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TRANSIENT AND STEADY-STATE ANALYSIS OF ELECTRICAL CIRCUITS SUPPLIED BY SEQUENCE OF RECTANGULAR PULSES BY Z - TRANSFORM

Zhivko Georgiev, Ivan Trushev, Atanas Chervenkov

Abstract*: Transient and steady-state process in series RC-circuits when the supplied voltage is a sequence of periodic rectangular pulses are analyzed. Analytical formulas for the voltage on the capacitor in any period of the supplied voltage are derived. The analysis is performed using z - transformation. The approach applied may also be used in other circuits and other forms of the supplied quantity.*

Key words*: transient, steady-state process, sequence of pulses, z - transformation*

1. INTRODUCTION

Very often in various electrical and electronic circuits, sources generate a sequence of identical pulses (eg. rectangular, triangular, saw-tooth, etc.). Both from theoretical and practical point of view it is interesting to analyze the transient and steady-state processes in such circuits. The most common approach in such cases is to use the techniques of Fourier series [1], [2]. Fourier series are valid only for periodic functions, thus it can be examined only the steady-state process. Moreover, unknown quantities are obtained in the form of series, which leads to cumbersome calculations and inconvenience in use.

This article offers an approach for analyzing both the transient and steady-state process, when the input signals are periodic non-sinusoidal functions. In this case the unknown quantities are described by analytical formulas valid both for the above mentioned two processes. This approach is based on the use of z - transformation. The approach will be applied for the specific electrical circuit and input signal, but the basic principles of the analysis can be applied in other circuits and signals.

A series *RC*-circuits is considered (Fig.1).

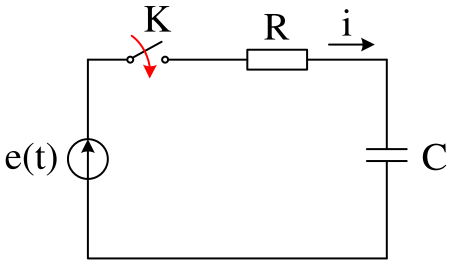

Fig.1. *A series RC-circuit*

Electromotive voltage $e(t)$ of the input source is a periodic sequence of rectangular pulses with an amplitude E, length $t_И$ and repetition period T (Fig.2).

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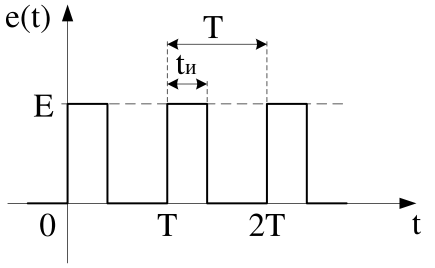

Fig.2. *Periodic series of rectangular pulses*

It can be described analytically by Heaviside function (continuous unit function) $u(t)$ as follows

$$e(t)=\sum_{н=0}^{\infty} E\left[u(t-nT)-u(t-nT-t_{И})\right], \tag{1}$$

and $u(t)$ is defined as follows

$$u(t)=\begin{cases}1, & t\geq 0\\ 0, & t<0\end{cases} \tag{2}$$

The instant of the closure of switch K coincides with the beginning of the first pulse. In general the capacitor voltage $u_C(t)$ satisfies the following differential equation [3], [4]

$$RC\frac{\mathrm{d}u_C}{\mathrm{d}t}+u_C=e(t)\,,\quad t\geq 0\,. \tag{3}$$

The time t, in the equation above is measured from the beginning of the first pulse.

2. THE MAIN IDEA FOR ANALYSIS

The main idea of the article is to determine the output response – capacitor voltage $u_C(t)$, of the electrical circuit (Fig.1) in discrete instants of time (samples) $t = 0$, T, $2T$, …, nT, after sampling of input signal $e(t)$. This can be done by using of the discrete impulse function, z-transform and the transfer function of z-transform [5], [6], [7], [8]. In this case, the output variable within the interval of repetition of the pulses is obtained by using the samples as the initial conditions.

The input signal is sampled so that the samples are multiple of the period T of the pulse sequence $e(t)$. Using constant approximation of the input signal in every period, for the discrete input signal (Fig.3) the following equation is obtained

$$e\left[n\right]=e(nT)=Eu\left[n\right], \tag{4}$$

where $u[n]$ is discrete unit function

$$u\left[n\right]=\begin{cases}1 \;\; за & n=0,1,2,...\\ 0 \;\; за & n=-1,-2,-3,...\end{cases}$$

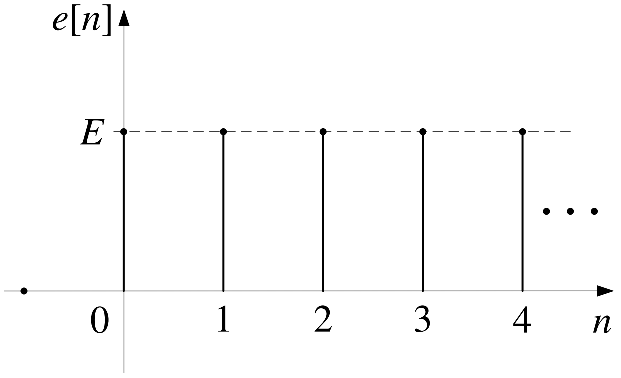

Fig.3. *Sampled input signal*

Thus the z-image of $e[n]$ is found

$$\mathrm{E}(z) = Z\left\{e\left[n\right]\right\} = Z\left\{Eu\left[n\right]\right\} = E\frac{z}{z-1} \tag{5}$$

3. CONTINUOUS AND DISCRETE IMPULSE CHARACTERISTICS

Transient response of the output voltage $u_C(t)$ for the considered electrical circuit is determined when the input signal is continuous unite function $u(t)$.

When the input signal is $e\left(t\right) = E.u(t)$, $E = const$, and the continuous initial conditions are zero, the transient response is defined by the following equation

$$h(t) = \frac{u_C(t)}{E}. \tag{6}$$

Taking into account the expression for $u_C(t)$

$$u_C(t) = E\left(1 - e^{-\frac{t}{\tau}}\right), \quad \tau = RC, \tag{7}$$

for continuous transfer function the following equation is found

$$h(t) = \left(1 - e^{-\frac{t}{\tau}}\right) \tag{8}$$

The continuous impulse characteristic can be obtained as follows [9]

$$h_{И}(t) = h(t) - h(t - t_{И}), \quad t \geq 0 \tag{9}$$

For the instant of time $t = 0$, it is valid $h_{И}(0) = h(0) - 0 = 0$.

After substitution in (9) the following expression for $h_{И}(t)$ is found

$$h_{И}(t) = \left(1 - e^{-\frac{t}{\tau}}\right) - \left(1 - e^{-\frac{t-t_{И}}{\tau}}\right) = e^{-\frac{t-t_{И}}{\tau}} - e^{-\frac{t}{\tau}} = \left(e^{\frac{t_{И}}{\tau}} - 1\right)e^{-\frac{t}{\tau}}, \quad t > 0 \tag{10}$$

$$h_И(t)=\begin{cases}\left(e^{\frac{t_И}{\tau}}-1\right)e^{-\frac{t}{\tau}}, & t>0\\ 0, & t=0\end{cases} \tag{11}$$

Discrete impulse characteristic is

$$\begin{aligned} &h_И\left[n\right]=h_И\left(nT\right), \quad за\ \ n=1,2,....\\ &h_И\left[0\right]=h_И(0)=0\end{aligned} \tag{12}$$

$$h_И\left[n\right]=\begin{cases}\left(e^{\frac{t_И}{\tau}}-1\right)e^{-\frac{nT}{\tau}}, & n=1,2,...\\ 0, & n=0\end{cases}. \tag{13}$$

4. DETERMINATION OF THE TRANSFER FUNCTION AT Z - TRANSFORMATION

It is well known that the transfer function $H_И(z)$ is the z – image of the discrete impulse characteristic

$$H_И(z)=Z\left\{h_И\left[n\right]\right\} \tag{14}$$

An expression for $H_И(z)$ is obtained below

$$H_И(z)=\sum_{n=0}^{\infty}h_И\left[n\right]z^{-n}=\sum_{n=1}^{\infty}h_И\left[n\right]z^{-n}=\left(e^{\frac{t_И}{\tau}}-1\right)\sum_{n=1}^{\infty}\left[e^{-\frac{nT}{\tau}}z^{-n}\right]=$$

$$=\left(e^{\frac{t_И}{\tau}}-1\right)\sum_{n=1}^{\infty}\left(e^{-\frac{T}{\tau}}z^{-1}\right)^{n}=\left(e^{\frac{t_И}{\tau}}-1\right)\left[\sum_{n=0}^{\infty}\left(e^{-\frac{T}{\tau}}z^{-1}\right)^{n}-1\right]=\left(e^{\frac{t_И}{\tau}}-1\right)\left(\frac{1}{1-e^{-\frac{T}{\tau}}z^{-1}}-1\right)=$$

$$=\left(e^{\frac{t_И}{\tau}}-1\right)\left(\frac{z}{z-e^{-\frac{T}{\tau}}}-1\right)=\left(e^{\frac{t_И}{\tau}}-1\right)\left(\frac{z-z+e^{-\frac{T}{\tau}}}{z-e^{-\frac{T}{\tau}}}\right)$$

$$H_И(z)=\left(e^{\frac{t_И}{\tau}}-1\right)e^{-\frac{T}{\tau}}\left(\frac{1}{z-e^{-\frac{T}{\tau}}}\right) \tag{15}$$

5. DETERMINATION OF THE IMAGE OF THE OUTPUT SIGNAL

For the image of the output signal the following equality is valid (Fig.4)

$$u_C(z)=\mathrm{E}(z)H_И(z) \tag{16}$$

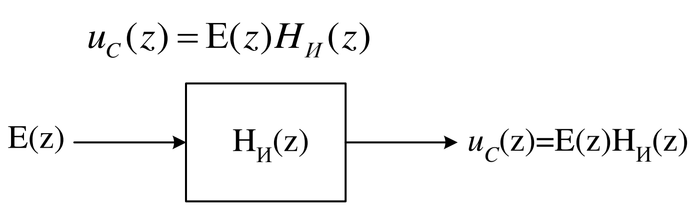

Fig.4. *Block scheme for determination the image of the output signal*

After substitution of (5) and (15) in (16) the following is obtained

$$u_C(z) = E\left(e^{\frac{t_{\text{И}}}{\tau}} - 1\right)e^{-\frac{T}{\tau}} \frac{z}{(z-1)\left(z - e^{-\frac{T}{\tau}}\right)} \tag{17}$$

After introducing the notation

$$a = E\left(e^{\frac{t_{\text{И}}}{\tau}} - 1\right)e^{-\frac{T}{\tau}} \tag{18}$$

the expression below is obtained

$$u_C(z) = \frac{az}{(z-1)\left(z - e^{-\frac{T}{\tau}}\right)} \tag{19}$$

6. DETERMINATION OF THE DISCRETE OUTPUT SIGNAL

To find the original $u_C[n]$, the expression (19) is decomposed into elementary terms

$$u_C(z) = \frac{az}{(z-1)\left(z - e^{-\frac{T}{\tau}}\right)} = \frac{c_1 z}{(z-1)} + \frac{c_2 z}{\left(z - e^{-\frac{T}{\tau}}\right)} \tag{20}$$

The aim is to find the unknown coefficients c_1 and c_2. This can be done by using the undetermined coefficients method. The expression (20) is divided to z and the following is obtained

$$\frac{u_C(z)}{z} = \frac{a}{(z-1)\left(z - e^{-\frac{T}{\tau}}\right)} = \frac{c_1}{z-1} + \frac{c_2}{z - e^{-\frac{T}{\tau}}} \tag{21}$$

The expression (21) is multiplied to $(z - 1)$ and the equality below is found

$$\frac{a(z-1)}{(z-1)\left(z - e^{-\frac{T}{\tau}}\right)} = \frac{c_1(z-1)}{z-1} + \frac{c_2(z-1)}{z - e^{-\frac{T}{\tau}}} \tag{22}$$

It is replaced z = 1 and the following is obtained

$$c_1 = \frac{a}{1 - e^{-\frac{T}{\tau}}} \tag{23}$$

For determination the expression for c_2, (21) is multiplied to $\left(z - e^{-\frac{T}{\tau}}\right)$

$$\frac{a\left(z-e^{-\frac{T}{\tau}}\right)}{(z-1)\left(z-e^{-\frac{T}{\tau}}\right)}=\frac{c_1\left(z-e^{-\frac{T}{\tau}}\right)}{z-1}+c_2 \tag{24}$$

After substituting $z=e^{-\frac{T}{\tau}}$, it is found

$$c_2=\frac{a}{e^{-\frac{T}{\tau}}-1}\,. \tag{25}$$

Expressions (23) and (25) are substituted in (20) and the following equation for $u_C(z)$ is obtained

$$u_C(z)=\frac{a}{\left(1-e^{-\frac{T}{\tau}}\right)}\frac{z}{(z-1)}-\frac{a}{\left(1-e^{-\frac{T}{\tau}}\right)}\frac{z}{\left(z-e^{-\frac{T}{\tau}}\right)}, \tag{26}$$

$$u_C(z)=\frac{a}{\left(1-e^{-\frac{T}{\tau}}\right)}\left[\frac{z}{(z-1)}-\frac{z}{\left(z-e^{-\frac{T}{\tau}}\right)}\right]. \tag{27}$$

The inverse transformations are used

$$Z^{-1}\left\{\frac{z}{(z-1)}\right\}=u[n]=1 \quad npu\ n\geq 0\,, \tag{28}$$

$$Z^{-1}\left\{\frac{z}{\left(z-e^{-\frac{T}{\tau}}\right)}\right\}=e^{-\frac{nT}{\tau}}\,, \tag{29}$$

thus the following expressions for $u_C[n]$ are obtained

$$u_C[n]=\frac{a}{\left(1-e^{-\frac{T}{\tau}}\right)}\left(1-e^{-\frac{nT}{\tau}}\right) \tag{30}$$

$$u_C[n]=E\frac{\left(e^{\frac{t_И}{\tau}}-1\right)e^{-\frac{T}{\tau}}}{\left(1-e^{-\frac{T}{\tau}}\right)}\left(1-e^{-\frac{nT}{\tau}}\right), \quad n=0,1,2,3,... \tag{31}$$

The determined formulas give the value of the capacitor voltage at the discrete instants of time, which are multiple of the period of the input signal. The expression (31) coincides to the formula (24) from article [10], taking into account that τ is reciprocal value of α. Furthermore, after little transformations [10], for the capacitor voltage the following expressions are obtained:

1) For an arbitrary period with the number of n, $n = 1, 2, 3\ldots$ at $0 \le t' \le t_И$

$$u_C(t',n) = E + E\left\{\frac{1-e^{\frac{t_И}{\tau}}}{1-e^{-\frac{T}{\tau}}}e^{-\frac{nT}{\tau}} - \frac{1-e^{-\frac{(T-t_И)}{\tau}}}{1-e^{-\frac{T}{\tau}}}\right\}e^{-\frac{t'}{\tau}}; \qquad (32a)$$

2) For an arbitrary period with the number of n, $n = 1, 2, 3\ldots$ at $t_И \le t' \le T$

$$u_C(t',n) = E\left\{\frac{1-e^{\frac{t_И}{\tau}}}{1-e^{-\frac{T}{\tau}}}e^{-\frac{nT}{\tau}} - \frac{1-e^{\frac{t_И}{\tau}}}{1-e^{-\frac{T}{\tau}}}\right\}e^{-\frac{t'}{\tau}}. \qquad (32b)$$

The expressions obtained above are general. They are valid both for the transient and the steady-state process. Taking into account the relation $t' = t-(n-1)T$ it is possible to go back to the initial time t in formulas (32).

As an application of the theory described above, the following example is considered.

Example: *The element parameters of the circuit on fig.1 are E=200mV, T = 10ms, $t_И$ = 5ms, R = 2$k\Omega$, C = 13μF. To analyze the circuit by PSPICE and to show as a graph the capacitor voltage $u_C(t)$. To calculate the value of the capacitor voltage for the time instant t = 47ms by PSPICE. To compare these results with results, obtained of analytical formulas.*

Solution: The graph of the capacitor voltage $u_C(t)$ obtained by PSPICE is shown on Fig.5. The value of the voltage for the time instant t = 47ms, calculated by PSPICE is $u_C(47ms)$ = 86.684mV. The time instant t = 47ms belongs to the fifth period of the input voltage, i.e. n = 5 and $t' = t-(n-1)T = 47-(5-1)10 = 7\,ms$.

The following values are calculated at first:

$$\tau = RC = 2.10^3.13.10^{-6} = 0.026,$$

$$T/\tau \approx 0.38461538,$$

$$5T/\tau = 1.9230769,$$

$$t_И/\tau \approx 0.19230769,$$

$$t'/\tau \approx 0.269230769.$$

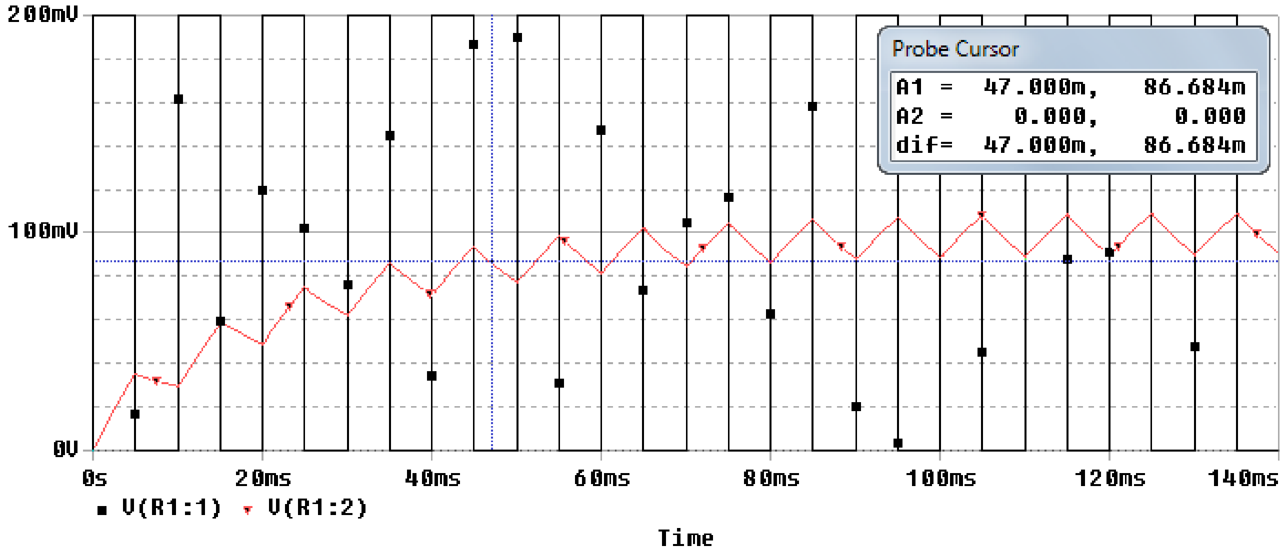

Fig.5. *The capacitor voltage graph obtained by PSPICE*

After that the obtained data are substituted in expression (32b):

$$u_C(0.007,5)=0.2\left\{\frac{1-e^{0.19230769}}{1-e^{-0.38461538}}e^{-1.9230769}-\frac{1-e^{0.19230769}}{1-e^{-0.38461538}}\right\}e^{-0.269230769}=$$

$$=0.2\left\{\frac{-0.2120433932}{0.3192875985}0.14615656-\frac{-0.2120433932}{0.3192875985}\right\}0.7639669354=$$

$$=0.2\left\{-0.09706463+0.66411409\right\}0.763966935=0.08664V=86.64\ mV\ .$$

It is obvious that there is a very good match between the results obtained by PSPICE and by using the analytical formulas (32).

The capacitor voltage graph obtained by PSPICE shows both - the transient and the steady-state voltage. As can be seen from the graph and from the formulas given above the steady-state solution is a periodic combination of exponential functions. The input voltage (right side of equation (1)), however, is a periodic sequence of rectangular pulses. This example illustrates well the fact that not always the type of steady-state solution coincides with the type of the input signal (the right side of the differential equation).

7. CONCLUSIONS

Z-transformation can be used as a stand-alone method for analysis of circuits in the effects of a sequence of pulses. In this case, the discrete transfer function, transient and pulse characteristics defined by z-transformation are used.

The article discusses some problems related to the study of the transient and the steady-state process in electrical circuits when the sources generate periodic non-sinusoidal signals. The statement refers to a specific circuit and input signal. Nevertheless, the proposed method for analyzing the transient and the steady-state modes, when the input signal is a sequence of rectangular pulses using z – transformation, is sufficiently general and can be applied to arbitrary first order circuits and periodic input signals (for example - bipolar rectangular impulses, triangular impulses, saw-tooth pulses, etc.). The basic principles that have been used here are presented. The method can also be applied for analysis of the circuits of second and higher order [11]. The results can be compared with the similar ones made in [12], where it was made analysis of the influence of the active and reactive elements in linear electrical over the type of transient processes with both methods - classical method for transient analysis and simulation with OrCAD Capture. This will be subject of the future work.

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