 + 0.66411409 \} 0.763966935 = 0.08664V = 86.64 \text{ mV} .
 \end{aligned}$$

It is obvious that there is a very good match between the results obtained by PSPICE and by using the analytical formulas (32).

The capacitor voltage graph obtained by PSPICE shows both - the transient and the steady-state voltage. As can be seen from the graph and from the formulas given above the steady-state solution is a periodic combination of exponential functions. The input voltage (right side of equation (1)), however, is a periodic sequence of rectangular pulses. This example illustrates well the fact that not always the type of steady-state solution coincides with the type of the input signal (the right side of the differential equation).

7. CONCLUSIONS

Z-transformation can be used as a stand-alone method for analysis of circuits in the effects of a sequence of pulses. In this case, the discrete transfer function, transient and pulse characteristics defined by z-transformation are used.

The article discusses some problems related to the study of the transient and the steady-state process in electrical circuits when the sources generate periodic non-sinusoidal signals. The statement refers to a specific circuit and input signal. Nevertheless, the proposed method for analyzing the transient and the steady-state modes, when the input signal is a sequence of rectangular pulses using z – transformation, is sufficiently general and can be applied to arbitrary first order circuits and periodic in-

put signals (for example - bipolar rectangular impulses, triangular impulses, saw-tooth pulses, etc.). The basic principles that have been used here are presented. The method can also be applied for analysis of the circuits of second and higher order [11]. The results can be compared with the similar ones made in [12], where it was made analysis of the influence of the active and reactive elements in linear electrical over the type of transient processes with both methods - classical method for transient analysis and simulation with OrCAD Capture. This will be subject of the future work.

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