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Investigation of the Influence of the Inertia of the Photon Counting System on the Acceptance Accuracy in Photon Counting Mode

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Abstract. A methodology for determining the minimum distance, ensuring a connection with a negligible influence of the inertia of the photon counting system, is proposed. An algorithm for applying the methodology has been developed. A numerical estimate of the discussed minimum distance for the two types of inertia of the photon counting system was made using data typical for Space Laser Communication Systems. Numerical examples illustrated by graphic material have been shown.

Introduction

Space Laser communication systems of the type "Earth-to-space" or "space-to-Earth" are characterized by large distances between the corresponding points, respectively, with high diffraction scattering of the optical energy, as well as with energy losses from the extinction of the radiation in the atmospheric part of the transmission medium. The conditions described above necessitate the registration of feeble optical signals in the receiving parts of the systems. There is no electric current in the anode circuits of the photoelectron multipliers but only separate single-electron pulses (SEP). Moreover, the average interval Δt between two neighbouring SEP along the time axis t significantly exceeds their durations. Each value of the received signal in the output of the photoelectronic multiplier is formed by estimating the random number of SEP m, which appear in a correspondingly large time interval t_r (time for one registration) [1 - 7]. In this connection, the names photon counting mode (PCM) and photon counting system (PCS) are used. In addition, the interval t_r is a step of sampling, of the signal $S(t)$ when it is received in the photon counting mode.

We must note the significant important role of the inertia of the real PCS - the inertia from the first genus type I (with non-prolonging recovery time) and the inertia from the second genus (with prolonging recovery time) [7].

Figure 1 illustrates the main components of PCS with their corresponding input and output characteristics for both types of inertia of PCS.

The photons of the received optical flow fall on the photocathode of the photoelectric multiplier. Their number for time t_r is *n*. In the out of the photocathode, the emitted electrons appear and their number for registration time t_r is ***m*,** where *m* is Poisson random quantity. The number of single-electron pulses in the out of the dynode system for t_r is *i* and the number of the standard pulses (SP) is ***k*.** Because of the Photon counting systems inertia, the random variables *i* and *k* are not Poisson distributed.

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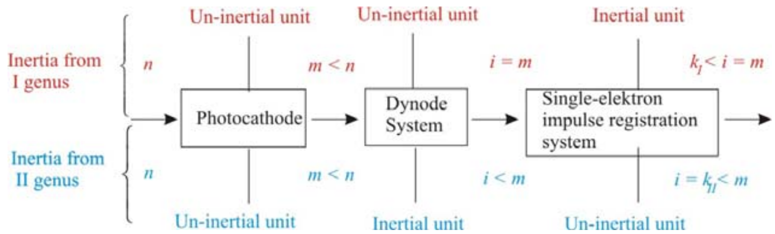

FIGURE 1

The present work aims to develop an algorithm for determining the minimum distance, providing a connection with the negligible influence of PCS inertia on the accuracy of PCM reception.

THEORETICAL ANALYSIS

Figure 2 depicts the simplified scheme of an optical communication system with reception in photon counting mode. The altitude above the ground is measured along the h -axis. It is assumed that $h=0$ coincides with the plane of the receiving aperture A_r. The altitude of the Earth's atmosphere H with general transparency τ_a in the vertical direction is introduced. Transmitting antenna A_t has τ_t transparency. The interference filter IF has a bandwidth $(\Delta\lambda)_{IF}$ and transparency relative to that of the receiving antenna τ_r, θ is the diffraction Gaussian laser beam divergence, $l_b(\lambda_0)$ – the spectral density of the background radiations energy, $\varepsilon = 0{,}865$, $\overline{\Phi_t}$ is the signal average optical flow. The signal optical flow Φ_{rs} and the background optical flow Φ_{rb} are observed at the IF output, for which we can write [1]:

$$\Phi_{rs} = \frac{2\tau_t\tau_a\tau_r A_r \overline{\Phi_t}}{\pi\,\varepsilon\,\theta^2 Z^2} \tag{1}$$

$$\Phi_{rb} = \pi\,\tau_r l_b(\lambda_0) A_r \theta_r^2 (\Delta\lambda)_{IF} \tag{2}$$

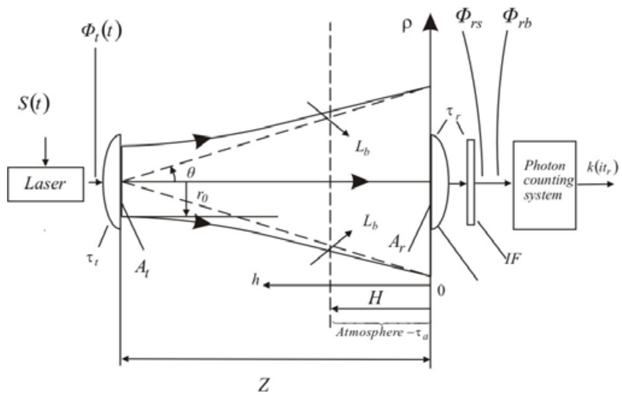

FIGURE 2

As a criterion for the estimation of $Z_{\min}$ we accept the condition

$$\frac{(SNR)_{I,II}[dB]}{(SNR)_0[dB]} = V \ , \quad V \le 1 \, , \tag{3}$$

where the signal-to-noise ratio is calculated for a Photon Counting System inertia of the first genus, of the second genus and for the hypothetical case of "zero" inertia ($k = m$). The value of V is set depending on the specific required accuracy of the reception in the PCM.

The input of background radiation in the analysis requires the representation of n, m and k by the sums

$$n = n_s + n_b ,\ m = m_s + m_b ,\ k = k_s + k_b . \tag{4}$$

(s - signal, b –background).

The random variables m_s and m_b are Poisson and statistically independent. The random variables k_s and k_b are also statistically independent, but not Poisson. In accordance with (4) we have:

$$\langle m \rangle = \langle m_s \rangle + \langle m_b \rangle , \tag{5}$$

$$\sigma_m^2 = \sigma_{m_s}^2 + \sigma_{m_b}^2 = \langle m_s \rangle + \langle m_b \rangle , \tag{6}$$

$$\langle k \rangle = \langle k_s \rangle + \langle k_b \rangle . \tag{7}$$

Statistical characteristics of k_I and k_{II} are described by the equations [7]

$$\langle k \rangle_I = \frac{\langle m \rangle}{1 + \frac{t_s}{t_r}\langle m \rangle} , \quad \left(\sigma_k^2\right)_I = \langle k \rangle_I \left(1 - \frac{t_s}{t_r}\langle k \rangle_I \right)^2 , \tag{8}$$

$$\langle k \rangle_{II} = \langle m \rangle \exp\left(-\frac{t_s}{t_r}\langle m \rangle \right) , \quad \left(\sigma_k^2\right)_{II} = \langle k \rangle_{II} \left(1 - 2\frac{t_s}{t_r}\langle k \rangle_{II} \right) \tag{9}$$

Since the signaling components of $\langle m \rangle$ and $\langle k \rangle$ are $\langle m_s \rangle$ and $\langle k_s \rangle$, respectively, the definition of SNR becomes

$$(SNR)_{I,II} = \frac{\langle k_s \rangle_{I,II}}{\sigma_m} = \frac{\langle k_s \rangle_{I,II}}{\sqrt{\langle m_s \rangle + \langle m_b \rangle}} , \tag{10}$$

and ignoring the inertia of the PCS –

$$(SNR)_0 = \frac{\langle m_s \rangle}{\sigma_m} = \frac{\langle m_s \rangle}{\sqrt{\langle m_s \rangle + \langle m_b \rangle}} \tag{11}$$

We start our analysis by finding the value of $Z_{\min,I}$, i.e. the value of $Z_{\min}$ in the case of inertia of SBF of the first genus.

Based on (8) for $\langle k_s \rangle_I$ we write :

$$\langle k_s \rangle_I = \frac{\langle m_s \rangle}{1 + \frac{t_s}{t_r}\left(\langle m_s \rangle + \langle m_b \rangle \right)} . \tag{12}$$

Taking into account (10) and (12) we find

$$(SNR)_I = \frac{\langle m_s \rangle}{\left[1 + \frac{t_s}{t_r}\left(\langle m_s \rangle + \langle m_b \rangle \right) \right] \sqrt{\langle m_s \rangle + \langle m_b \rangle}} ,$$

respectively

$$(SNR)_I[dB] = 20\lg \frac{\langle m_s\rangle}{\left[1+\frac{t_s}{t_r}\left(\langle m_s\rangle+\langle m_b\rangle\right)\right]\sqrt{\langle m_s\rangle+\langle m_b\rangle}}\,. \tag{13}$$

We represent (11) as

$$(SNR)_0[dB] = 20\lg \frac{\langle m_s\rangle}{\sqrt{\langle m_s\rangle+\langle m_b\rangle}} \tag{14}$$

Substituting (13) and (14) into (3) we obtain

$$\lg \frac{\langle m_s\rangle}{\left[1+\frac{t_s}{t_r}\left(\langle m_s\rangle+\langle m_b\rangle\right)\right]\sqrt{\langle m_s\rangle+\langle m_b\rangle}}\left[\lg \frac{\langle m_s\rangle}{\sqrt{\langle m_s\rangle+\langle m_b\rangle}}\right]^{-1} = V_I\,,$$

which, after an elementary mathematical transformation, comes down to

$$\left(\frac{\langle m_s\rangle}{\sqrt{\langle m_s\rangle+\langle m_b\rangle}}\right)^{1-V_I} = 1+\frac{t_s}{t_r}\left(\langle m_s\rangle+\langle m_b\rangle\right). \tag{15}$$

For the mathematical expectation of the random number m of the electrons emitted by photocathode for time t_r we use the natural relation [1]

$$\langle m\rangle = \eta\, n = \frac{\eta\, t_r \lambda_0 \overline{\Phi_r}}{h\,c}\,,$$

where η is the quantum efficiency of the photodetector, λ_0 is the optical wavelength.

Substituting expressions for $\langle m_s\rangle$ и $\langle m_b\rangle$ in (15) we write down

$$\langle m_s\rangle = \frac{\eta\, t_r \lambda_0 \overline{\Phi_{rs}}}{h\,c}\,, \tag{16}$$

$$\langle m_b\rangle = \frac{\eta\, t_r \lambda_0 \Phi_{rb}}{h\,c}\,, \tag{17}$$

We substitute (16), (17), (1) and (2) in (15), and obtain

$$\left\{\alpha\,\varphi\left(Z_{\min,I}\right)\Big/\sqrt{\alpha\left[\varphi\left(Z_{\min,I}\right)+\beta\right]}\right\}^{1-V_I} = 1+\gamma\,\alpha\left[\varphi\left(Z_{\min,I}\right)+\beta\right], \tag{18}$$

where

$$\alpha = \frac{\eta\,\tau_r\, t_r \lambda_0 A_r}{\pi\, h\,c}\,, \tag{19}$$

$$\varphi\left(Z_{\min,I}\right) = \frac{2\tau_t \tau_a \overline{\Phi_t}}{\varepsilon\,\theta^2\left[Z_{\min,I}\right]^2}\,, \tag{20}$$

$$\beta = \pi^2 l_b(\lambda_0)\,\theta_r^2\,(\Delta\lambda)_{IF}\,, \tag{21}$$

$$\gamma = \frac{t_s}{t_r}. \tag{22}$$

We now assume PCS with second genus and find expression $Z_{\min,II}$ for $Z_{\min}$. It is easy to repeat the analysis above. Formal differences result only from the fact that we use (9) instead of (8) as a basic expression.

Based on (5), (7), (9), and (10) we find:

$$(SNR)_{II}[dB] = 20\lg \frac{\langle m_s\rangle \exp\left[-\frac{t_s}{t_r}\left(\langle m_s\rangle + \langle m_b\rangle\right)\right]}{\sqrt{\langle m_s\rangle + \langle m_b\rangle}}. \tag{23}$$

We replace (23), (14) in (3) and after elementary transformation we get

$$\left(\langle m_s\rangle \Big/ \sqrt{\langle m_s\rangle + \langle m_b\rangle}\right)^{1-V_{II}} = \exp\left[\frac{t_s}{t_r}\left(\langle m_s\rangle + \langle m_b\rangle\right)\right]$$

$$\left(\langle m_s\rangle \Big/ \sqrt{\langle m_s\rangle + \langle m_b\rangle}\right)^{1-V_{II}} = \exp\left[\frac{t_s}{t_r}\left(\langle m_s\rangle + \langle m_b\rangle\right)\right]. \tag{24}$$

With expressions (16), (17), (1) and (2) we transform (24) into the equation

$$\left\{\alpha\,\varphi\left(Z_{\min,II}\right) \Big/ \sqrt{\alpha\left[\varphi\left(Z_{\min,II}\right) + \beta\right]}\right\}^{1-V_{II}} = \exp\left[\gamma\left(\alpha\varphi\left(Z_{\min,II}\right) + \alpha\beta\right)\right], \tag{25}$$

determined by substitutions (19) - (22) (of course, in (20) we replace $Z_{\min,I}$ with $Z_{\min,II}$).

NUMERICAL RESULTS AND DISCUSSIONS

For the numerical estimation of $Z_{\min,I}$, the following typical for SCS input data have been used: $\lambda_0 = 0{,}53\,\mu m$; $\overline{\Phi_t} = 0{,}5\,W$; $\theta = 3\,mrad$; $\tau_t = 0{,}6$; Z – current value; $S_M = 20\,km$; $\tau_r = 0{,}4$; $A_r = 0{,}02\,m^2$; $\eta = 0{,}1$; $t_1 = 8\,ns$; $t_s = 15\,ns$; $t_r = 10\,ms$, $V_I = V_{II} = 0{,}9$; $\theta_r = 5\,mrad$; $(\Delta\lambda)_{IF} = 100\,\text{Å}$; $l_b(\lambda_0) = 10^{-6}\,\frac{W}{m^2 sr\,\text{Å}}$

Substituting the indicated numerical values in (19) - (22) we calculate:

$$\alpha = 6{,}79.10^{13}\,\frac{m^2}{W}, \quad \varphi\left(Z_{\min,I}\right) = 5{,}22.10^{-2} \Big/ \left(Z_{\min,I}[km]\right)^2\,\frac{W}{m^2}, \quad \beta = 2{,}47.10^{-8}\,\frac{W}{m^2}, \quad \gamma = 1{,}5.10^{-7}.$$

Taking these results into account, (18) is transformed into expression:

$$\left[\frac{3{,}75.10^9}{\left(Z_{\min,I}[km]\right)^2} \Big/ \sqrt{\frac{3{,}75.10^6}{\left(Z_{\min,I}[km]\right)^2} + 1{,}68}\right]^{0,1} = 1 + 0{,}15\left[\frac{3{,}75.10^6}{\left(Z_{\min,I}[km]\right)^2} + 1{,}68\right] \tag{26}$$

We are looking for a graphic solution of (26). For this purpose we denote the left and right sides of (26) by $Left_I\left(Z_{\min,I}\right)$ and $Right_I\left(Z_{\min,I}\right)$. The calculated values are shown in Table 1

TABLE 1

$Z_{\min,I}$	$Left_I(Z_{\min,I})$	$Right_I(Z_{\min,I})$	$Z_{\min,II}$	$Left_{II}(Z_{\min,II})$	$Right_{II}(Z_{\min,II})$
500	*2,272*	*3,502*	*950*	*2,107*	*2,399*
600	*2,227*	*2,815*	*1000*	*2,093*	*2,258*
700	*2,187*	*2,400*	*1050*	*2,079*	*2,143*
800	*2,152*	*2,131*	*1100*	*2,066*	*2,048*
900	*2,121*	*1,947*	*1150*	*2,054*	*1,969*
1000	*2,093*	*1,815*	*1200*	*2,042*	*1,902*

The resulting dependences of $Left_I$ and $Right_I$ on distance $Z_{\min,I}$ are graphically presented in Fig. 3. The abscissa of their intersection gives the solution of (26), namely: $Z_{\min,I} = 787\,km$.

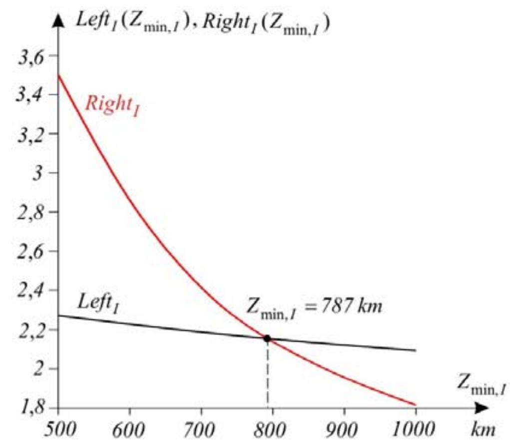

FIGURE 3. Graphical solution of equation (26) for determining $Z_{\min,I}$

Now we verify whether the condition for receiving the optical signal in the PCM, displayed in [1] for $Z = Z_{\min,I}$, is fulfilled.

$$\Phi_r \le \Phi_{r,\min}^{(CR)} = \frac{3hc}{\eta \lambda_0 t_1}, \tag{27}$$

where $\Phi_{r,\min}^{(CR)}$ is the minimum received optical flow, granting the current reception mode.

For this case we calculate $\Phi_{r,\min}^{(CR)} = 1{,}407\,nW$ and, in accordance with (1) and (2) we obtain $\Phi_r = 0{,}278\,nW$

We have $0{,}278 < 1{,}407$, i.e. condition (27) is satisfied.

To determine $Z_{\min,II}$ we represent (25) in the form

$$Left_{II}(Z_{\min,II}) = Right_{II}(Z_{\min,II})..$$

and we put in (19) - (22) the above numerical input data. The results of the corresponding calculations are shown in Table 1.

The resulting dependences of $Left_{II}$ and $Right_{II}$ on distance $Z_{\min,II}$ are graphically presented in Fig. 4, which represents the graphical solution of equation (25) for the received input data. We find $Z_{\min,II} = 1089\,km$.

Again, we verify the fulfillment of (27), but in the case of $Z = Z_{\min,II}$.

We calculate $\Phi_r = 0,175\, nW$ .Therefore $\Phi_r = 0,175\, nW < \Phi_{r,\min}^{(CR)} = 1,407\, nW$, i.e. condition (27) is satisfied.

Comparing the results of the two parts of our analysis, we find inequality $Z_{\min,II} > Z_{\min,I}$. It is a natural consequence of the fact that $(SNR)_{II}[dB]$ decreases with a decrease of Z faster than $(SNR)_I[dB]$.

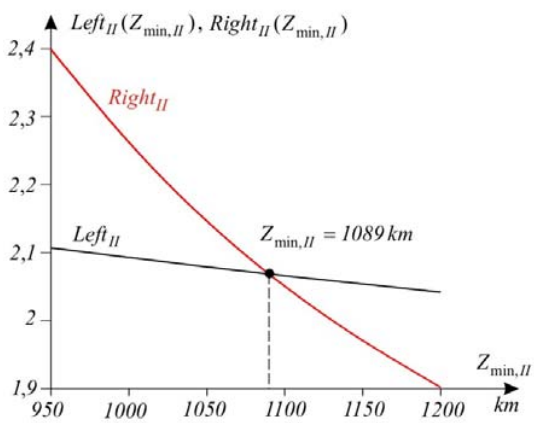

FIGURE 4. Graphical solution of equation (25) for determining $Z_{\min,II}$

CONCLUSION

An algorithm for determining the minimum distance $Z_{\min}$, ensuring a connection with a negligible influence of the inertia of the photon counting system in Space laser communication systems, is proposed. Graphical solutions for determining $Z_{\min,I}$ (inertia from I genus) and $Z_{\min,II}$ (inertia from II genus) for the used data have been obtained. As a result of the analysis it is established $Z_{\min,II} > Z_{\min,I}$.

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