# Hygrometers calibration: features of measurement results processing

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***Abstract*—The procedure of hygrometers calibration is considered. The calibration scheme and a mathematical model describing it are presented. A procedure for measurement uncertainty evaluating at hygrometers calibration has been developed. The issues of increasing the hygrometer calibration productivity associated with the processing of measurement results are investigated. Recommendations on the use of the characteristics of the range of observations sample for the evaluating numerical value of the measurand and its type A standard uncertainty are given.**

***Keywords— hygrometer, calibration, uncertainty, standard deviation, range***

## I. Introduction

Various types of hygrometers are widely used to measure of the gas's humidity. Hygrometers must be calibrated to traceability ensure of humidity measurement results to SI units. At the same time, in accordance with the requirements of ISO/IEC 17025:2019 [1], the laboratory must have a procedure for measurement uncertainty evaluating.

The most common hygrometer calibration scheme involves comparing the readings of the calibrated and reference hygrometers using a comparison device – a humidity generator. Such a scheme requires taking into account the correlation between the measuring results of the calibrated and reference hygrometers while simultaneously measuring the humidity in the generator chamber [2].

The need to calibrate a large number of hygrometers used for environmental monitoring in a large industrial plant such as Kozloduy NPP requires an increase in calibration performance. The latter can be achieved by simplifying of the measurement results processing.

The report substantiates the possibility of using the characteristics of the range of observations for evaluating the numerical value of the measurand and its standard uncertainty of type *A*.

## II. Hygrometer calibration orocedure

### *A. Calibration scheme and measurement model*

The calibration scheme for hygrometers by comparison method is shown in Fig. 1.

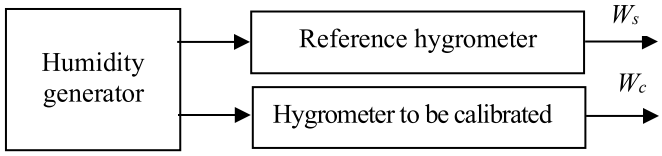


Fig. 1 – Scheme for comparing the reference hygrometers and hygrometer to be calibrated using a comparison device

The scheme provides for comparing the readings of the calibrated and reference hygrometers using a comparison device – a humidity generator. In this case, the mathematical model of calibration has the following form:

$$\Delta = (W_c + \delta_c) - (W_s + \Delta_s + \delta_{dr}) + \delta_{gr}, \tag{1}$$

where $\Delta$ is systematic error of the hygrometer to be calibrated; $W_c$ is the humidity measured by the hygrometer to be calibrated; $W_s$ – humidity measured by a reference hygrometer; $\delta_c$ is the correction for the resolution of the hygrometer to be calibrated; $\Delta_s$ is the systematic error of the reference hygrometer at the calibration point of the hygrometer to be calibrated; $\delta_{dr}$ is the correction for calibration instability (drift) of the reference hygrometer since its previous calibration; $\delta_{gr}$ is the correction for the humidity gradient in the humidity generator chamber.

### *B. Standard procedure for evaluating the result and measurement uncertainty at hygrometer calibration*

The estimate of relative humidity, measured by a hygrometer to be calibrated during repeated measurements, is the arithmetic mean of its $n_c$ readings $W_{ci}$, % RH:

$$\overline{W}_c = \frac{1}{n_c}\sum_{i=1}^{n_c} W_{ci},$$

The estimate of relative humidity measured by a reference hygrometer during repeated measurements is the arithmetic mean of its $n_s$ readings $W_{si}$, %RH:

$$\overline{W}_s = \frac{1}{n_s}\sum_{i=1}^{n_s} W_{si}.$$

The correction for the resolution of the hygrometer to be calibrated has a zero mathematical expectation: $\hat{\delta}_c = 0$.

An estimate of the systematic error of the reference hygrometer at the calibration point of the hygrometer to be calibrated $\hat{\Delta}_s$ is calculated from the bias specified on the calibration certificate of the reference hygrometer.

The correction for calibration instability (drift) of a reference hygrometer since its previous calibration has a zero mathematical expectation: $\hat{\delta}_{dr} = 0$.

The correction for the humidity gradient in the humidity generator chamber has a zero mathematical expectation: $\hat{\delta}_{gr} = 0$.

Taking into account of the input quantities estimates given above, the estimate of the measurand will be equal to:

$$\hat{\Delta} = \bar{W}_c - \bar{W}_s + \hat{\Delta}_s \qquad (1)$$

The standard uncertainty of the observed variability of the hygrometer to be calibrated readings is determined by the formula:

$$u_A(\bar{W}_c) = \sqrt{\frac{1}{n_c(n_c-1)}\sum_{i=1}^{n_c}(W_{ci} - \bar{W}_c)^2}\,.$$

The standard uncertainty of the correction estimate, due to the resolution of the hygrometer to be calibrated, is characterized by a rectangular distribution and is calculated by the formula:

$$u_B(\delta_c) = \frac{b}{2\sqrt{3}}\,,$$

were $b$ is resolution of the hygrometer to be calibrated.

The standard uncertainty of the observed variability of the reference hygrometer readings is given by:

$$u_A(\bar{W}_s) = \sqrt{\frac{1}{n_s(n_s-1)}\sum_{i=1}^{n_s}(W_{si} - \bar{W}_s)^2}\,.$$

The standard uncertainty of the systematic error estimate (bias) of a reference hygrometer is calculated by the formula:

$$u_B(\Delta_s) = \frac{U_s}{k_s}\,,$$

where $U_s$ is the expanded uncertainty of the reference hygrometer, taken from the calibration certificate for the calibration point of the hygrometer to be calibrated (if the value for the calibration point is not directly stated on the certificate, it is determined by linear interpolation between two known adjacent uncertainty values for the corresponding measurement range of the reference hygrometer); $k_s$ is the coverage factor taken from the calibration certificate of the reference.

The standard uncertainty of the correction estimate due to the instability (drift) of the reference hygrometer is characterized by a rectangular distribution and is calculated by the formula:

$$u_B(\delta_{dr}) = \frac{\theta_{dr}}{\sqrt{3}}\,,$$

where $\theta_{dr}$ is the limit corresponding to the normalized stability specified in the specification of the reference hygrometer.

The standard uncertainty of the correction estimate due to the humidity gradient in the humidity generator chamber is characterized by a rectangular distribution and is calculated by the formula:

$$u_B(\delta_{gr}) = \frac{\theta_{gr}}{\sqrt{3}}\,,$$

where $\theta_{gr}$ is the humidity gradient limits in the humidity generator chamber.

Since the measurement model is linear, and the contributions of all input quantities do not correlate with each other, the standard uncertainty of the measurand at calibration is calculated by the formula:

$$u(\Delta) = \sqrt{u_A^2(\bar{W}_c) + u_B^2(\delta_c) + u_A^2(\bar{W}_s) + u_B^2(\Delta_s) + u_B^2(\delta_{dr}) + u_B^2(\delta_{gr})}\,. \qquad (2)$$

The expanded measurement uncertainty for each calibration point is defined as:

$$U(\Delta) = k \cdot u(\Delta)\,, \qquad (3)$$

where $k$ is coverage factor, will be calculated as a Student's coefficient for a confidence level 0,9545 and the effective degrees of freedom $\nu_{eff}$, determined by the Welch–Satterthwaite formula:

$$\nu_{eff} = \frac{u^4(\Delta)}{\frac{u_A^4(W_c)}{n_c-1} + \frac{u_A^4(W_s)}{n_s-1} + \frac{u_B^4(\Delta_s)}{\nu_s}}\,, \qquad (4)$$

where $\nu_s$ is the degrees of freedom specified in the calibration certificate of the reference hygrometer.

The uncertainty budget for the considered procedure is given in Table. 1.

TABLE I. UNCERTAINTY BUGET FOR HYGROMETER CALIBRATION WHISOUT TAKING INTO ACCOUNT CORRELATION

| $X_i$ | $x_i$ | $u(x_i)$ | $\nu_i$ | $c_i$ | $u_i(y)$ |
|---|---|---|---|---|---|
| $W_c$ | $\bar{W}_c$ | $u_A(\bar{W}_c)$ | $n_c - 1$ | 1 | $u_A(\bar{W}_c)$ |
| $\delta_c$ | 0 | $u_B(\delta_c)$ | $\infty$ | 1 | $u_B(\delta_c)$ |
| $W_s$ | $\bar{W}_s$ | $u_A(\bar{W}_s)$ | $n_s - 1$ | -1 | $-u_A(\bar{x}_s)$ |
| $\Delta_s$ | $\hat{\Delta}_s$ | $u_B(\Delta_s)$ | $\nu_s$ | -1 | $-u_B(\Delta_s)$ |
| $\delta_{dr}$ | 0 | $u_B(\delta_{dr})$ | $\infty$ | -1 | $-u_B(\delta_{dr})$ |
| $\delta_{gr}$ | 0 | $u_B(\delta_{gr})$ | $\infty$ | 1 | $u_B(\delta_{gr})$ |
| $Y$ | $y$ | $u(y)$ | $\nu_{eff}$ | $k$ | $U$ |
| $\Delta$ | (1) | (2) | (4) | $t_{0,9545;\nu_{eff}}$ | (3) |

*C. Accounting for the correlation between the readings of the reference hygrometer and hygrometer to be calibrated*

With the simultaneous measurement of the value reproduced by the humidity generator with the reference higrometer and higrometer to be calibrated, an observed correlation between their readings may occur [2].

In this case, the estimate of the measurand should be calculated as:

$$\hat{\Delta} = \overline{W_c - W_s} + \hat{\Delta}_s, \qquad (5)$$

where

$$\overline{W_c - W_s} = \frac{1}{n}\sum_{i=1}^{n}(W_{ci} - W_{si}),\ n = n_c = n_s.$$

The combined standard measurement uncertainty at the hygrometer calibration in this case will be equal to:

$$u(\Delta) = \sqrt{u_A^2(\overline{W_c - W_s}) + u^2(\delta_c) + u^2(\Delta_s) + u^2(\delta_{dr}) + u^2(\delta_{gr})}, \qquad (6)$$

where $u_A(\overline{W_c - W_s})$ is standard uncertainty of the difference between the simultaneously observed readings of the reference hygrometer and hygrometer to be calibrated, determined by the formula:

$$u_A(\overline{W_c - W_s}) = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}[(W_{ci} - W_{si}) - (\overline{W_c - W_s})]^2}.$$

The effective degrees of freedom $\nu_{eff}$ for this case will be equal to:

$$\nu_{eff} = \frac{u^4(\Delta)}{\frac{u_A^4(\overline{W_c - W_s})}{n-1} + \frac{u_B^4(W_s)}{\nu_s}}. \qquad (7)$$

The uncertainty budget for the considered procedure is given in Табл. II.

TABLE I. UNCERTAINTY BUGET FOR HYGROMETER CALIBRATION WITH TAKING INTO ACCOUNT CORRELATION

| $X_i$ | $x_i$ | $u(x_i)$ | $\nu_i$ | $c_i$ | $u_i(y)$ |
|---|---|---|---|---|---|
| $W_c - W_s$ | $\overline{W_c - W_s}$ | $u_A(\overline{W_c - W_s})$ | $n-1$ | 1 | $u_A(\bar{W}_c)$ |
| $\delta_c$ | 0 | $u_B(\delta_c)$ | $\infty$ | 1 | $u_B(\delta_c)$ |
| $\Delta_s$ | $\hat{\Delta}_s$ | $u_B(\Delta_s)$ | $\nu_s$ | -1 | $-u_B(\Delta_s)$ |
| $\delta_{dr}$ | 0 | $u_B(\delta_{dr})$ | $\infty$ | -1 | $-u_B(\delta_{dr})$ |
| $\delta_{gr}$ | 0 | $u_B(\delta_{gr})$ | $\infty$ | 1 | $u_B(\delta_{gr})$ |
| $Y$ | $y$ | $u(y)$ | $\nu_{eff}$ | $k$ | $U$ |
| $\Delta$ | (5) | (6) | (7) | $t_{0,9545;\nu_{eff}}$ | (2) |

## III. EXAMPLE OF MEASUREMENT RESULTS PROCESSING AT HYGROMETER CALIBRATION

Let us consider the of the measurement uncertainty evaluation at calibration of a Testo 650 type hygrometer with a resolution of 0,1 %RH by comparing the readings with the readings of a reference Testo 400 type hygrometer using a humidity generator type "Huminator" at a point of 25 %RH.

The reading of the calibrated hygrometer is 26.8%RH. Reference hygrometer readings are given in Table. III.

TABLE II. REFERENCE HYGROMETER READUNGS

| № | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $W_{si}$, %RH | 26,13 | 26,11 | 26,12 | 26,1 | 26,14 |
| № | 6 | 7 | 8 | 9 | 10 |
| $W_{si}$, %RH | 26,10 | 26,11 | 26,13 | 26,14 | 26,12 |

The correction for bias of the reference hygrometer from the calibration certificate is 0,1 %RH with an expanded uncertainty of 1,2 %RH. The limit of instability of the reference hygrometer is 1,44%RH. The chamber humidity gradient limit is 1,4%RH.

The result of measurement uncertainty calculation at calibrating the Testo 650 hygrometer without taking into account the correlation is shown in Table. IV.

TABLE III. UNCERTAINTY BUDGET OF THE TESTO 650 HYGROMETER CALIBRATION

| $X_i$ | $x_i$, %RH | $u(x_i)$, %RH | $\nu_i$ | $c_i$ | $u_i(y)$, %RH |
|---|---|---|---|---|---|
| $W_c$ | 26,8 | 0 | 9 | 1 | 0 |
| $\delta_c$ | 0 | 0,0289 | $\infty$ | 1 | 0,0289 |
| $W_s$ | 26,119 | 0,00471 | 9 | -1 | - 0,00407 |
| $\Delta_s$ | 0,1 | 0,6 | $\infty$ | -1 | - 0,6 |
| $\delta_{dr}$ | 0 | 0,8314 | $\infty$ | -1 | - 0,8314 |
| $\delta_{gr}$ | 0 | 0,8083 | $\infty$ | 1 | 0,8083 |
| $Y$ | $y$, %RH | $u(y)$, %RH | $\nu_{eff}$ | $k$ | $U$, %RH |
| $\Delta$ | 0,781 | 1,306 | $\infty$ | $t_{0,9545;\nu_{eff}}$ | 2,6 |

## IV. INCREASING HYGROMETER CALIBRATION PERFORMANCE

Hygrometer calibration performance can be improved by simplifying the processing of measurement results. This can be achieved by using the middle of the range of humidity readings by the reference hygrometer and the hygrometer to be calibrated and by standard uncertainty estimating by way of range.

### A. *Obtaining of the measurand estimate*

The estimate of the measured humidity can be determined through the middle of the range of the sample using the formula:

$$\hat{W} = \frac{W_{\max} + W_{\min}}{2},$$

where $W_{\max}$, $W_{\min}$ are the minimum and maximum humidity values in the sample, respectively.

Dependences of different estimates of humidity by a reference hygrometer on the sample size *n* (Fig. 2) obtained as a result of the experiment were investigated.

It can be seen from the figure that with an increase of the sample size, the arithmetic mean of the sample increases, while the middle of the range decreases. For *n*=60, the difference in estimates may be 0,0035 %RH. This difference can be ignored if the resolution of the reference hygrometer is greater than 0,01 %RH.

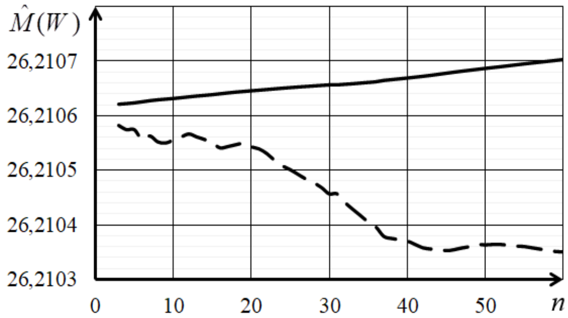


Fig. 2 Dependence of estimates of mathematical expectation on $n$:
— average; — — the middle of the range

*B. Determination of the standard uncertainty of the observed variability of hygrometer readings*

An estimate of the standard uncertainty of the observed variability of hygrometer readings can be obtained through its range ($W_{\max} - W_{\min}$):

$$u_A(\hat{W}) = \frac{W_{\max} - W_{\min}}{\alpha\sqrt{n}},$$

where the coefficient $\alpha$ is equal to the ratio of the sample range ($W_{\max} - W_{\min}$) to its standard deviation $s$ and depends both on the distribution law of the observed variability of readings and on the number of measurements taken $n$ [3].

We have investigated the law of distribution of the observed variability of the reference hygrometer readings. For this, 6392 measurements of humidity were carried out with a reference hygrometer. A histogram of the variability of these readings after eliminating outliers is shown in Fig. 3.

Despite the visual similarity of the histogram with the normal distribution law, Pearson's chi-squared test gave a negative result: the value of $\chi^2$ = 825 at $\chi_0^2$ = 18.46 for the probability of 0,999 and the number of degrees of freedom 4. A similar situation arose when using triangular, double exponential and lognormal distributions as hypothetical.

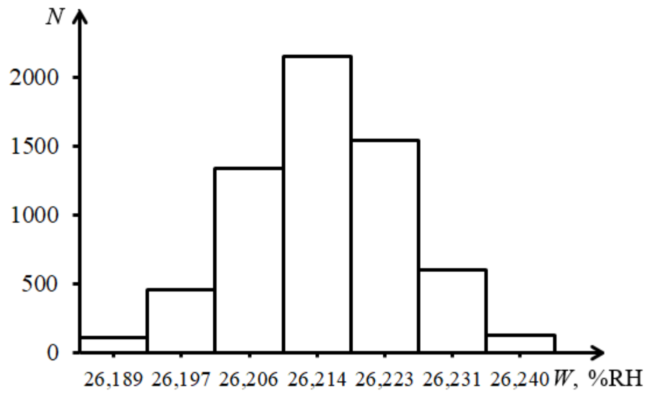


Fig. 3 Histogram of reference hygrometer readings variability

On Figure 4 shows the dependences of the coefficients $\alpha$ on the sample size $n$ for different laws of distribution of the observed scatter of the readings of a reference hygrometer, including the real one obtained experimentally.

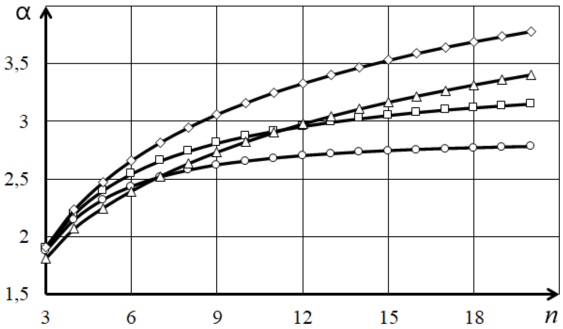


Fig. 4 Dependences of parameter α on $n$ for different PDF:
Δ – real; O – arcsine; ◊ – normal; □ – uniform

It can be seen from the figure that the real dependence is not approximated by any of the given theoretical ones.

For a real dependence $\alpha(n)$, an approximation was found in the form:

$$\alpha = 0{,}8508 \cdot \ln(n) + 0{,}862.$$

Using this approximation, we calculate $u_A(\hat{W}_s)$ for the given in Table. III data. The maximum humidity value is 26,14%RH, the minimum is 26,10%RH. That's why

$$u_A(\hat{W}_s) = \frac{26{,}14 - 26{,}10}{[0{,}8508 \cdot \ln(10) + 0{,}862]\sqrt{10}} = 0{,}00449 \text{ \%RH}.$$

The budgeted value (Table IV) is 0,00471 %RH, i.e. the error in determining the type A standard uncertainty through the sampling range was 4,75%.

## Conclusions

1. The procedure for the measurement uncertainty evaluation at hygrometer calibration by the method of comparison with a reference hygrometer is given, taking into account the possible correlation between the readings.

2. An example of measurement uncertainty evaluation at hygrometer calibration in the metrological laboratory NPP "Kozloduy" is considered.

3. The possibility of using the characteristics of the range of a sample of observations for evaluation the numerical value of the measurand and its standard uncertainty of type *A* is substantiated.

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