



NONLINEAR ADAPTIVE OBSERVER DESIGN IN COMBINED ERROR NONLINEAR ADAPTIVE CONTROL

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Abstract

The paper deals with the design of nonlinear adaptive observers and parameter estimators in adaptive observer canonical form. The closed loop system nonlinear adaptive control law is designed by the combined error adaptive control approach. The nonlinear adaptive observer is designed in the adaptive observer canonical form. The parameter estimator is Lyapunov stable. The proposed approach is applied to an inverted pendulum driven by salient-pole permanent magnet synchronous motor. The closed loop adaptive system time responses are simulated for illustration.

Keywords: nonlinear adaptive observers, nonlinear adaptive systems, parameter estimators.

Introduction

The nonlinear adaptive state observers with unknown parameter estimator are generally used for adaptive closed loop system design. The input-output signals and the system structure is the only available information for state and parameters estimation. The nonlinear adaptive state observer design requires an appropriate nonlinear canonical form [1, 2, 6, 8] being usually the adaptive observer canonical form (AOCF) [6]. The stringent necessary conditions for transformation into AOCF limit the class of the transformable nonlinear systems. The known approaches in this field require persistent excitation of the system [1, 2, 6, 7] in order to achieve exact estimation, but this conflicts with the closed loop system control goals. In that sense, the adaptive closed loop system has to compensate the parameter estimation errors to achieve the desired performance or alternatively, an exact parameters estimator without persistency of excitation has to be designed. Various methods considering closed loop adaptive control with nonlinear adaptive observers [3, 4, 5, 13], based on classical methods for nonlinear adaptive control [9, 10] deal with the problem.

The paper presents an approach for closed loop adaptive system design with nonlinear adaptive state and parameter observers applied for nonlinear adaptive control of a salient-pole permanent magnet synchronous motors (PMSM) [11, 12] via a transformation into AOCF. The nonlinear adaptive tracking control design is accomplished by the adaptive combined error approach [14].

Problem Statement

The original objective nonlinear system is firstly

presented in nonlinear observer canonical form

$$\dot{\boldsymbol{\eta}} = \mathbf{A}\boldsymbol{\eta} + \mathbf{F}(\mathbf{y}, \mathbf{u})\boldsymbol{\theta}, \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (1a)$$

$$\mathbf{y} = \mathbf{C}\boldsymbol{\eta} \quad (1b)$$

where the matrix pair (\mathbf{A}, \mathbf{C}) is in single-output Brunovski canonical form, $\boldsymbol{\theta} \in \mathbb{R}^p$ is the unknown parameter vector and $\mathbf{F}(\mathbf{y}, \mathbf{u}) \in \mathbb{R}^{n \times p}$. The filtered transformation for second order systems has the following form

$$\mathbf{z}(\boldsymbol{\eta}) = \boldsymbol{\eta} - \begin{bmatrix} \mathbf{0} \\ \mathbf{m} \end{bmatrix} \boldsymbol{\theta}, \quad (2a)$$

$$\dot{\mathbf{m}} = \lambda \mathbf{m} + \boldsymbol{\beta} \mathbf{F}(\mathbf{y}, \mathbf{u}), \quad \mathbf{m}(0) = \mathbf{m}_0 \quad (2b)$$

where $\mathbf{m} \in \mathbb{R}^{1 \times p}$, $\lambda = -b_2/b_1$, $\boldsymbol{\beta} = [-b_2/b_1, 1]$. It transforms system (1) into the adaptive observer canonical form

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}\boldsymbol{\omega}\boldsymbol{\theta}, \quad \mathbf{z}(0) = \mathbf{z}_0, \quad (3a)$$

$$\mathbf{y} = \mathbf{C}\mathbf{z}. \quad (3b)$$

The vector $\boldsymbol{\omega} \in \mathbb{R}^{1 \times p}$ reads $\boldsymbol{\omega} = \mathbf{m} + \mathbf{C}\mathbf{F}(\mathbf{y}, \mathbf{u})$ and $\mathbf{b} \in \mathbb{R}^{n \times 1}$. The adaptive nonlinear observer for the system (3) has the form

$$\dot{\hat{\mathbf{z}}} = \mathbf{A}\hat{\mathbf{z}} + \mathbf{b}\boldsymbol{\omega}\hat{\boldsymbol{\theta}} + \mathbf{N}(\mathbf{y} - \hat{\mathbf{y}}), \quad \hat{\mathbf{z}}(0) = \hat{\mathbf{z}}_0, \quad (4a)$$

$$\hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{z}}, \quad (4b)$$

where the observer gain matrix \mathbf{N} makes the matrix $\mathbf{A}_o = (\mathbf{A} - \mathbf{N}\mathbf{C})$ stable. The observer error $\tilde{\mathbf{z}} = \mathbf{z} - \hat{\mathbf{z}}$ has the dynamics

$$\dot{\tilde{\mathbf{z}}} = \mathbf{A}_o \tilde{\mathbf{z}} + \mathbf{b}\boldsymbol{\omega}\tilde{\boldsymbol{\theta}}, \quad \tilde{\mathbf{z}}(0) = \tilde{\mathbf{z}}_0. \quad (5)$$

The adaptive parameter estimation dynamics $\dot{\hat{\boldsymbol{\theta}}}$ is designed via the direct Lyapunov method. A Lyapunov function candidate

$$V_o = \tilde{\mathbf{z}}^T \mathbf{P} \tilde{\mathbf{z}} + \tilde{\boldsymbol{\theta}}^T \Gamma^{-1} \tilde{\boldsymbol{\theta}}$$



is defined, whose total time derivative assuming that the parameters are constant ($\dot{\theta} = 0$) will be

$$\dot{V}_o = \tilde{z}^T (A_o^T P + P A_o) \tilde{z} + 2\tilde{\theta}^T (\omega b^T P \tilde{z} - \Gamma^{-1} \dot{\hat{\theta}}). \quad (6)$$

The choice

$$b = C P^{-1}, \quad (7)$$

where P is the solution of the Lyapunov equation

$$A_o^T P + P A_o = -Q \quad (8)$$

and parameter estimator dynamics

$$\dot{\hat{\theta}} = \Gamma \omega^T (y - \hat{y}) = \Gamma \tau_o, \quad \hat{\theta}(0) = \hat{\theta}_o, \quad (9)$$

reduces the derivative (6) into

$$\dot{V}_o = -\tilde{z}^T Q \tilde{z} - \tilde{\theta}^T (\dot{\hat{\theta}} - \Gamma \tau_o).$$

Hence, the observer error \tilde{z} will be asymptotically stable while the parameter errors $\tilde{\theta}$ are only Lyapunov stable. The original system has to be presented in second order nonlinear companion form

$$\ddot{y} - f(x) \bar{\theta} = \theta_p u, \quad (10)$$

where x is the state vector and $\theta = [\bar{\theta}^T, \theta_p]^T$ with $\bar{\theta}^T = [\theta_1, \theta_2, \dots, \theta_{p-1}]$. The task is to stabilize the combined error

$$e = \dot{y} - \dot{y}_r + \lambda(y - y_r) = \dot{y} - \dot{y}_d$$

with dynamics

$$\dot{e} = f(x) \bar{\theta} + \theta_p u - \dot{y}_d, \quad (11)$$

where y_r is the reference trajectory and λ is a dynamics tuning positive constant. For that purpose a Lyapunov function candidate is defined as

$$V_c = \frac{1}{2} e^2 + \frac{|\theta_p|}{2\gamma_\delta} \tilde{\delta}^2 + \frac{1}{2} \tilde{\theta}^T \Gamma_\theta^{-1} \tilde{\theta} \quad (12)$$

where $\delta = 1/\theta_p$ with estimate $\hat{\delta}$ and error $\tilde{\delta} = \delta - \hat{\delta}$, whose total time derivative considering (11) reads

$$\dot{V}_c = e(f(x) \bar{\theta} + \theta_p u - \dot{y}_d) - \frac{|\theta_p|}{\gamma_\delta} \tilde{\delta} \dot{\tilde{\delta}} - \tilde{\theta}^T \Gamma_\theta^{-1} \dot{\tilde{\theta}}. \quad (13)$$

With the control

$$u = \hat{\delta} \bar{u}, \quad (14)$$

$$\bar{u} = \dot{y}_d - f(x) \hat{\theta} - k e, \quad (15)$$

where $k > 0$, and parameter estimator dynamics

$$\dot{\hat{\delta}} = -\gamma_\delta \text{sign}(\theta_p) \bar{u} e \quad (16)$$

$$\dot{\hat{\theta}} = \Gamma_\theta f^T(x) e = \Gamma_\theta \tau_c \quad (17)$$

the derivative (13) becomes

$$\dot{V}_c = -k e^2 - \tilde{\theta}^T (\dot{\hat{\theta}} - \Gamma_\theta \tau_c). \quad (18)$$

The unified Lyapunov function reducing the parameter estimates is defined as

$$V = V_o + V_c, \quad (19)$$

whose time derivative reads

$$\dot{V} = -\tilde{z}^T Q \tilde{z} - \tilde{\theta}^T (\dot{\hat{\theta}} - \Gamma \tau_o) - k e^2 - \tilde{\theta}^T (\dot{\hat{\theta}} - \Gamma_\theta \tau_c).$$

If the gain matrix Γ is chosen to be

$$\Gamma = \begin{bmatrix} \Gamma_\theta & \mathbf{0} \\ \mathbf{0} & \gamma_p \end{bmatrix},$$

then the above derivative transforms to

$$\dot{V} = -k e^2 - \tilde{z}^T Q \tilde{z} - \tilde{\theta}^T (\dot{\hat{\theta}} - \Gamma (\tau_o + [\tau_c^T, 0]^T)). \quad (20)$$

The unified parameter estimator dynamics is

$$\dot{\hat{\theta}} = \Gamma (\tau_o + [\tau_c^T, 0]^T), \quad (21)$$

which reduces (20) to

$$\dot{V} = -k e^2 - \tilde{z}^T Q \tilde{z}.$$

The asymptotic stability of the combined error and the state observer error follows from the LaSalle-Yoshizawa convergence theorem. The estimate \hat{x} is used in all the above dependencies instead of the unknown vector x . In this way the elimination of the tracking error is achieved when the observer error $\tilde{x} = x - \hat{x}$ converges to zero. The transformation (2) is parameter dependent, hence the exact estimation of the original state x is possible only if $\tilde{\theta} = 0$, but for second order nonlinear systems in nonlinear companion form the following dependency holds

$$\dot{y} = z_2 + \omega \theta = x_2.$$

The state observer is asymptotically stable and hence \hat{y} converges to y asymptotically, so $\dot{\hat{y}} \rightarrow \dot{y}$ or

$$\dot{z}_2 + \omega \hat{\theta} = \dot{\hat{x}}_2 \rightarrow x_2.$$

Thus, it is guaranteed that $\hat{x} \rightarrow x$, which completes the nonlinear adaptive control design.

Application of the Approach

The approach is applied to a salient pole PMSM driven inverted pendulum system. The PMSM in rotating dqo coordinates, controlled in current mode, has a state space system model

$$\dot{x}_1 = x_2 \quad (22)$$

$$\dot{x}_2 = -\theta_1 x_2 - \theta_2 \sin y + \theta_3 u_1 u_2 + \theta_4 u_1$$

where x_1 is the mechanical angle, x_2 is the rotor speed, u_1, u_2 are the control currents i_d, i_q . The model parameters are

$$\theta_1 = \frac{b}{J}, \quad \theta_2 = \frac{mgl}{J}, \quad \theta_3 = \sqrt{\frac{3}{2}} \frac{n_p \Psi_{pm} (L_d - L_q)}{J},$$

$$\theta_4 = \sqrt{\frac{3}{2}} \frac{n_p \Psi_{pm}}{J},$$

where m and l are the pendulum mass and length, J – total moment of inertia, g – gravity acceleration, n_p – number of pole pairs, Ψ_{pm} –

magnets flux linkages magnitude, b – viscous friction coefficient and L_d , L_q are the motor inductances. The model (22) is firstly presented in the nonlinear observer canonical form (1)

$$\dot{\eta}_1 = \eta_2 - \theta_1 y$$

$$\dot{\eta}_2 = -\theta_2 \sin y + \theta_3 u_1 u_2 + \theta_4 u_1$$

via the transformation

$$\mathbf{x}(\boldsymbol{\eta}) = \begin{bmatrix} \eta_1 \\ \eta_2 - \theta_1 y \end{bmatrix}.$$

Hence, the filters (2) will be

$$\dot{m}_1 = \lambda(m_1 - y)$$

$$\dot{m}_2 = \lambda m_2 - \sin(y)$$

$$\dot{m}_3 = \lambda m_3 + u_1 u_2$$

$$\dot{m}_4 = \lambda m_4 + u_1$$

Then, the vector $\boldsymbol{\omega}$ takes the form

$$\boldsymbol{\omega} = [m_1 - y, m_2, m_3, m_4]^T.$$

The system in nonlinear adaptive observer canonical form (3) reads

$$\dot{z}_1 = z_2 + (m_1 - y)\theta_1 + m_2\theta_2 + m_3\theta_3 + m_4\theta_4$$

$$\dot{z}_2 = b_2((m_1 - y)\theta_1 + m_2\theta_2 + m_3\theta_3 + m_4\theta_4)/b_1$$

$$y = z_1$$

with nonlinear adaptive state observer (4)

$$\dot{\hat{z}}_1 = \hat{z}_2 + (m_1 - y)\hat{\theta}_1 + m_2\hat{\theta}_2 + m_3\hat{\theta}_3 + m_4\hat{\theta}_4 + n_1\tilde{y}$$

$$\dot{\hat{z}}_2 = b_2((m_1 - y)\hat{\theta}_1 + m_2\hat{\theta}_2 + m_3\hat{\theta}_3 + m_4\hat{\theta}_4)/b_1 + n_2\tilde{y}$$

$$\hat{y} = \hat{z}_1,$$

observer error (5)

$$\dot{\tilde{z}}_1 = -n_1\tilde{z}_1 + \tilde{z}_2 + (m_1 - y)\tilde{\theta}_1 + m_2\tilde{\theta}_2 + m_3\tilde{\theta}_3 + m_4\tilde{\theta}_4$$

$$\dot{\tilde{z}}_2 = b_2((m_1 - y)\tilde{\theta}_1 + m_2\tilde{\theta}_2 + m_3\tilde{\theta}_3 + m_4\tilde{\theta}_4)/b_1 - n_2\tilde{z}_1$$

and adaptive parameter estimator dynamics (9) with observer tuning function

$$\boldsymbol{\tau}_\omega = [m_1 - y, m_2, m_3, m_4]^T (y - \hat{y}).$$

The unified parameter estimator dynamics (21) with adaptive control tuning function

$$\boldsymbol{\tau}_c = [-\hat{x}_2, -\sin y, u_1 u_2]^T e$$

takes the form

$$\dot{\hat{\theta}}_1 = \gamma_1((m_1 - y)\tilde{y} - e\hat{x}_2),$$

$$\dot{\hat{\theta}}_2 = \gamma_2(m_2\tilde{y} - e\sin y),$$

$$\dot{\hat{\theta}}_3 = \gamma_3(m_3\tilde{y} + e u_1 u_2),$$

$$\dot{\hat{\theta}}_4 = \gamma_4 m_4 \tilde{y},$$

$$\dot{\hat{\delta}} = -\gamma_\delta \text{sign}(\theta_2) \bar{u}_1 e.$$

The complete adaptive control law for $u_2 = 0$ is

$$\bar{u}_1 = \dot{y}_d + \hat{x}_2 \hat{\theta}_1 + \sin y \hat{\theta}_2 - k e$$

$$u_1 = \hat{\delta}(\dot{y}_d + \hat{x}_2 \hat{\theta}_1 + \sin y \hat{\theta}_2 - k e),$$

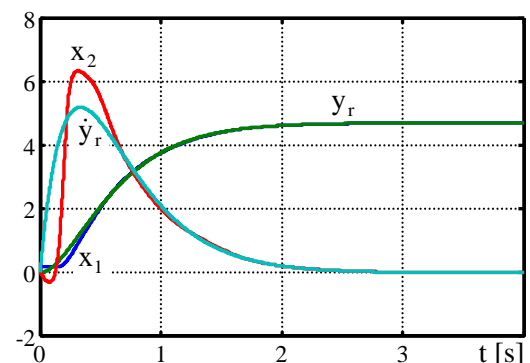
where

$$\ddot{y}_d = \ddot{y}_r - \lambda(\hat{x}_2 - \dot{y}_r).$$

The above dependencies are used for dynamic simulation of the closed loop nonlinear adaptive system with nonlinear adaptive state and parameter observers.

Simulation and System Time Responses

The dynamic simulation is carried out for Lenze MDSKS071-03 PM synchronous servomotor with parameters: nominal power – $P_N = 2$ kW, nominal torque – $T_N = 5.7$ Nm, nominal AC voltage – $U_N = 330$ V, nominal current – $I_N = 4.2$ A, torque constant – $k_T = 1.37$ Nm/A, stator resistance – $R_s = 3.55$ Ω , moment of inertia – $J = 6 \times 10^{-4}$ kg.m², inductances $L_d = 19.15$ mH, $L_q = 4.2$ mH, and number of pole pairs $n_p = 3$. The pendulum attached to the shaft is determined by the mass $m = 0.5$ kg, link length $l = 0.5$ m, and the gravity constant $g = 9.81$ ms⁻². The closed loop nonlinear adaptive system with nonlinear state observer and parameter estimator responses are for initial conditions $\mathbf{x}_0 = [0.2, 0]^T$, $\mathbf{z}_0 = [0.2, 0.0002]^T$, $\hat{\mathbf{z}}_0 = [0, 0]^T$, $\mathbf{m}_0 = [0, 0, 0, 0]^T$, $\hat{\boldsymbol{\theta}}_0 = [0, 0, 0, 0]^T$, $\hat{\delta}_0 = 0$. The reference trajectory used is the state of a second order linear system with double real pole $p_{12} = -3$. The observer gain matrix $\mathbf{N} = [100, 2500]^T$ sets a double pole -50 for the observer dynamics. The parameter estimates convergence gains are $\gamma_1 = 0.0003$, $\gamma_2 = 50$, $\gamma_3 = 5$, $\gamma_4 = 10^4$, $\gamma_\delta = 10^{-3}$.



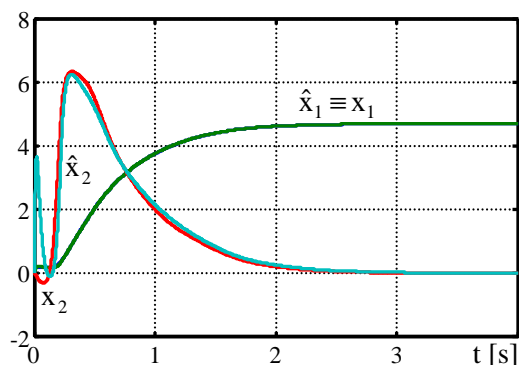


Figure 1. Adaptive system and state observer responses

Figure 1 shows the closed loop adaptive system trajectory tracking and the adaptive state observer responses. The time evolution of the parameters and their estimates is given on figure 2. The torque creating current i_d and the currents in the abc frame

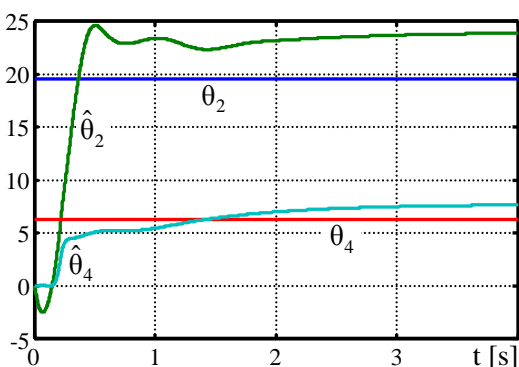
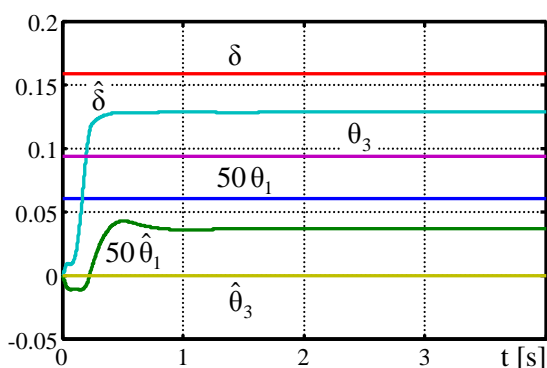


Figure 2. Parameter observer dynamics

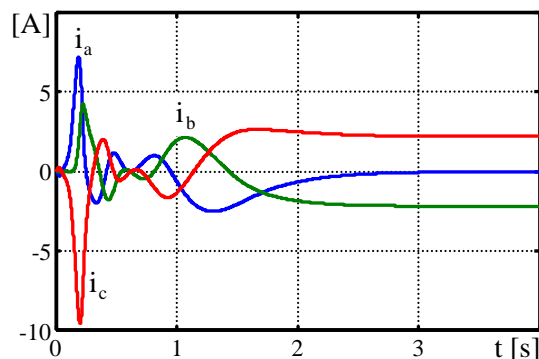
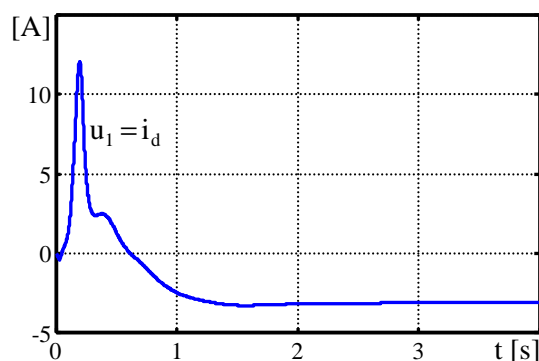


Figure 3. Control currents dynamics

are displayed on figure 3. The time response of the electromagnetic torque is depicted on figure 4. The closed loop adaptive system tracking and combined errors are shown on figure 5. The control current

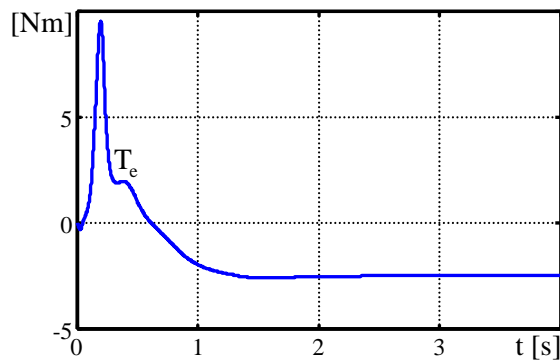


Figure 4. Electromagnetic torque

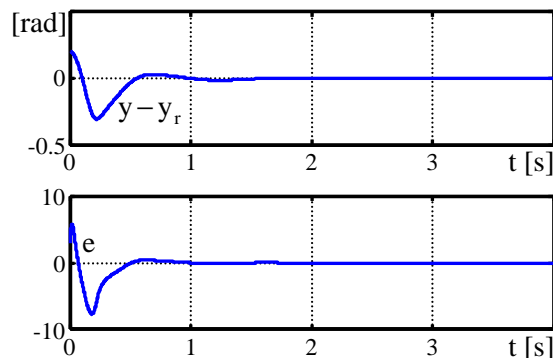


Figure 5. Tracking and combined errors



$u_2 = i_q$ is asymptotically stabilized at zero, because the PMSM operates at lower than the rated speed $\omega_N = 356$ rad/s. The parameter estimates are only Lyapunov stable while the state observer and the combined error are asymptotically stable. The estimation of the true parameters depends on the reference trajectory excitation capability. The convergence of the estimates to the true parameters is guaranteed only if the reference trajectory is persistently exciting. The convergence of the combined error depends on the tuning parameters k and λ . All the responses are for $k=10$ and $\lambda=15$. With these values the tracking of the reference trajectory by the closed loop adaptive system is done with the prescribed overdamped performance specifications.

Conclusions

The paper deals with the adaptive control of a salient-pole PMSM driven inverted pendulum with adaptive state and parameter observers. The task solved is tracking of a reference trajectory. The nonlinear adaptive tracking control is accomplished by the adaptive combined error approach. The adaptive parameter observer is designed via the direct Lyapunov method and ensures Lyapunov stability of the parameter errors only. The nonlinear adaptive state observer is designed into nonlinear adaptive observer canonical form and provides asymptotically stable state tracking even with a Lyapunov stable parameter estimator. The closed loop nonlinear adaptive tracking control system with integrated nonlinear adaptive state and parameter observers achieves the prescribed overdamped tracking performance specifications by asymptotically stable state tracking and Lyapunov stable parameter estimation.

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