Approximation of Furie Experimental Data of Straighness Standard

Rositsa Miteva
Department of Precise Engineering and Measuring Techniques
Technical University of Sofia
Sofia, Bulgaria
rosimiteva@hotmail.com

Dimitar Diakov
Department of Precise Engineering and Measuring Techniques
Technical University of Sofia
Sofia, Bulgaria
diakov@tu-sofia.bg

Hristiana. Nikolova
Department of Precise Engineering and Measuring Techniques
Technical University of Sofia
Sofia, Bulgaria
hnikolova@tu-sofia.bg

Georgi Dukendjiev
Department of Precise Engineering and Measuring Techniques
Technical University of Sofia
Sofia, Bulgaria
duken@tu-sofia.bg

Abstract: **The development and implementation of standards of form and location is a promising field in metrological theory and practice. Integration with information technology enables the creation of a new type of virtual-mechanical standards with new capabilities and characteristics. The choice of mathematical models for interpolation is essential to achieve the accuracy required for the measurement.**

The paper presents an analysis and study of a mathematical model for interpolation and approximation of elements of a virtual-mechanical standard for straightness by means of Fourier analysis.

Keywords—standards of form, virtual standard, coordinate measurements, interpolation, approximation

I. INTRODUCTION

Deviations and tolerances of form and location are considered both for the surfaces of the parts and for their other geometrical features [1]. In various specific cases, these can be parts of surfaces, profiles, edges, axes, planes of symmetry, centers, etc. In this context, the real surface (measured, true surface) can be considered, which by definition is the surface that bounds the body (and separates it) from the environment. Then, the real profile will be a section of the real surface with a previously oriented plane or is a line of intersection of real surfaces [13, 16]. Strictly, the geometric feature determined by measurement differs from the true one due to measurement errors [4, 7, 12, 14].

Deviations of form of the real surfaces (features) of the details are estimated by comparing them with the corresponding geometric (approximating) elements having the form of the nominal ones. Quantitatively, these deviations are defined as the largest distance from the points of the real (extracted) element (real extracted profiles and surfaces) to the geometric (approximated, associated) element along the normal to it.

Of course, the accurate assessment of these deviations, the development of sufficiently effective solutions, oriented to the creation of modern methods and means in this field, are primarily related to the creation of the necessary models defining the main features [8, 10, 17]. In this regard, the following three groups of features can be introduced, united according to their functional nature:

• approximating elements of real surfaces and profiles;

• median elements;

• enveloped elements.

The purpose of creating the mathematical models of the first group of elements is to reduce the study of various numerical characteristics and qualitative properties of real objects to working with models whose theoretical apparatus is fully known and convenient to work [6, 9, 11]. The use of average elements has a number of advantages: equivalence of points on the real surface or profile when calculating the element, less influence of single local deviations, easy calculation with appropriate software, good connection with performance characteristics (e.g. in press assemblies or in air bearings). Median elements are also used preferentially in coordinate measurements, due to the simple processing of the measurement results and the uniqueness of the mathematical solution. Enveloped elements conform to the surface conditions for zero clearance assemblies. In addition, a number of deviations can be defined through them. The correct assessment of the nature of the profile is important for the selection of suitable mathematical models for approximation, with which to achieve the accuracy of the geometric dimensions required for the measurement. Of particular importance is the experience and intuition of the researcher conducting the study.

Despite the above-mentioned advantages of median elements, it is necessary to note that the approximating models can be defined by appropriately chosen optimality criteria regarding the determining quantitative factors, which increases the accuracy in solving the relevant metrological task. Therefore, a combined approach based on the median and approximating models is taken.

II. MATHEMATICAL MODEL FOR APPROXIMATING THE REAL PROFILE

The used in model for approximating the real profile is based on the theoretical formulations of the Fourier series [5,

978-1-6654-8569-2/22/$31.00 ©2022 IEEE

6, 8]. This model has a number of advantages in this particular case over other approximating methods, as it provides the best overlap both for functions with a higher frequency and for those with highly pronounced extrema.

This method is primarily used in for the analysis and synthesis of linear dynamic systems, as well as in the study of various types of signals, including measurement ones. The idea in this case is to use "trigonometric sums", i.e. sums of harmonically related sines and cosines or periodic complex exponents to describe the real profiles of the surfaces of interest or the real curves defining the real axes of mechanical parts and assemblies.

The Fourier method is based on the properties of the periodic function, which for some positive value of the period T can be represented by the following equation:

$$y(x) = y(x + T) \quad (1)$$

for each *x*.

In fact, *y(x)* defines the function describing the real profile along the *x* coordinate. The primary period T_0 of *y(x)* is the smallest positive non-zero value of *T* for which equation (1) is satisfied, and the quantity $2.\pi/T_0$ is the primary frequency.

Fourier series expansion is a special case of series expansion in orthogonal functions.

If a decomposition interval $-\pi < t < \pi$ is set, then the Fourier series describing the actual function *f(x),* for which there is an integral $\int_{-\pi}^{\pi}|f(\tau)|d\tau$ is an infinite trigonometric series [1]:

$$\frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kt + b_k \sin kt) \equiv \sum_{k=-\infty}^{\infty} c_k e^{ikt} \quad (2)$$

The coefficients are determined by the Euler-Fourier formulas:

$$\left.\begin{aligned} a_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\tau)\cos k\tau\, d\tau, \\ b_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(\tau)\sin k\tau\, d\tau, \\ c_k = \bar{c}_{-k} &= \frac{1}{2}(a_k - i\, b_k) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\tau)\, e^{-ik\tau} d\tau \end{aligned}\right\} \quad (3)$$

$(k = 0, 1, 2, \dots)$

The methods for Fourier analysis of continuous functions discussed above are largely applicable in other cases as well. These methods are the basis of the analysis and synthesis of the discretely represented functions, which are characteristic above all for the results obtained in static measurements. Formulas based on the Fourier method for processing discrete data sets can be successfully used to numerically approximate any real profile, axis or surface.

It should be noted that the actual profiles differ from the theoretical description discussed above. The difference lies in the following two circumstances:

- in most cases, the real profile or surface can be defined by means of aperiodic functions, and not by means of the periodic ones discussed above;

- due to the discrete nature of the measurement process, it is incorrect to use the theoretical Fourier apparatus for continuous series.

In fact, the results obtained in the measurement process determine the interval in which the function $y(x_k)$ is calculated. In this case, the formulas defining the mathematical apparatus for processing the results according to the Fourier method will have the following form [1].

We have *m* values of the function $y(x_k)= y_k$ at $x_k = kT/m$ (k = 0, 1, 2, …, *m*-1); its necessary to approximate *y(x)* for the interval (0, *T*) with a trigonometric polynomial

$$Y(x) = \frac{1}{2}A_0 + \sum_{j=1}^{n} A_j cos\, j\frac{2\pi x}{T} + B_j sin\, j\frac{2\pi x}{T} \quad (4)$$

$\left(n < \frac{m}{2}\right)$

so that to minimize the sum of the squares of the deviations:

$$\Delta = \sum_{k=0}^{m-1}[Y(x_k) - y_k]^2 \quad (5)$$

The coefficients could be found by using the following formulas:

$$A_j = \frac{2}{m}\sum_{k=0}^{m-1} y_k \cos j\frac{2\pi k}{m} \quad (6)$$

$$B_j = \frac{2}{m}\sum_{k=0}^{m-1} y_k \sin j\frac{2\pi k}{m} \quad (7)$$

where $0 \leq j < \frac{m}{2}$

When we have a special case $n = \frac{m}{2}$, formulas 4, 6 and 7 together with

$$A_n = A_{\frac{m}{2}} = \frac{1}{m}\sum_{k=0}^{m-1}(-1)^k y_k \quad (8)$$

give us the trigonometric interpolation $Y(x_k) = y(x_k)$ for any B_n.

III. Experimental Setup and Research Methodology

For the purpose of the research, a model standard line with a length of 560 mm was produced by the company "NIK - 47" OOD - Plovdiv. The examination of the profile of the standard line is carried out using a straightness measurement module in comparison with a standard for straightness [2, 3, 5].

The experimental measuring setup for researching the standard line (Fig. 1 a, b) consists of the granite vibration-insulated support 1, on which the guide for straight movement 2 is mounted, located on aerostatic bearings, supplied with a constant pressure *P* and realizing the progressive movement of the movable module. The console 4 with fixed measuring transducers 5 is on the carriage of the moveable module. The movement of the module is checking with the help of the incremental scale 8. The movement of the moveable module is carried out with the help of the drive module 7. On the granite support, the reference standard line 10 is established together with the sample under test 6 (line standard, which will be calibrated). By means of adjustable supports, the standards are oriented nominally parallel to the direction of movement of the movable module. During the movement of the module 3, the measuring tips of the transducers 5 traverse the controlled profile of stadards 10 and 6 within the limits of the normalized section. The displacement of the points of the profile relative to the movement trajectory of the transducers and the readings of the incremental scale 8 are read with the electronic unit 9 [15].

The two profiles are measured in parallel – of the reference and of the sample standard. Measurements are made using linear incremental gages Heidenhein MT 1281, ID359355-02SN26981030 P, Inspection Certificate DIN 55350-18-4.4.2/02.09.2009 and straightness standard DEA630. Processing is done with Putty Configurator software.

Subsequence of five cycles of measurements are performed with a step of 5 mm at a length of the normalized section of 540 mm for each of the profiles [4]. The measurements were carried out at an ambient temperature of 20.7±0.1°C and a pressure in the airbearings – 0.32 MPa.

The measurement results are processed by introducing corrections to exclude the own non-linearity of the reference line [3]. The data are then processed with specialized software [6] to determine the deviation from straightness with respect to the datum line - median line.

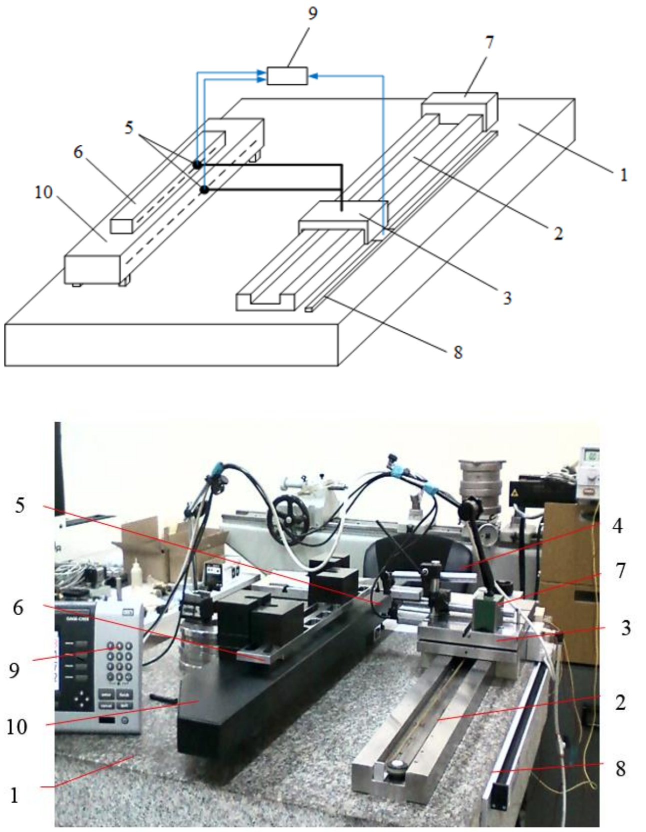

Fig. 1 Measurement of straightness of reference line with virtual-mechanical standard

From the obtained measurement information, a profilograms of the measured profiles is constructed and the deviation from straightness relative to the corresponding approximating straight line is estimated.

IV. Experimental Study

Experimental studies were carried out according to the methodology presented in the previous point. The results of the experimental studies were obtained using an associated base line - median line.

After processing the primary measurement information with specialized software, the results presented in fig. 2, where at an median datum line the deviation from straightness of the sample line profile is EFL = 8.2 μm.

The post-processed data is further used to interpolate the 19-point profile over a 30 mm interval for the normalized section of 540 mm length.

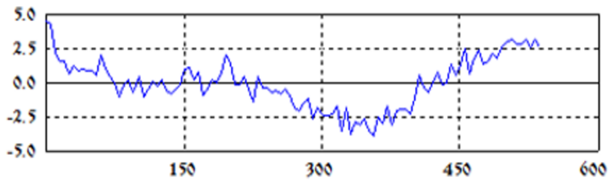

Fig. 2. Deviation from straightness of profile relative to median datum line

For a six-harmonic Fourier series approximation, Δ_{max} = 2.7 μm (33% of EFL). In the graph on fig. 3 function *y* is the approximating function with Fourier series at 6 harmonics, and the set profile through 30 mm is denoted by y_1.

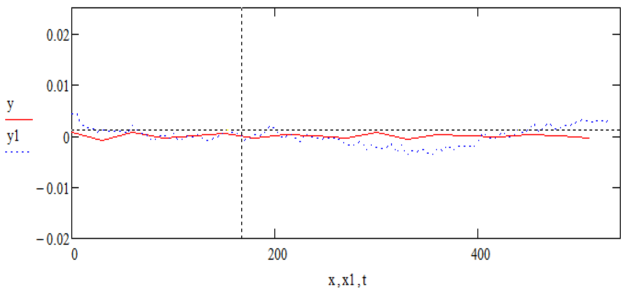

Fig. 3. Fourier series approximation of the profile at 19 points and 6 harmonics

When setting a step of 10 mm (55 points) and 12 harmonics Δ_{max} = 1.6 μm (19 % of EFL). As can be seen from the experimental results, the accuracy of approximation with the Fourier series increases with increasing number of harmonics, and it can be chosen in the interval $0 \le j < \frac{m}{2}$, where *m* is the number of interpolation nodes. At 26 harmonics Δ_{max} = 0.16 μm (2% of EFL). The data are summarized in Table. 1.

TABLE 1 Fourier Series Approximation at *EFL*=8.2 μm

	Profile, number of points	Harmonic	Δ_{max}, μm	Δ_{max}, %
EFL = 8.2 μm	19	6	2.7	33
	55	12	1.6	19
		26	0.16	2

V. Conclusion

In conclusion, it can be said that with a proper assessment of the nature of the profile, appropriate mathematical approximation models should be used to achieve the accuracy required for the measurement. The Fourier series approximation is weakly affected by the nature of the profile, so it is recommended to use it in the absence of information. It is necessary to analyze and select an appropriate number of interpolation nodes and number of harmonics to achieve the intended accuracy.

Acknowledgment

This work has been accomplished with financial support by the Grant № BG05M2OP001-1.002-0011 "MIRACle (Mechatronics, Innovation, Robotics, Automation, Clean technologies)", financed by the Science and Education for Smart Growth Operational Program (2014-2020) co-financed by the European Union through the European structural and Investment funds and supported by the European Regional Development Fund within the Operational Programme “Science and Education for Smart Growth 2014 - 2020” under the Project CoE “National center of mechatronics and clean technologies“ BG05M2OP001-1.001-0008-C01".

References

[1] Diakov D. Otnosno otsenyavaneto na otkloneniyata na formata i orientatsiyata. Sb. dokladi ot 10-ti Natsionalen Nauchen Simpozium s mezhd.uchastie “MMO’2000”, Sozopol, 2000 g., (s. 87-92), ISBN 954-438-229-3.

[2] Diakov D., I. Kalimanova, V. Bogev, G. Kasabova. Sistema za izmervane na otklonenieto ot pravolineynost. 12-ti Natsionalen Nauchen Sbornik dokladi ot 12-ti Natsionalen Nauchen Simpozium s mezhd. uchastie“MMO’2002”, Sozopol, 2002 g., s. 147-152, ISBN 954-438-229-5.

[3] Diakov, D., I. Kalimanova, A. Georgiev, Stend za izsledvane na tochnostnite parametri na koordinatni mikropozitsionirashti sistemi. Sbornik dokladi na Konferentsia s mezhdunarodno uchastie mashinoznanie i mashinni elementi, 2008 g., ISBN 978-954-580-260-7, s. 254-260.

[4] Dichev D., F. Kogia, Hr. Koev and D. Diakov. Method of analysis and correction of the error from nonlinearity of the measurement instruments, Journal of Engineering Science and Technology Review. Volume 9, Number 6, 2016 ISSN: 1791-2377, p.116-121

[5] Dichev, D., I. Zhelezarov, N. Madzharov. Dynamic error and methods for its elimination in systems for measurement of moving objects parameters. Transactions of Famena, vol. 45, issue 4, 2021, pp 55-70. DOI: 10.21278/TOF.454029721

[6] Korn G, T. Korn, Spravochnik po matematike dlya nauchnыh rabotnikov i inzhenerov, izd. Nauka, M. 1973.

[7] Kupriyanov O., R. Trishch, D. Dichev, T. Bondarenko. Mathematic Model of the General Approach to Tolerance Control in Quality Assessment. In 3rd Grabchenko’s International Conference on Advanced Manufacturing Processes (InterPartner-2021), September 7-10, Lecture Notes in Mechanical Engineering, 2021, pp. 415-423. DOI 10.1007/978-3-030-91327-4_41

[8] Lysenko V., Mathematical model of complicated form surfaces coordinate measurement. 11th National Scientific Symposium "MMA", Sozopol, September, 2001.

[9] Miteva R. Virtualno-mekhanichni etaloni na formata i razpolozhenieto, Softtreĭd, 2019 g., ISBN 978-954-334-213-6.

[10] Miteva R., Izsledvane na modeli za aproksimatsiya na materialen etalon s izpolzvaneto na virtualno-mehanichen etalon za pravolineynost, Sbornik dokladi ot Mezhdunarodna nauchna konferentsiya „Uniteh 2019“, Gabrovo, 15-16.11.2019, tom 2, s. 330-334, ISSN 1313-230X

[11] Piskunov N.. Differentsialʹnoye i integralʹnoye ischisleniya. Mifril. Sankt-Petrburg, 1996. 416 s. Sveshnikov A.. Prikladnyye metody teorii sluchaynykh funktsii. Sudpromgiz. Sankt-Petrburg, 1981.

[12] Radev Hr. i dr., Metrologiya i izmervatelna tehnika, tom II, S., Softtreyd, 2010

[13] Sladek J., Coordinate metrology, Accuracy of Systems and Measurements, 2016, Spinger-Verlag GmbH, Berlin ISBN978-3-662-48463-0.

[14] Tonev D., B. Sotirov, Extending the production tolerance of a gauge and determining its trueness, 2020, Sibiu, Romania, Land Forces Academy “Nicolae Balcescu”, International Conference Knowledge-Based Organization, pp. 173-178, ISSN: 1843-682X.

[15] Vassilev V., System for measurement geometric parameters of large-scaled, 2019, Softtrade, ISBN 978-954-334-215-0.

[16] Whitehouse D. J., Surfaces and their Measurement, London, Hermes Penton Ltd., 2002

[17] Zhelezarov, I. S. Povtoryaemost i vazproivodimost na izmervatelni sistemi. UNITEH 2010. Gabrovo. 2010 g., s. II445 – II448. ISSN 1313-230X.