H_∞ Reduced Order Control of a Thermal Plant

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***Abstract* — The current paper investigates the possibility of *H*∞ controller order reduction for a simple dynamic plants, which are predominand in the industry. A design procedure is carried out for a thermal plant model, received by system identification. Two full order controllers with integral part are obtained having series and parallel form. A mixed *S/KS* sensitivity structure is employed. Using Hankel singular values additional 5 reduced order controllers are obtained. Finally based on reduced plant model 4 new controllers are obtained. All control algorithms are implemented in digital form based on discrete state space equations, on a Programmable Logic Controller. Simple programs are developed for matrix addition and multiplication based on user defined PLC type. Additionally some programs are implemented for matrix transfer between a MATLAB client and SCADA OPC server. Experiments are conducted with a real plant. All eleven closed loop systems are compared by the singular values of their sensitivity functions. Performance characteristics are calculated and arranged on table. For the plant employed, the results show strong possibility of order reduction without significant loss of performance. The parallel form controller makes it possible to reduce the controller further that the series form.**

***Keywords*— Hankel Singular values, H∞ theory, reduced order controller, PLC**

I. Introduction

Although there exists already a solution for the task of finding a fixed order H_∞ controller [1], the problem of order reduction based on full order controllers is still an interesting one. Often the obtained controllers are of high order, dictated by the plant order and the use of weighting fiters [2]. The higher the filter order is the higher the controller order gets. Significant part of industrial plants are self-regulating and are characterized by simple dynamics. In that case the use of high order controller might not be waranted. On top of that higher order leads to more calculations. Keeping in mind the limited precision of number representation, one would like to avoid the need for too many calculations, to minimize the accumulation of numerical errors.

II. H_∞ Controller Design

The plant to be controlled is shown of Fig. 1. It is a thermal plant consisting of 4 pieces of aluminum. At each piece a thermosensor is attached. On three of them a heater is used for control and on the fourth a fan. An identification procedure is conducted, based on the subspace method [3] and a discrete multivariable plant with the three heaters as inputs is received. The outputs to be controlled are the temperatures of three of the aluminum pieces. The model order is nine. Validation tests are shown on Fig. 2 and Fig. 3

Fig. 1. Thermal plant

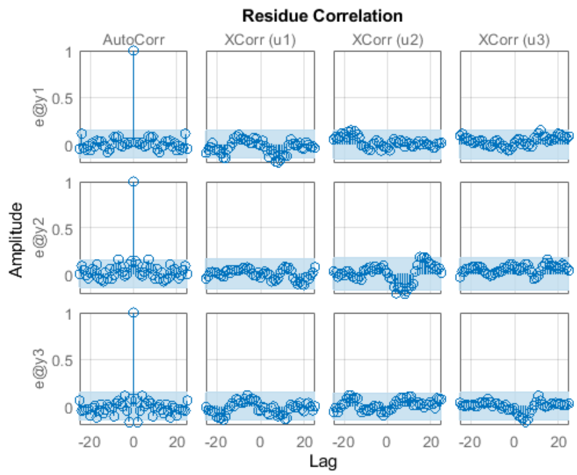

Fig. 2. Correlation test

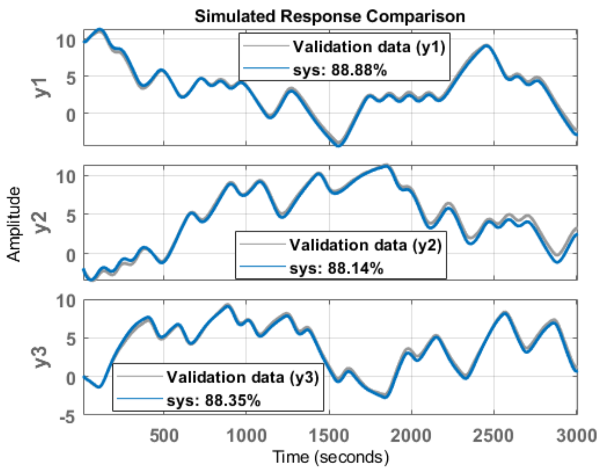

Fig. 3. Comparison test

The comparison test shows around 88% overlap, between the received model and the experimental data. The residual error has characteristics of a white noise, meaning the plant parameter estimates are not biased. Additionally, time and frequency responses of the plant are shown on Fig. 4 and Fig. 5.

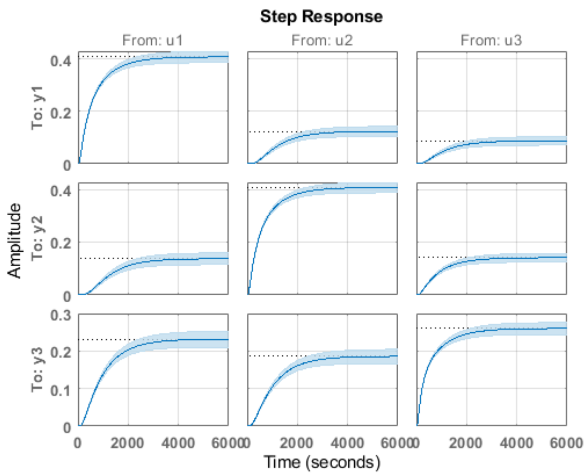

Fig. 4. Step response of the plant

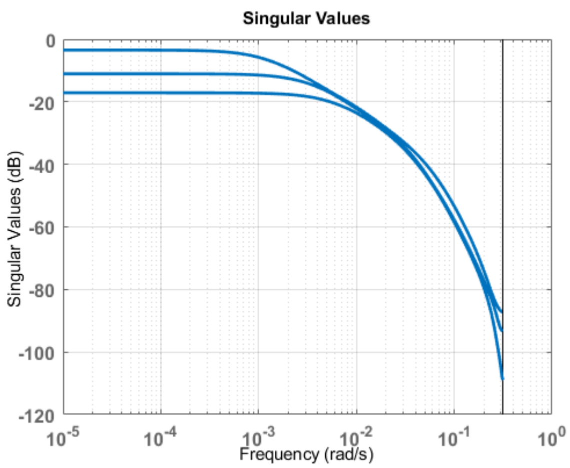

Fig. 5. Frequency response of the singular values of the plant

The plant needs at least 2000s to get to a steady state. There are no oscillations. The temperature rises monotonically. One could note that the dynamic is a simple one. But the small proportional gain could lead to input saturation of the closed loop system.

The current work uses mixed S/KS sensitivity scheme for the design of H∞ controllers [4]. Two possible arrangements of the controller and the integral part are tried, as shown on Fig. 6 and Fig 7.

On Fig. 6 the controller and the integral part are arranged in a series form and on Fig. 7 a parallel form is used. For both cases w represents the external input to the loop, z are the error signals or the signals acting as quality measures, u is the control signal, y is the feedback signal, K is the controller to be designed and P represents the extended plant. The extended plant consists of the plant to be controlled and the transfer matrices of the weighting filters W_p and W_u.

The task at hand is to find a stabilizing controller K such that (1):

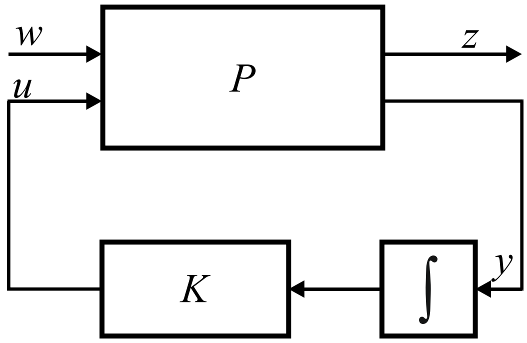

Fig. 6. Hinf synthesis of series integral controller

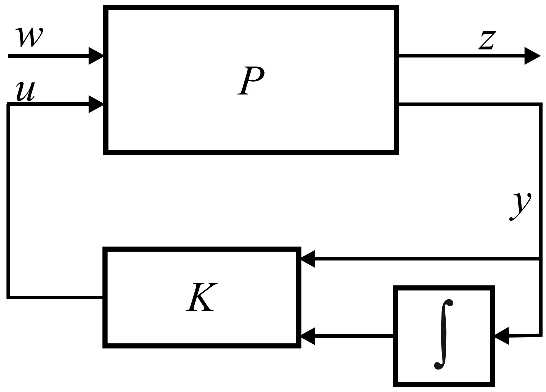

Fig. 7. Hinf synthesis of parallel integral controller

$$\min_{K} \max_{\omega} \bar{\sigma}\left(F_l(j\omega)\right), \forall \omega, \quad (1)$$

where F_l is (2):

$$F_l = \begin{bmatrix} S_o \\ KS_o \end{bmatrix}. \quad (2)$$

S_o denotes the output sensitivity function and KS_o denotes the sensitivity at the plant input. The current work uses the disturbance at the plant output as external signal w, the plant output and input as error signals to be minimized. The input to the controller is the control error, due to existence of disturbance at the plant output. This problem is know as the regulation problem.

The weighting filters matrices are set as diagonal, with elements as shown in (3) and (4):

$$W_p(s) = \frac{\frac{s}{1.4}+0.011}{s+0.011e^{-3}}, \quad (3)$$

$$W_u(s) = 0.01\left(\frac{\frac{1}{0.1}s+1}{\frac{1}{10}s+1}\right). \quad (4)$$

The design procedure leads to values 2.04 and 1.9768 for γ for the series and parallel form respectively, which means that the controllers are neither optimal nor suboptimal. The order of both controllers is 18 without the integral part, which further increases it by 3, making the whole control algorithm of order 21. It is not unusual for the H_∞ design procedure to lead to controllers of high order. The order equals the order of the extended plant plus the order of the weighting filters. Settings even tighter performance limits by higher order transfer functions on the diagonal of weighting filters would increase the controller order even further.

For order reduction the *balred()* function of MATLAB is used. By default the method outlined in [5] is used. The task at hand is to reduce the order of the controller K by keeping the absolute error (5) small.

$$\|K(s) - K_r(s)\|_\infty \,, \qquad (5)$$

where K_r is the reduced order controller. The obtained controllers are in a balanced realization form. It is related to the observability and controllability gramians. The reduced order controllers have their DC gain matched.

Fig. 8 shows the Hankel singular values of the series form controller.

Judging from the absolute error bound and the state contribution the order could be reduced to atleast 14. Furthermore the order could be reduced to 10. Any further reduction leads to steady state errors, shown by a simulation of the closed loop system.

Fig. 9 shows the Hankel singular values of the parallel form controller.

For comparison reasons the same order reduction (14 and 10) is applied. Additionaly the parallel form makes it possible to reduce the order even further than the series form. A reduced controller of order six is obtained. It is even possible to reduce the order to 3, but the output signals tend to have a high overshoot so it is avoided. Looking at Fig. 8 and Fig. 9 one could see an interesting difference. The contribution of states for the series controller are more evenly distributed, excluding the last 4 states. The contributions for the parallel controller are mainly concentrated into the first 3 states, than the next 4 and from now on the contributions dicrease linearly.

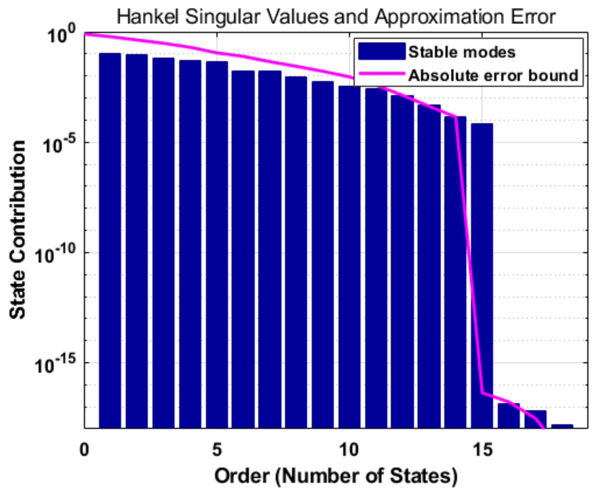

Fig. 8. Hankel singular values for series controller

Lastly a design procedure is carried for a H_∞ controllers in a series and parallel form based on reduced order (6 and 3) models of the plant. Correlation test for these models show that the residuals are not white noise, meaning that their parameters are biased or incorrect in the sense of system identification, nonetheless the results for transient behavoir of the closed loop systems still have a good performance. The resulting controllers are of order 15 and 12 and the values for γ are 2.04, 2.038, 1.977, 19747 for the series and parallel respectively for the 6th and 3rd order reduced models. The changes to γ are marginal.

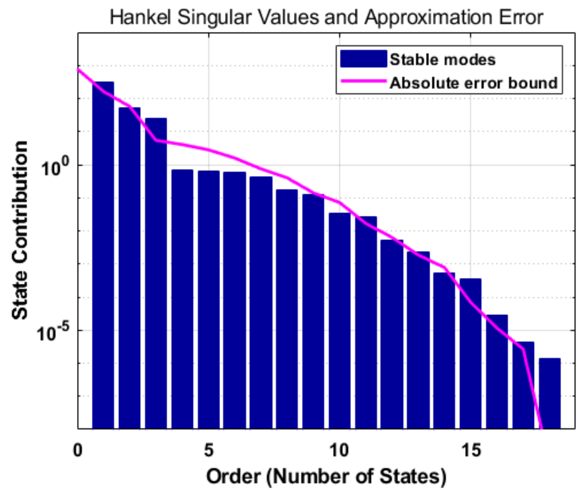

Fig. 9. Hankel singular values for parallel controller

Different sensitivity functions of the closed loop systems for the eleven controllers are calculated. To avoid visual clutter, only the singular values of the closed loop systems with the highest and the lowest order controllers are shown. Fig. 10 to Fig. 12 represent the singular values of the sensitivity functions for the series form.

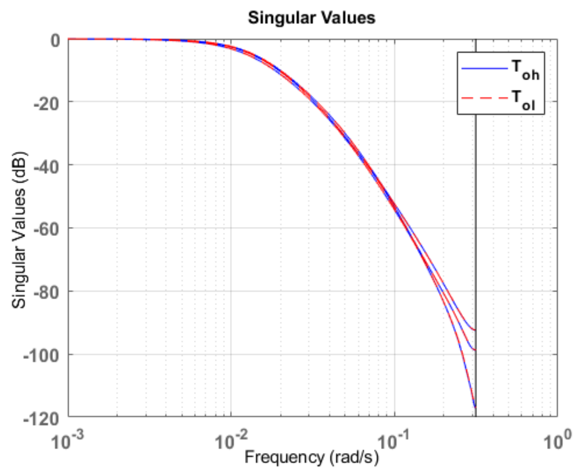

Fig. 10. Output complementary sensitivity singular values - series

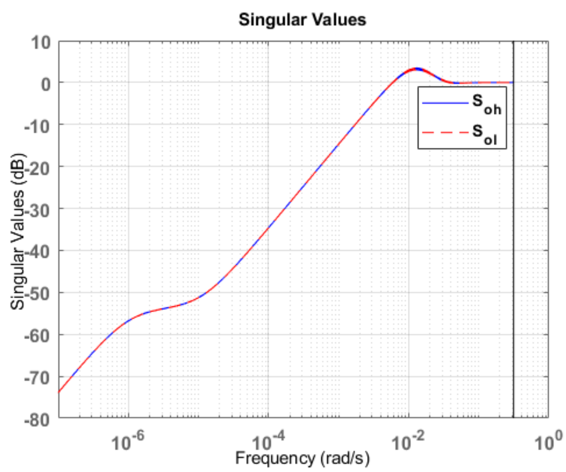

Fig. 11. Output sensitivity function singular values - series

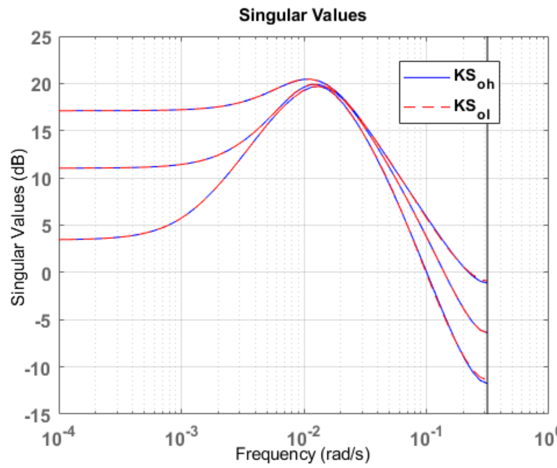

Fig. 12. Sensitivity at plant input singular values – series

Index h means highest order, index l means lowest order. One could observe the lack of difference, with the exception of a negligible deviation in the high frequency region for the KS_o transfer matrix. The feedback loop should be effective at around 0.004 rad/s. The maximum of the sensitivity function is at 1.47, which is under the proposed limit of 2. The control signals could exhibit amplitudes ten times the reference or disturbance change.

Fig 13 to Fig. 15 show the singular values of the sensitivity functions for the parallel form.

One could observer a bigger difference between the highest and the lowest possible order for the parallel form. Nonetheless the deviations are not significant. On top of that the profile for the singular values is almost the same for the series and parallel form, but one should not forget the possibility of higher order reduction on the parallel side.

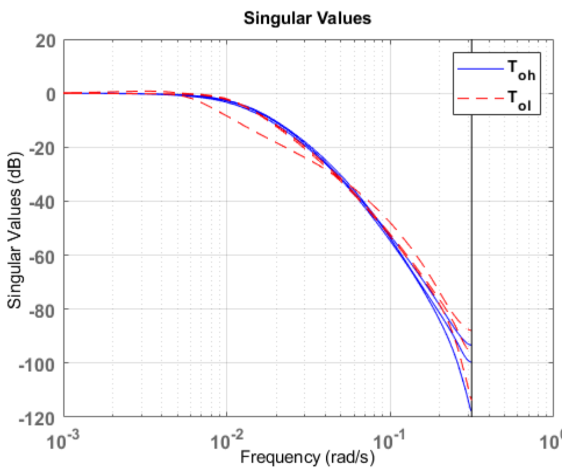

Fig. 13. Output complementary sensitivity singular values – parallel

III. IMPLEMENTATION

The control algorithms are executed by a Programmable Logic Controller (PLC). All software is developed by Siemens TIA Portal V18. The algorithms themselves are implemented in a discrete state space form as show in (6) and (7).

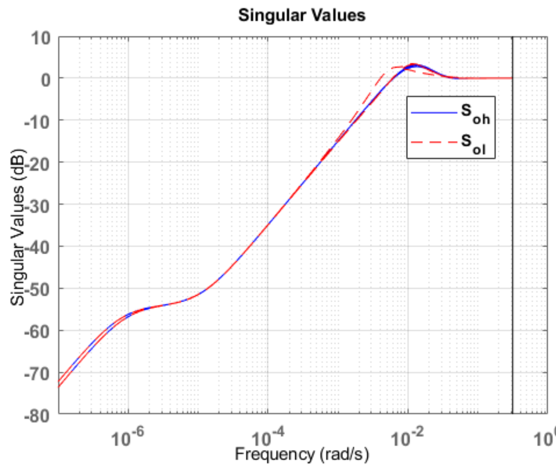

Fig. 14. Output sensitivity function singular values – parallel

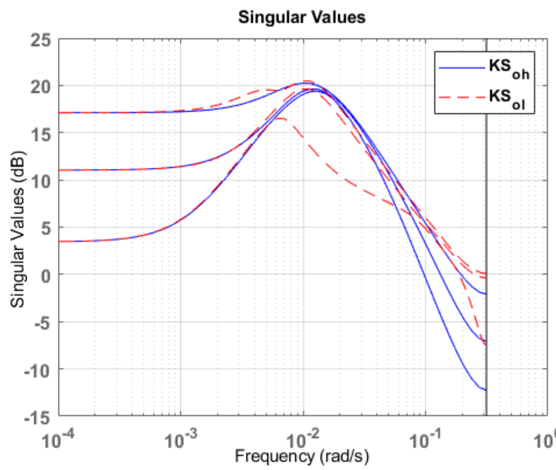

Fig. 15. Sensitivity at plant input singular values – parallel

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c e_i(k) \\ u(k) &= C_c x_c(k) + D_c e_i(k) \end{aligned}, \tag{6}$$

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c[e(k) \quad e_i(k)]^T \\ u(k) &= C_c x_c(k) + D_c[e(k) \quad e_i(k)]^T \end{aligned}, \tag{7}$$

where A_c, B_c, C_c, D_c are the controller matrices, x_c is the state vector, u is the control signal, e and e_i are the control error and the integral control error respectivly. The integral part is implemented by backward Euler difference approximation [6]. The sample time is set to ten seconds. Both the series and the parallel controller possess an anti wind-up mechanism by use of back calculation [7].

Since matrix operations are not readily available, one needs to develop simple functions to supplement the state space controller implementation. By inspection of (6) and (7) only "addition" and "multiplication" operations are needed. In the current work this functions are created in such a way that they operate on a user defined PLC type. This type is called "MATRIX" and consists of a two-dimensional array for the matrix elements, and two unsigned integers for the row and column dimensions.

The matrices of the controllers are calculated with MATLAB *hinfsyn()* function. An OPC connection is established between the SCADA system and the MATLAB client. This connection is used to pass the parameters of the matrices, calculated in MATLAB to the PLC. Because for the current implementation of the OPC by Siemens, multi-dimensional arrays are not supported, one needs to implement additional functions on the client side and the SCADA side. This functions rearrange a matrix in a vector and vice-versa. Additionaly to index the array elements Tag multiplexing is needed on the SCADA side.

Finally a SCADA system is implemented using WinCC Unified V18. It is used for data visualisation, automatic and manual control, as well as for matrix transfer and archival of signals and data.

IV. EXPERIMENTAL RESULTS

All the eleven closed loop systems were tested. For the reference signal, a step is applied at all loops. A different amplitude is used for every reference signal. For the first loop the reference signal is 60 degree, 50 for the second and 5 for the third. The results are shown on Fig.16 to Fig. 21. One could see a small variance of the initial conditions of every test. This is due to inability to control the environement and the ambient temprature in the room. The outputs are plotted together just to show the small difference, despite their different order.

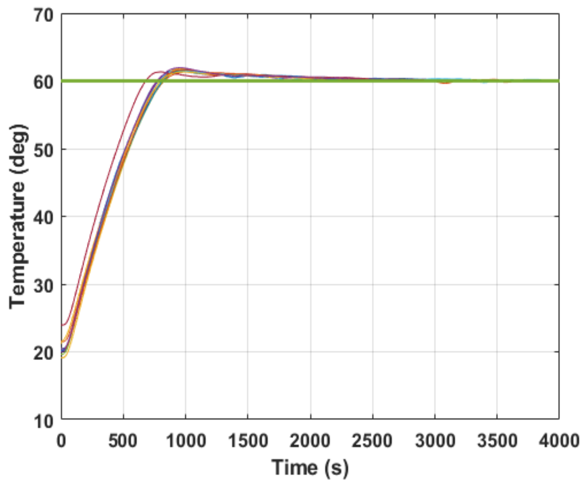

Fig. 16. Transient response of first output

All three outputs of every closed loop system increase monotonically with a slight overshoot. There is no big variation of the input signal. For the first 500 to 600 seconds the control signal is saturated, then it converges to a stationary value. To assess the quality of the closed loop systems, some criteria are calculated and arranged in TABLE I. Notations "m3" and "m6" represent the plant order, three and six respectively. S_e and S_u are defined as (8) and (9).

$$S_e = \sqrt{\frac{1}{T}\sum_0^T e^2(t)}, \tag{8}$$

$$S_u = \sqrt{\frac{1}{T}\sum_0^T u^2(t)}, \tag{9}$$

where T is the time frame of the experiment, e is the control error and u is the control signal. The rise time, settling time and overshoot are calculated using *stepinfo()* MATLAB function. Criterion (8), (9) and overshoot have small differences. Bigger variations one could observe in rise time and settling time, but they could be attributed to the difference of initial conditions.

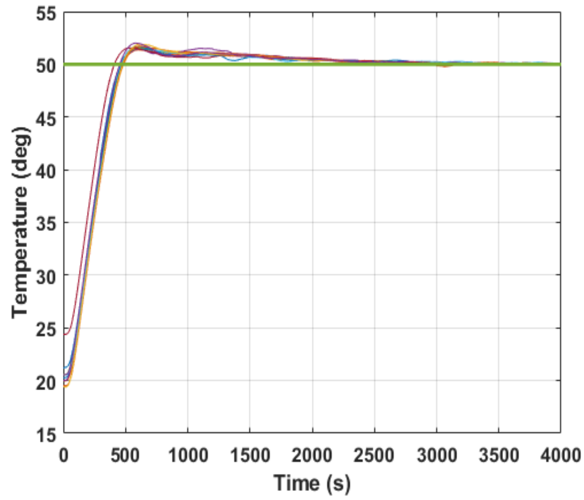

Fig. 17. Transient response of second output

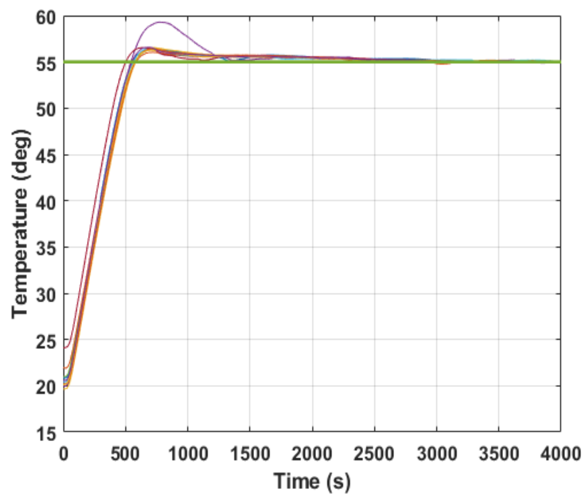

Fig. 18. Transient response of third output

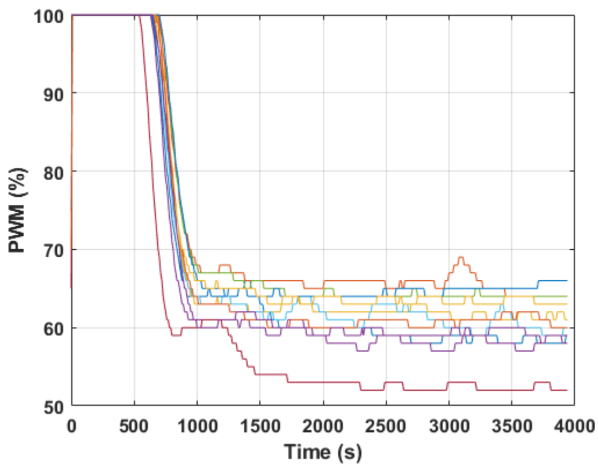

Fig. 19. Control signal of first input

V. CONCLUSION

The current work investigated the possibility of H_∞ controller order reduction for a thermal plant. Two full order controllers were designed, four more were designed by use of reduced model order and 5 more by reduction balanced realizations based on Hankel singular values. All algorithms were implemented in a PLC and test were carried with a real system. The resulting closed loop systems were compared by their singular values of the respective sensitivity functions, transient behavior and by some quality criteria.

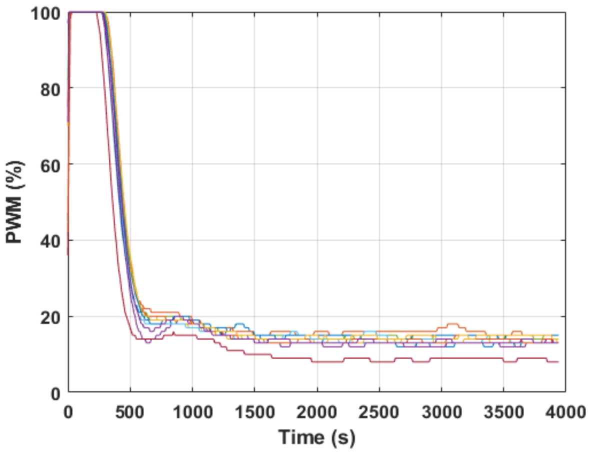

Fig. 20. Control signal of second input

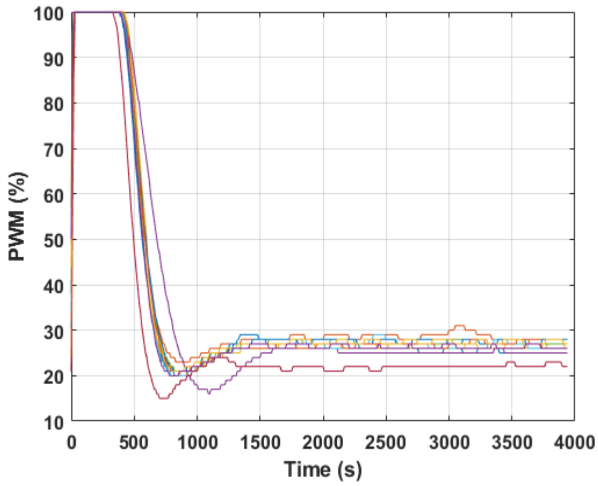

Fig. 21. Control signal of third input

The results showed marginal difference between the highest and the lowest order controller, meaning that order reduction does not lead to significant loss in performance.

TABLE I. PERFORMANCE INDEX

Criterion Type and order	*Rise Time, s*			*Settling time, s*			*Overshoot, %*			*Se [-]*			*Su [-]*		
Serial 18	638.91	368.52	451.54	1144.2	1115.2	827.82	2.47	3.14	2.37	**57.28**	48.97	53.36	73.43	35.05	43.98
Serial 14	613.83	364.01	446.1	1141.4	1125.2	851.84	2.57	3.38	2.63	57.43	48.98	53.36	70.76	34.05	43.09
Serial 10	**529.4**	**313.36**	**389.88**	**840.33**	**756.75**	756.49	2.22	3.1	2.79	57.94	49.26	53.71	**63.94**	**29.94**	**38.77**
Serial m6	628.36	380.26	457.55	1281.4	1381	849.34	2.74	3.41	2.56	57.36	48.94	53.34	70.62	34.84	43.53
Serial m3	634.34	365.03	451.94	1201.6	803.04	809.52	2.68	3.37	2.43	**57.28**	48.96	**53.31**	73.41	34.8	44.08
Parallel 18	604.83	351.26	433.62	1057	988.8	892.45	2.61	2.9	2.63	57.52	49.05	53.46	69.78	33.69	42.72
Parallel 14	634.09	369.15	477.67	1076.2	821.04	**542.65**	2.18	**2.74**	**1.88**	57.3	**48.9**	53.33	74.19	35.33	44.4
Parallel 10	626.49	381.21	463.72	1204.3	1318.8	953.68	2.94	3.5	2.8	57.3	48.94	53.32	71.44	35.08	43.82
Parallel 6	604.99	358.95	442.49	1200.5	1405.9	1171.9	2.42	3.02	7.8	57.48	49.06	53.71	68.7	33.63	44.4
Parallel m6	616.29	376.66	448.14	1072.4	1155	793.53	**2.17**	3.82	2.29	57.41	48.92	53.33	72.45	35.22	43.62
Parallel m3	606.03	356.85	438.53	1173.4	1261	788.68	3.23	4.0	2.85	57.49	49.05	53.43	69.67	34.14	43.02

Even the controllers designed by use of reduced order models, having biased parameters, showed performance in line with the rest of the pack. On top of that the parallel form made it possible to reduce the order further that the series form controller without much performance degradation. However the results shown should not be used to draw a general conclusions, simply because this is just one example with a simple dynamic plant. Nonetheless the current work shows a little insight of the practical aspects and the problem of reduced order controllers.

ACKNOWLEDGMENT

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