

# Increasing the accuracy of making threaded gauges based on statistical methods

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**Abstract** - A method for improving the volumetric accuracy of coordinate technological machines is proposed in the article. The method is based on the derived mathematical dependencies, determining the components of the geometric inaccuracies for each coordinate axis. These dependencies give an opportunity software correction of the volume error of the machine, which is implemented by a measurement-computing platform whose input vector represents data from interferometric laser systems. The effectiveness of the method has been proven by statistical methods through a real technological process for making gauges for the control of external metric threads.

**Keywords** - volumetric accuracy; coordinate technological machines; geometric errors; threaded gauges

## I. INTRODUCTION

In modern industry, almost all products represent assembled units, in which one of the main types of detachable connections is the threaded joint [1, 2]. Since the operational characteristics of the threaded joint are largely determined by the accuracy of manufacturing the geometric parameters of the thread, the task of increasing the accuracy of measurement and control of these elements arises, which leads with itself corresponding requirements for the quality of threaded gauges [3, 4]. The latter have very narrow tolerance fields of the main parameters, which leads to significant difficulties in their production because even also small inaccuracies of a technological nature lead to large amounts of reject [5, 6]. That is why, tasks related to the accuracy of making of thread gauges are of great importance in engineering theory and practice.

In connection with the increasingly widespread distribution of threaded parts, nowadays the use of multi-coordinate technological metal cutting machines, machining centers, 3D printers and coordinate measuring machines

capable of producing and measuring them is increasing [7, 8]. The accuracy of making and measurement of all these machines and systems is characterized with the notion of volumetric accuracy [9-11]. The most important metrological indicator of volumetric accuracy is the volumetric error, which is a vector in the working space of the machine between the nominal and the actual position of the cutting tool [12-15]. Deviations in the volumetric accuracy of the machines listed above are mostly due to geometric errors, kinematic errors, thermal deformations, force deformations, vibrations, etc [16-25]. Also, in the process of operation, it is possible for clearances to appear and the initial parameters for the coordinate position to change by small values [26].

Since for each position of the cutting tool a corresponding vector can be determined about the basic coordinate system of the machine, it follows that the geometric accuracy of the coordinate axes is of extremely important. In practice, these errors are realized as deviations in the parallelism, perpendicularity and angle of the guides along each one coordinate axis [27].

Increasing the accuracy of machines by improving the quality of production and assembling of individual elements and units are practically exhausted, and further development of this approach is difficult and expensive. On the other hand, however, the modern rapid development of computer and microprocessor engineering gives new opportunities for improving the accuracy of technological and measurement systems. In this regard, software applications can be created for the correction of the systematic components of the errors of the guides, based on a measurement-computing platform whose input vector is data from interferometric laser systems. It is precisely on the basis of this approach that the main aspects of the present work related to improving the accuracy

of making threaded gauges for external metric threads will be developed.

## II. A MODEL FOR DETERMINING THE VOLUMETRIC GEOMETRIC ERROR

The reasons for the appearance of a volumetric geometric error are due to errors in the movements of the machine. Thus, for example, the cutting tool, in addition to the desired purely progressive rectilinear movement, parallel to the axes, additionally performs two transverse movements (also rectilinear) and three rotary ones, which, although undesirable, can hardly be completely avoided. To these can be added positioning inaccuracies also, due above all to errors in the machine's measuring systems.

For effective error correction of technological machines, it is necessary to develop a mathematical model defining both the errors made during movement and positioning of the cutting tool along each of the coordinate axes, as well as the volumetric geometric error due to them. For this purpose, in Fig.1, the ideal coordinate system of the machine is defined with axes  $X, Y, Z$ , which provide the corresponding ideal volumetric accuracy. When setting a motion of the cutting tool parallel to the  $Z$  axis, it should move from position  $A$  to point  $B$ . Due to the imperfection of the multi-coordinate system, the elements of which have unwanted positioning errors, deviations from straightness of movement and rotation around the coordinate axes, the working body will deviate from the desired point  $B$  to point  $C$ . The vector  $\vec{BC}$  determines the volumetric error of the machine. In fact, as a consequence of the listed above of the undesirable geometric deviations, the movement of the cutting tool will carry out parallel to the axes of the real coordinate system  $O_1x_1y_1z_1$ . All geometric deviations of a three-coordinate machine are presented in Table 1.

$$\Delta_{xyz} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = E_{xyz} + \Delta_{str} + A \cdot L_{xyz}, \quad (1)$$

where  $E_{xyz}$  is a matrix defining the positioning errors along the individual coordinate axes;  $\Delta_{str}$  – a matrix defining the projections of the deviations from straightness of each one of the coordinate axes relative to the other two axes;  $A$  – matrix with the cosines of the angles defining the rotation of the real coordinate system  $O_1x_1y_1z_1$  relative to the ideal one  $OXYZ$ ;  $L_{xyz}$  – the coordinates of the working tool.

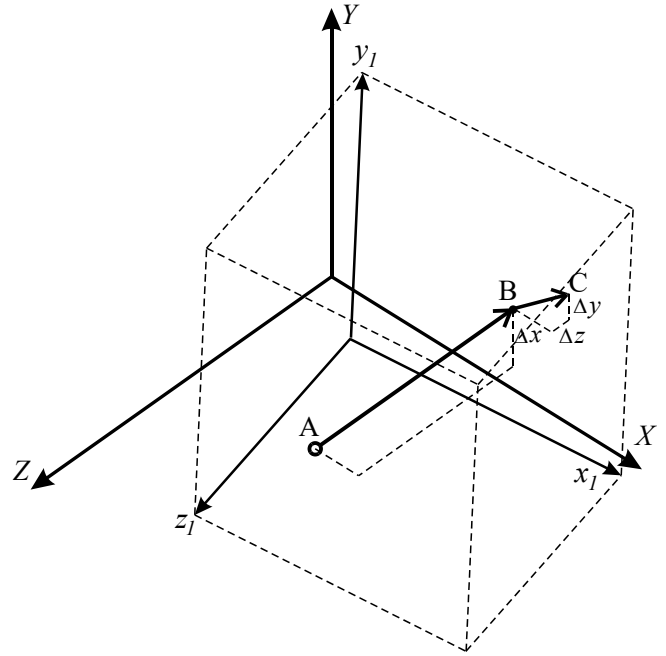


Fig.1. Volumetric error of coordinate technological machines

No. in order	Deviation name	Designation
1	Scale error of axis $OX$	$\epsilon_x$
2	Scale error of axis $OY$	$\epsilon_y$
3	Scale error of axis $OZ$	$\epsilon_z$
4	Deviation from straightness of axis $O_1x_1$ in the direction of axis $OY$	$\Delta_{xy}^x$
5	Deviation from straightness of axis $O_1x_1$ in the direction of axis $OZ$	$\Delta_{xz}^x$
6	Deviation from straightness of axis $O_1y_1$ in the direction of axis $OX$	$\Delta_{yx}^y$
7	Deviation from straightness of axis $O_1y_1$ in the direction of axis $OZ$	$\Delta_{yz}^y$
8	Deviation from straightness of axis $O_1z_1$ in the direction of axis $OX$	$\Delta_{zx}^z$
9	Deviation from straightness of axis $O_1z_1$ in the direction of axis $OY$	$\Delta_{zy}^z$
10	Angular rotation of the guide along axis $O_1x_1$ relative to axis $OY$ (Fig.2)	$\psi_x$
11	Angular rotation of the guide along axis $O_1x_1$ relative to axis $Oz_1'$	$\theta_x$
12	Angular rotation of the guide along axis $O_1x_1$ relative to axis $Ox_1''$	$\gamma_x$
13	Angular rotation of the guide along axis $O_1y_1$ relative to axis $OY$	$\psi_y$
14	Angular rotation of the guide along axis $O_1y_1$ relative to axis $Oz_1'$	$\theta_y$
15	Angular rotation of the guide along axis $O_1y_1$ relative to axis $Ox_1''$	$\gamma_y$
16	Angular rotation of the guide along axis $O_1z_1$ relative to axis $OY$	$\psi_z$
17	Angular rotation of the guide along axis $O_1z_1$ relative to axis $Oz_1'$	$\theta_z$
18	Angular rotation of the guide along axis $O_1z_1$ relative to axis $Ox_1''$	$\gamma_z$

Table 1

The matrix defining the coordinates of the radius-vector  $\vec{BC}$  of the volumetric error of the machine has the form

The matrix  $A$  is determined by the multiplication (Fig.2)

$$A = A_\gamma \cdot A_\theta \cdot A_\psi. \quad (2)$$

$$A_\psi = \begin{bmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{bmatrix}. \quad (3)$$

The second rotation is related to the matrix

$$A_\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (4)$$

The rotation of the system  $Ox''_1y''_1z''_1$  about the axis  $Ox''_1$  at an angle  $\gamma$  is defined by the third matrix

$$A_\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix}. \quad (5)$$

It is obtained the following record for  $A$  after substituting the above three matrices into (2):

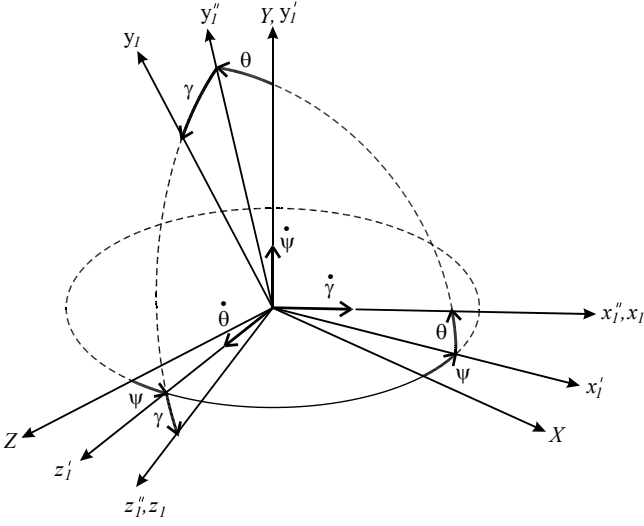


Fig.2. Coordinate systems defining Euler angles

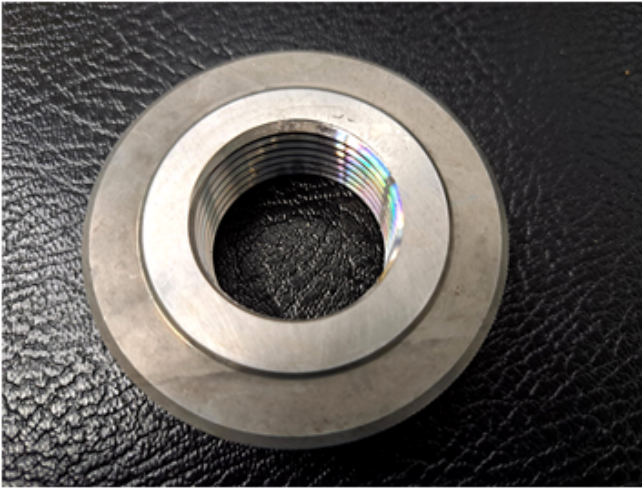


Fig.3. GO ring gauge

The system  $OXYZ$  positions in the intermediate coordinate system  $Ox''_1y''_1z''_1$  at the first rotation around the axis  $OY$ , where the matrix  $A_\psi$  has the form

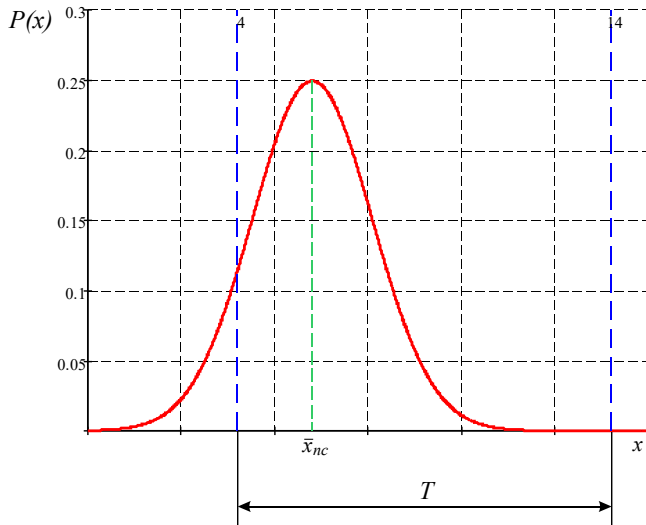


Fig.4. Location of the statistical distribution when making gauges without correction

$$A = \begin{bmatrix} c\psi \cdot c\theta & s\theta & -s\psi \cdot c\theta \\ s\psi \cdot s\gamma - & c\theta \cdot c\gamma & c\psi \cdot s\gamma + \\ -c\psi \cdot s\theta \cdot c\gamma & & +s\psi \cdot s\theta \cdot c\gamma \\ s\psi \cdot c\gamma + & -c\theta \cdot s\gamma & c\psi \cdot c\gamma - \\ +c\psi \cdot s\theta \cdot s\gamma & & -s\psi \cdot s\theta \cdot s\gamma \end{bmatrix}, \quad (6)$$

where  $c\psi = \cos\psi$ ,  $s\psi = \sin\psi$ ,  $c\theta = \cos\theta$ ,  $s\theta = \sin\theta$ ,  $c\gamma = \cos\gamma$ ,  $s\gamma = \sin\gamma$ .

In practice, the angles  $\psi$ ,  $\theta$  and  $\gamma$  are sufficiently small, because of which only small quantities of the second order can be accounted for in (6), i.e.

$$A = \begin{bmatrix} 1 & \theta & -\psi \\ \psi \cdot \gamma - \theta & 1 & \gamma + \psi \cdot \theta \\ \psi + \theta \cdot \gamma & -\gamma & 1 \end{bmatrix}. \quad (7)$$

The equations determining the errors along the coordinate axes  $OX$ ,  $OY$ , and  $OZ$  are obtained based on (1) and (7).

$$\begin{aligned} \Delta x &= \varepsilon_x + \Delta_{yx}^y + \Delta_{zx}^z + L_y(\psi_y\gamma_y - \theta_y) + L_z(\psi_z + \theta_z\gamma_z) \\ \Delta y &= \varepsilon_y + \Delta_{xy}^x + \Delta_{zy}^z + L_x\theta_x - L_z\gamma_z \\ \Delta z &= \varepsilon_z + \Delta_{xz}^x + \Delta_{yz}^y - L_x\psi_x + L_y(\gamma_y + \psi_y\theta_y), \end{aligned} \quad (8)$$

where  $L_x, L_y$  и  $L_z$  are the coordinates of the cutting tool in the system  $O|x|y|z|$ ;  $\psi_x, \theta_x, \gamma_x$  are Euler angles analogous to  $\psi, \theta, \gamma$  and defining the angular rotation of a coordinate system related to the guide of the machine along the  $OX$  axis;  $\psi_y, \theta_y, \gamma_y$  are Euler angles analogous to  $\psi, \theta, \gamma$  and defining the angular rotation of a coordinate system related to the guide of the machine along the  $OY$  axis;  $\psi_z, \theta_z, \gamma_z$  are Euler angles analogous to  $\psi, \theta, \gamma$  and defining the angular rotation of a coordinate system related to the guide of the machine along the  $OZ$  axis.

### III. EXPERIMENT AND RESULTS

The experimental researches were carried out on the basis of a technological procedure for the making of gauges for the control of external metric threads M30×1-6g. The final technological operation - grinding, is performed on a CNC Universal Internal Cylindrical Grinding Machine. The study covers GO ring gauges, which are presented in Fig.3.



Fig.5. Photographic material of the performed measurements

The first part of the experiment involves making 50 GO ring gauges for control of the one mentioned above external metric thread. The controlled parameter of the gauges is the average diameter, which is measured by a Bore gauge "Xtreme 3 Holematic Smart" with specially installed for the purpose of the research a Threads measuring head. The making of the gauges is carried out under one and the same cutting modes.

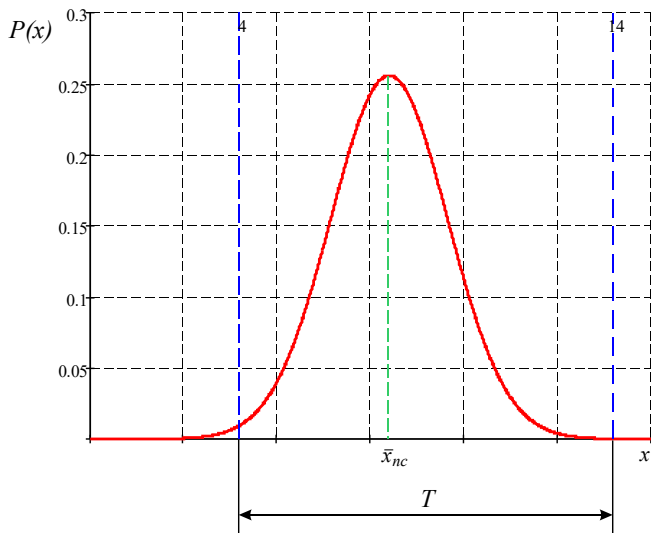
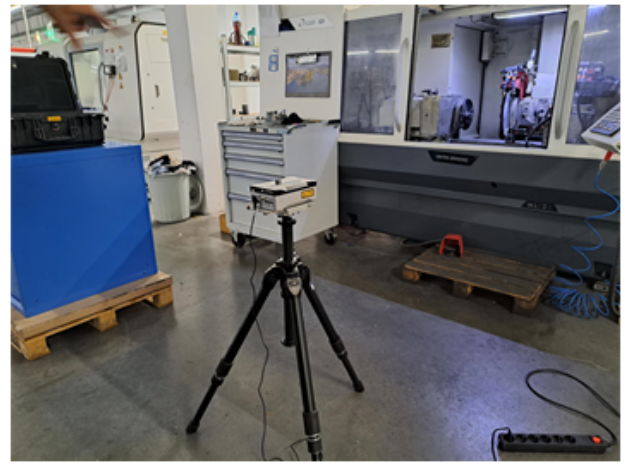


Fig.6. Location of the statistical distribution when making gauges with correction

A statistical analysis was performed on the data obtained when measuring the mean diameter of a excerption of 50 pieces of calibers. The type of statistical distribution, the arithmetic mean value  $\bar{x}_{nc}$  and the root mean square value  $\sigma_{nc}$  are determined. The result of the study is shown in a graphical format in Fig. 4, where the location of the curve of the differential law of distribution regarding the tolerance of the mean diameter of the made gauge is presented. It can be seen that the arithmetic mean value  $\bar{x}_{nc}$  is shifted to the left of the center of the tolerance field. This leads to a part of the statistical curve going beyond the tolerance limits, which is expressed in practice by the corresponding percentage of reject, which in this case is 10,6%.

The second part of the experimental work is based on the correction concept of machine errors proposed in this work. For this purpose, measurements of the parameters involved in the right-hand side of equations (8) were performed. Deviation measurements along each one of the machine's



three linear axes were taken using an XL-80 laser measurement system (Fig.5). Based on equations (8), the correcting values for each one of the coordinate axes were obtained, which were entered into the software platform of the machine. Fifty pieces of new calibers were made, which a statistical distribution of the mean diameter measurement data is shown in Fig.6. It sees from this figure that the arithmetic mean value  $\bar{x}_{wc}$  of this distribution approaches the center of the tolerance field, unlike  $\bar{x}_{nc}$ . This leads to a reduction in the volume of reject, the percentage value of which in this case is 0,52%. As can be seen from Figures 4 and 6, the curves defining the statistical distributions in the two excerptions are approximately the same, which means that the proposed correction methodology is sensitive above all to systematic errors and cannot influence the reduction of random errors.

In order to confirm the effect of the proposed concept, a variance analysis was performed. The statement that there is no statistically credible difference in the compared statistical indicators, which in this case are the mean values  $\bar{x}_{nc}$  and  $\bar{x}_{wc}$ , is accepted as a null hypothesis  $H_0$  in the analysis. According to the alternative hypothesis  $H_1$ , the ascertained difference in the empirical data is statistically credible and can be generalized to the general aggregates. To conduct the analysis, the Student's  $t$ -criterion was chosen for independent excerptions that have a normal distribution. After calculating the empirical value  $t_{emp}$ , it is found that the alternative hypothesis  $H_1$  is confirmed. This means that the conclusions made about the effectiveness of the proposed concept are statistically credible.

#### IV. CONCLUSIONS

A concept of correction of instrumental errors of technological machines is presented in the work. The total accuracy of the machine is characterized by the concept of volumetric error. The main constituent of the volumetric error is determined by the presence of geometric inaccuracies of the coordinate axes. The mathematical dependences determining the constituents of the geometric inaccuracies for each coordinate axis, which in practice represent deviations in the parallelism, perpendicularity and angle of the guides of the machine, are derived. The models of the constituents of the geometrical errors are developed so that their measurement with a laser measurement system is possible.

The concept proposed in this work has been verified in a real technological process for the making of gauges for the control of external metric threads. Due to the narrow tolerance fields of these elements, even small geometric errors of the coordinate axes lead to significant amounts of reject. The conducted statistical analysis of the presented method shows its good effectiveness in reducing the amount of reject.

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