Using of Four-Dimensional Electromagnetic Potentials in Cases of Movement. Part II: Examples

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***Abstract*—This work shown application of four-dimensional potential in case of moving. Two cases for application of four-dimensional potential for determining of the vectors of the induced electric field and magnetic field by motion in Minkovski space are considered. The electromagnetic field vectors are defined as elements of dual Maxwell's tensor. The elements of four-dimensional electric potential are electric vector-potential and scalar magnetic potential, respectively. A variable magnetic charge model is used in case of determination of electric vector-potential. The relation between the four-dimensional electric potential and the corresponding field tensors is used for determination of electrical field components in case of movement.**

***Keywords*—electromagnetic field, Maxwell's tensor, electromagnetic duality, four-dimensional potential, magnetic charge**

I. Introduction

In the present work, two type of four-dimensional potential are used, based on the vector potential and the scalar potential with four-dimensional relativistic approach. The components of the first four-dimensional magnetic potential $\vec{\Psi}_\mu$ are magnetic vector potential $\vec{A}_\mu$ and scalar electric potential V_ε. The components of the second four-dimensional electric potential $\vec{\Psi}_\varepsilon$ are or electric vector potential $\vec{A}_\varepsilon$ and scalar magnetic potential V_μ , respectively. This approach is based on the use of known exciters of the electromagnetic field. Also, the corresponding dual exciters of the electromagnetic field are introduced.

A four-dimensional magnetic potential is used to determine the induced electric field in the case of motion of an electric charges relative to a stationary observer [1].

The purpose of the present work is to show the application of four-dimensional electric potential for determination of the induced electric field in the cases of movement of exciters with constant and alternating magnetic flux.

II. Analysis with Four-Dimensional Electric Potential

Four-dimensional electric potential $\vec{\Psi}_\varepsilon$ is used [1, 2, 3]

$$\{\vec{\Psi}_\varepsilon\} = \left\{A_{\varepsilon x}\ ,\ A_{\varepsilon y}\ , A_{\varepsilon z}\ ,\ \frac{j}{c}V_\mu\right\}, \tag{1}$$

where: $A_{\varepsilon x}$, $A_{\varepsilon y}$, $A_{\varepsilon z}$ are components of the electric vector-potential $\vec{A}_\varepsilon$; V_μ is scalar magnetic potential; j is imaginary unit and c is the speed of light.

The electric vector-potential $\vec{A}_\varepsilon$ from next condition of electric field density $\vec{D}$ is determined

$$\vec{D} = -rot\vec{A}_\varepsilon \tag{2}$$

Using Maxwell's second equation for electrical intensity [2], the Poisson's equation for the electric vector potential $\vec{A}_\varepsilon$ is obtained

$$\nabla^2\vec{A}_\varepsilon = \varepsilon\frac{\partial\vec{B}}{\partial t} \tag{3}$$

For a homogeneous environment, when $\gamma = const$, $\varepsilon = const$ (γ is electric conductance and ε is electric permittivity) and an exciter with magnetic induction $\vec{B}$ and change of the magnetic induction $\frac{\partial\vec{B}}{\partial t}$, centered in volume V the Poisson's equation solution is searched [2].

For a magnetic loop with a relatively small cross section s, the electric vector potential $\vec{A}_\varepsilon$ is

$$\vec{A}_\varepsilon = \frac{\varepsilon}{4\pi}\iint_{(s)}\frac{\partial\vec{B}}{\partial t}d\vec{s}\oint_{(l)}\frac{d\vec{l}}{r} = \frac{\varepsilon}{4\pi}\frac{d\Phi}{dt}\oint_{(l)}\frac{d\vec{l}}{r} \tag{4}$$

In this case the change of the magnetic induction $\frac{\partial\vec{B}}{\partial t}$ and change of the magnetic flux $\frac{\partial\Phi}{\partial t}$ can be considered as "*magnetic exciter*".

The scalar magnetic potential V_μ , excited by a radial magnetic field (created by a solenoid with a very small cross section $s \to 0$ and magnetic permeability μ is determined as

$$V_\mu = \int_{(l)}\vec{H}\,d\vec{l} = \frac{\Phi}{4\pi\mu r} \tag{5}$$

This dependence suggests that magnetic flux Φ plays the role of "*magnetic charge*".

The components of electromagnetic vectors - electric induction $\vec{D}$ (respectively, electric field intensity $\vec{E}$ in isotropic medium) and magnetic field intensity $\vec{H}$ as elements of dual Maxwell tensor $F_{jk}^{(\varepsilon)}$ are determined [2, 4]

$$F_{jk}^{(\varepsilon)} = \\ = \begin{Bmatrix} F_{11} = 0 & F_{12} = -\varepsilon E_z & F_{13} = \varepsilon E_y & F_{14} = \frac{-j}{c}H_x \\ F_{21} = \varepsilon E_z & F_{22} = 0 & F_{23} = -\varepsilon E_x & F_{24} = \frac{-j}{c}H_y \\ F_{31} = -\varepsilon E_y & F_{32} = \varepsilon E_x & F_{33} = 0 & F_{34} = \frac{-j}{c}H_z \\ F_{41} = \frac{j}{c}H_x & F_{42} = \frac{j}{c}H_y & F_{43} = \frac{j}{c}H_z & F_{14} = 0 \end{Bmatrix} \tag{6}$$

The relation between the four-dimensional electric potentials $\vec{\Psi}_\varepsilon$ and the field tensors $F_{jk}^{(\varepsilon)}$ is determined by the dependence [2, 3, 6]