Relative Influence of Some Stochastic Factors on Bit-Error Rate of Ground-to-Ground Free Space Optics

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Abstract – **In this paper an equation for full Bit-Error Rate calculation of ground-to-ground free space optics according to partial Bit-Error Rates, caused by signal current fluctuations and quantum noises of signal and background currents has been suggested. Fluctuations of signal current are produced by atmosphere transmittance fluctuations, mechanical vibrations of antennas and atmospheric turbulence. Relative influence of this factors on full Bit-Error Rate is investigated.**

Keywords – **Bit-Error Rate, Free Space Optics, Atmospheric Transmittance Fluctuation, Atmospheric Turbulence and Quantum Noise.**

I. INTRODUCTION

The interest in Free-Space Laser Communications (Free-Space Optics – FSO) is increased in the last years. The great importance of Internet in modern world and some disadvantage of microwave and cable networks lead to widely usage of FSO in local area networks. This is caused to a great extent by the fact that FSO's technical and economical parameter most fully correspond to the requirements of that type of networks – quick and easy installation, portability, relatively low price and high speed information flows. Through Free-Space Optics the "last mile" problem is resolved. Furthermore FSO are irreplaceable for connections of distant light-guide systems, in cases when for infrastructure or economical reasons is impossible to use another light-guide.

The utilization of FSO is related to some important problems, which decision is subject to a number of researches [1-8]. They are result of simultaneous influence of many factors that affect the spatial structure of the laser beam - the atmospheric transmittance fluctuations, the propagation direction fluctuations caused by mechanical vibrations [6] of antennas and atmospheric turbulence [6,9-11]. Jointly influence of these factors and their stochastic character on one hand and the quantum noises of signal and background currents on the other have formed Bit-Error Rate (BER) of FSO.

The role of each of this factors in total effect of their jointly influence has not been examined in the literature. Therefore in this paper relative influence of mentioned factors on BER has been investigated.

II. THEORETICAL ANALYSIS

FSO shown in Fig.1 consists of laser and pulse-code modulator (PCM) with output optical power Φ_L, transmitting antenna TA with transparency τ_1, radiated Gaussian laser beam with initial radius r_0, receiving antenna RA with radius R_2 and transparency τ_2, interference filter IF and photodetector PhD with radius R_d.

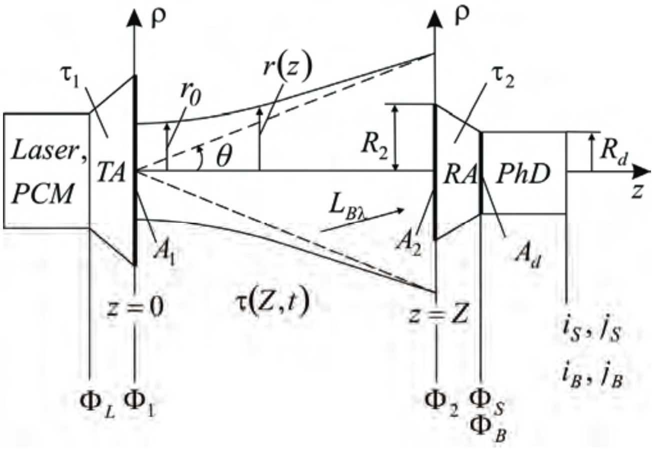

Fig. 1

Let us assume that the values of these parameters of laser communication system – the frequency bandwidth of pulse-code signal Δf, the working wavelength λ, the horizontal link altitude H and the distance between the transmitter and the receiver Z are known.

The average value of signal current over quantum fluctuations is given by [14]

$$\langle i_S \rangle(Z,\rho) = U(Z)\tau(Z)\psi(Z,\rho),$$

$$U(Z) = \frac{2\tau_1\tau_2 S_i \Phi_L R_2^2}{\varepsilon r^2(Z)},$$

$$\varepsilon = 1 - \frac{1}{exp(2)} \approx 0{,}865, \tag{1}$$

where $\tau(Z)$ is the atmospheric transmittance and the function

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$$\psi(Z,\rho)=\exp\left[-2\frac{\rho^2}{r^2(Z)}\right] \qquad (2)$$

gives casual linear deviation ρ of laser beam in relation to receiving antenna centre, caused by mechanical vibration of antennas and atmospheric turbulence [14].

Gaussian laser beam radius in receiving antenna plane is presented with the equation

$$r(Z)=r_0\sqrt{1+\left(\frac{K\lambda}{\pi r_0^2}\right)^2 Z^2}\ , \qquad (3)$$

where the coefficient K provides a simple and adequate account of both real beam divergence and additional beam expansion resulting from the radiance scatter from relatively small (in relation to $2r(Z)$) aerosol inhomogeneties and turbulence eddies [12].

The absolute photodetector's sensitivity is computed from relation

$$S_i=\frac{e\eta\lambda}{hc}\, , \qquad (4)$$

where η is quantum efficiency of the photodetector and $e=1{,}6.10^{-19}\,\mathrm{C}$, $h=6{,}626.10^{-34}\,\mathrm{J.s}$, $c=3.10^8\,\mathrm{m/s}$.

Signal-to-Noise Ratio (SNR) of the laser communication system is represented as

$$SNR=\frac{\langle i_S\rangle^2(Z)}{\sigma_{i_S}^2(Z)+\sigma_{j_S}^2(Z)+\sigma_{j_B}^2(Z)}\, . \qquad (5)$$

The averaging of $\langle i_S\rangle$ over fluctuations of atmospheric transmittance τ and over function $\psi(Z,\rho)$ can be given by expression

$$\langle\langle i_S\rangle\rangle(Z)=\langle i_S\rangle(Z,\langle\rho\rangle=0)=$$
$$=\langle i_S\rangle(Z)=U(Z).\langle\tau\rangle(Z).\langle\psi\rangle(Z). \qquad (6)$$

Dispersion of signal current determined by the fluctuations of τ and $\psi(Z,\rho)$ is

$$\sigma_{i_S}^2(Z)=$$
$$=U^2(Z)\left[\sigma_\tau^2(Z)\sigma_\psi^2(Z)+\langle\psi\rangle^2(Z)\sigma_\tau^2(Z)+\langle\tau\rangle^2(Z)\sigma_\psi^2(Z)\right]. \qquad (7)$$

Schottky's formula gives the variances of the quantum fluctuations of signal and background currents

$$\sigma_{j_S}^2(Z)=2e\langle i_S\rangle(Z)\Delta f \qquad (8)$$

and

$$\sigma_{j_B}^2=2e\pi^2\tau_2 S_i L_{B\lambda}R_2^2R_d^2\frac{(\Delta\lambda)_{IF}\Delta f}{f_{eq}^2}\, , \qquad (9)$$

where $L_{B\lambda}$ is the brightness of background radiation, $(\Delta\lambda)_{IF}$ is the bandwidth of the interference filter before photodetector and f_{eq} is equivalent focal length of the receiving antenna.

Statistical moments of atmospheric transmittance τ is given by relations [13]

$$\langle\tau\rangle(Z)=\exp(-\langle\alpha\rangle Z), \qquad (10)$$

$$\sigma_\tau^2(Z)=\langle\alpha\rangle^2\Delta_\alpha^2 HZ\exp(-2\langle\alpha\rangle Z).$$

For calculation of coefficient of extinction $\langle\alpha\rangle$ the following expression is used [15]

$$\langle\alpha\rangle,\mathrm{km}^{-1}=\frac{3{,}91}{S_M,\mathrm{km}}\left(\frac{\lambda,\mu\mathrm{m}}{0{,}55}\right)^{-0{,}585(S_M,\mathrm{km})^{1/3}}\, , \qquad (11)$$

where S_M is meteorological visibility.

With $\Delta_\alpha\in[0{,}05\,;0{,}5]$ is represented the variation coefficient of spatial heterogeneity of α [16] and

$$\Delta_\alpha=\frac{\sigma_\alpha}{\langle\alpha\rangle}\, . \qquad (12)$$

Eq. (5) can be transformed like

$$SNR=\frac{1}{\dfrac{1}{\dfrac{\langle i_S\rangle^2(Z)}{\sigma_{i_S}^2(Z)}}+\dfrac{1}{\dfrac{\langle i_S\rangle^2(Z)}{\sigma_{j_S}^2(Z)}}+\dfrac{1}{\dfrac{\langle i_S\rangle^2(Z)}{\sigma_{j_B}^2}}}=$$
$$=\frac{1}{\dfrac{1}{SNR_{i_S}}+\dfrac{1}{SNR_{j_S}}+\dfrac{1}{SNR_{j_B}}}\, , \qquad (13)$$

where SNR_{i_S}, SNR_{j_S} and SNR_{j_B} are signal-to-noise ratios, caused by examining factors – fluctuation of signal current due to the atmospheric transmittance and laser beam direction fluctuations, quantum fluctuations of signal current and quantum fluctuations of background current.

With respect to Eqs. (6), (7), (8) and (13) for SNR_{i_S}, SNR_{j_S} and SNR_{j_B} is obtained

$$SNR_{i_S}=\frac{\langle i_S\rangle^2(Z)}{\sigma_{i_S}^2(Z)}=$$

$$=\frac{\langle\tau\rangle^2(Z).\langle\psi\rangle^2(Z)}{\sigma_\tau^2(Z).\sigma_\psi^2(Z)+\langle\psi\rangle^2(Z).\sigma_\tau^2(Z)+\langle\tau\rangle^2(Z).\sigma_\psi^2(Z)}\, , \qquad (14)$$

$$SNR_{j_S}=\frac{\langle i_S\rangle^2(Z)}{\sigma_{j_S}^2(Z)}=\frac{\langle i_S\rangle(Z)}{2e\Delta f}=$$

$$=\frac{U(Z).\langle\tau\rangle(Z).\langle\psi\rangle(Z)}{2e\Delta f}, \quad (15)$$

$$SNR_{j_B}=\frac{\langle i_S\rangle^2(Z)}{\sigma_{j_B}^2} \quad (16).$$

In Eq. (14) SNR_{i_S} depends only on atmospheric communication channel condition, but SNR_{j_S} and SNR_{j_B} from Eqs. (15) and (16) depend on both atmospheric communication channel condition and power parameters of communication system.

According to [14] partial Bit-Error Rates, produced of mentioned three factors can be calculated by equations

$$BER_{i_S}=\frac{1}{2}\operatorname{erfc}\left(\frac{SNR_{i_S}}{2\sqrt{2}}\right),$$

$$BER_{j_S}=\frac{1}{2}\operatorname{erfc}\left(\frac{SNR_{j_S}}{2\sqrt{2}}\right), \quad (17)$$

$$BER_{j_B}=\frac{1}{2}\operatorname{erfc}\left(\frac{SNR_{j_B}}{2\sqrt{2}}\right).$$

The Bit-Error Rate of system can be expressed with partial BERs by means of the equation

$$BER=\frac{1}{2}\operatorname{erfc}\left\{\sum_{k=i_S;j_S;j_B}\frac{1}{[\operatorname{erfc}\operatorname{inv}(2.BER_k)]^2}\right\}^{-\frac{1}{2}}=\frac{1}{2}\operatorname{erfc}\left(\frac{SNR}{2\sqrt{2}}\right). \quad (18)$$

In Eq. (18) $\mathrm{X}=\operatorname{erfcinv}(\mathrm{Y})$ is the inverse function of $\mathrm{Y}=\operatorname{erfc}(\mathrm{X})$, which is integrated in MATLAB 7.1.

III. NUMERICAL RESULTS

The results of calculation of BER_{i_S}, BER_{j_S} and BER in dependence on meteorological visibility S_M and dispersion of linear deviation of laser beam in transmitter's plane σ_ρ^2 are represented on Fig. 2 and Fig. 3. The following typical for FSO input data have been assumed: $Z=10\,\mathrm{km}$; $H=50$ m; $\Delta_\alpha=0{,}35$; $\tau_1=0{,}7$; $\tau_2=0{,}6$; $\Phi_L=10\,\mathrm{mW}$; $R_2=10\,\mathrm{cm}$; $\Delta f=1\,\mathrm{GHz}$; $\lambda=1{,}55\,\mu\mathrm{m}$; $\eta=0{,}7$; $r_0=3\,\mathrm{mm}$; $K=10$; $L_{B\lambda}=0{,}01\,\mathrm{W/m^2 sr\,\mathring{A}}$; $R_d=5\,\mu\mathrm{m}$; $f_{eq}=0{,}5\,\mathrm{m}$; $(\Delta\lambda)_{IF}=20\,\mathring{A}$.

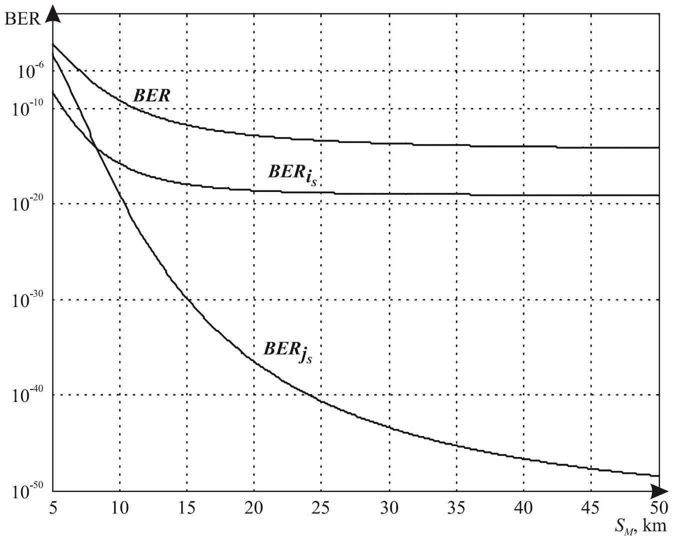

Fig. 2. Dependence of BER_{i_S}, BER_{j_S} and BER on meteorological visibility S_M for $\sigma_\rho^2=5{,}76\,\mathrm{m}^2$

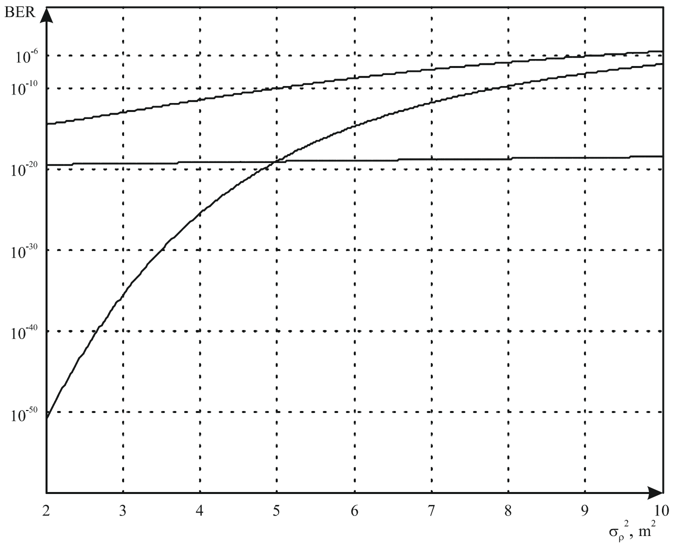

Fig. 3. Dependence of BER_{i_S}, BER_{j_S} and BER on dispersion σ_ρ^2 for $S_M=10\,\mathrm{km}$

IV. CONCLUSION

The calculations with MATLAB 7.1. show that BER_{j_B} is too small and therefore is not displayed on Fig. 2 and Fig. 3. This is confirmed by [7]. The exception is the case of direct sunlight hitting the photodetector. Direct sunlight may cause break of communication link. That can be escape with close to North-South orientation of antennas.

Fig. 2 illustrates the natural decrement of Bit-Error Rate with the meteorological visibility S_M increase. That is strongly expressed in BER_{j_S} because of more weaker influence of quantum noise in case of high signal current values. The value of BER follows BER_{i_S} but is five to six orders greater. Despite too small values of BER_{j_S} in case of high meteorological visibility, it produces sensible influence on full Bit-Error Rate.

In Fig. 3 BER_{i_S} is practically permanent, while BER_{j_S} has changed in wide number of values. When the dispersion of linear deviation of laser beam σ_ρ^2 has great values, BER_{j_S} vastly exceeds BER_{i_S} and has a decisive role in forming of BER. That can be explained with increasing of quantum noises when the values of signal current are smaller in case of greater linear deviation σ_ρ^2.

From investigations can be concluded that signal current fluctuations due to atmospheric transmittance, mechanical vibrations of antennas and atmospheric turbulence and quantum noises of detection has a great part in Bit-Error Rate forming. Therefore they can be taken into consideration in FSO planning. Influence of quantum fluctuations caused by background current can be ignored, when the suitable orientation of communication system antennas is used.

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