NONLINEAR ADAPTIVE OBSERVERS IN ADAPTIVE TUNING FUNCTIONS TRACKING CONTROL

R. Mishkov, V. Petrov

Control Systems Department, Technical University Sofia, Branch Plovdiv, St. Canko Dyustabanov 25, Plovdiv 4000, Bulgaria, phone: +35932659584

r.mishkov@gmail.com, v.spasovv@gmail.com

Abstract: The paper presents a nonlinear adaptive closed loop system design with nonlinear adaptive state and parameter observers and tuning functions based nonlinear adaptive control for trajectory tracking. The closed loop nonlinear adaptive system is asymptotically stable with respect to the tracking and state estimation errors and Lyapunov stable for the parameters estimation errors. This is achieved by a nonlinear damping technique. An advantage of the approach is that the overdamped performance of the closed loop nonlinear adaptive system is guaranteed in its whole range of operation. The approach is applied to a permanent magnet synchronous motor driven inverted pendulum for illustration.

Key words: nonlinear systems, adaptive control, adaptive observers, adaptive parameter estimators, Lyapunov stability.

INTRODUCTION

Generally a nonlinear adaptive closed loop system design incorporates an adaptive nonlinear state observer and an unknown parameters estimator. The known system information includes the system structure, the input and the output signals only. The nonlinear adaptive state observer design requires transformation of the original nonlinear system into an appropriate nonlinear canonical form [1, 2, 6, 8] being usually adaptive observer canonical form (AOCF) [6]. The necessary conditions for transformation into AOCF are stringent and limit the class of the transformable nonlinear systems. Another problem is that most often the transformation is dependent on the parameters, which requires exact parameter estimation. The known approaches in this field, put the requirement for persistent system excitation [1, 2, 6, 7] to achieve exact estimation, but it conflicts with the closed loop system control goals. In that sense, a relevant solution of the objective nonlinear adaptive control task can be achieved by unification of the nonlinear adaptive control, nonlinear adaptive state observer, and nonlinear adaptive parameter estimator designs in one approach. Various methods considering closed loop adaptive control with adaptive nonlinear observers [3, 4, 5, 13], based on classical methods for nonlinear adaptive control [9, 10] deal with the problem. The paper presents a unified approach for closed loop adaptive system design with nonlinear adaptive state and parameter observers applied for nonlinear adaptive control of a non-salient pole permanent magnet synchronous motors (PMSM) [11, 12] via a transformation into AOCF. The nonlinear adaptive tracking control design is accomplished by the adaptive tuning functions approach [5, 10].

PROBLEM STATEMENT

The original objective nonlinear system is firstly presented in observer canonical form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{F}(y,u)\boldsymbol{\theta}\,,\ \ \mathbf{x}(0) = \mathbf{x}_0 \tag{1a}$$

$$y = \mathbf{C}\mathbf{x} \tag{1b}$$

where the matrix pair $(\mathbf{A},\mathbf{C})$ is in single-output Brunovski canonical form, $\boldsymbol{\theta} \in R^{p}$ is the unknown parameter vector and $\mathbf{F}(y,u) \in R^{n\times p}$. The filtered transformation for second order systems has the following form

$$\mathbf{z}(\mathbf{x}) = \mathbf{x} - \begin{bmatrix} \mathbf{0} \\ \mathbf{m} \end{bmatrix}\boldsymbol{\theta}\,, \tag{2a}$$

$$\dot{\mathbf{m}} = \lambda\mathbf{m} + \boldsymbol{\beta}\mathbf{F}(y,u)\,,\ \ \mathbf{m}_0 = \mathbf{m}(0) \tag{2b}$$

where $\mathbf{m} \in R^{1\times p}$, $\lambda = -b_2/b_1$, $\boldsymbol{\beta} = [-b_2/b_1, 1]$. It transforms system (1) into the adaptive observer canonical form

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}\boldsymbol{\omega}^{T}\boldsymbol{\theta}\,,\ \ \mathbf{z}(0) = \mathbf{z}_0\,, \tag{3a}$$

$$y = \mathbf{C}\mathbf{z}\,, \tag{3b}$$

The vector $\boldsymbol{\omega}^{T} \in R^{1\times p}$ reads $\boldsymbol{\omega}^{T} = \mathbf{m} + \mathbf{C}\mathbf{F}(y,u)$ and $\mathbf{b} \in R^{n\times 1}$ is a Hurwitz polynomial. The adaptive nonlinear observer for the system (3) has the form

$$\dot{\hat{\mathbf{z}}} = \mathbf{A}\hat{\mathbf{z}} + \mathbf{b}\boldsymbol{\omega}^{T}\hat{\boldsymbol{\theta}} + \mathbf{N}(y - \hat{y})\,,\ \ \hat{\mathbf{z}}(0) = \hat{\mathbf{z}}_0\,, \tag{4a}$$

$$\hat{y} = \mathbf{C}\hat{\mathbf{z}}\,, \tag{4b}$$

with the observer gain matrix $\mathbf{N}$. The observer error $\tilde{\mathbf{z}} = \mathbf{z} - \hat{\mathbf{z}}$ has the dynamics

$$\dot{\tilde{\mathbf{z}}} = (\mathbf{A} - \mathbf{N}\mathbf{C})\tilde{\mathbf{z}} + \mathbf{b}\boldsymbol{\omega}^{T}\tilde{\boldsymbol{\theta}}\,,\ \ \tilde{\mathbf{z}}(0) = \tilde{\mathbf{z}}_0\,. \tag{5}$$

The adaptive parameter estimation dynamics $\dot{\hat{\boldsymbol{\theta}}}$ is designed via the direct Lyapunov method. A positive definite Lyapunov function candidate

$$V_o = \frac{1}{2}\tilde{\mathbf{z}}^{T}\mathbf{P}\tilde{\mathbf{z}} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{T}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}$$

is defined whose total time derivative assuming that the parameters are constant $(\dot{\boldsymbol{\theta}} = \mathbf{0})$ will be

$$\dot{V}_o = \tilde{\mathbf{z}}^{T}[(\mathbf{A} - \mathbf{N}\mathbf{C})^{T}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{N}\mathbf{C})]\tilde{\mathbf{z}} + \tilde{\boldsymbol{\theta}}^{T}(\boldsymbol{\omega}\mathbf{b}^{T}\mathbf{P}\tilde{\mathbf{z}} - \boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}) \tag{6}$$

with $(\mathbf{A} - \mathbf{N}\mathbf{C})$ stable. The observer error dynamics (5) is with strictly positive real transfer function, the Kalman-Yakubovic lemma holds, hence

$$\mathbf{b}^{T}\mathbf{P} = \mathbf{C}\,, \tag{7}$$

where $\mathbf{P}$ is the solution of the Lyapunov equation

$$(\mathbf{A} - \mathbf{N}\mathbf{C})^{T}\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{N}\mathbf{C}) = -\mathbf{Q}\,. \tag{8}$$

The adaptive parameter estimator dynamics is

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\tau}_0\,,\ \ \hat{\boldsymbol{\theta}}_0 = \hat{\boldsymbol{\theta}}(0)\,, \tag{9}$$

where $\boldsymbol{\tau}_0 = \boldsymbol{\omega}(y - \hat{z}_1)$ is the observer tuning function that will later be used in the adaptive control synthesis. Considering (7) and (8) the derivative (6) becomes

$$\dot{V}_o = -\tilde{\mathbf{z}}^T \mathbf{Q} \tilde{\mathbf{z}} .$$

According to the La-Salle-Yoshizawa convergence theorem for non-autonomous nonlinear systems the observer error $\tilde{\mathbf{z}}$ will be asymptotically stable while the parameter errors $\tilde{\boldsymbol{\theta}}$ are only Lyapunov stable. Let the number of unknown parameters be $p = 3$. Then the adaptive tuning functions tracking control design starts with the derivative of the output for system (3)

$$\dot{y} = \omega_2\theta_2 + \hat{z}_2 + \overline{\boldsymbol{\omega}}^T\boldsymbol{\theta} + \tilde{z}_2 \tag{10}$$

where $z_2 = \hat{z}_2 + \tilde{z}_2$ and $\overline{\boldsymbol{\omega}} = [\omega_1, 0, \omega_3]^T$. Let ω_2 be the filtered transformation signal m_2 and its dynamics

$$\dot{m}_2 = -\lambda m_2 + u ,$$

includes the control u. That is why, m_2 will be the virtual control. The tracking problem is turned into a stabilization task by defining the error coordinates transformation

$$e_1 = y - y_r \tag{11a}$$

$$e_2 = m_2 - \hat{\delta}\dot{y}_r - \alpha \tag{11b}$$

where $\hat{\delta}$ is the estimate of $\delta = 1/\theta_2$, α is the stabilizing function and y_r is the reference trajectory. The tracking error e_1 dynamics obtained from (10) and (11a) considering (11b) reads

$$\dot{e}_1 = (e_2 + \hat{\delta}\dot{y}_r + \alpha)\theta_2 + \hat{z}_2 + \overline{\boldsymbol{\omega}}^T\boldsymbol{\theta} + \tilde{z}_2 - \dot{y}_r .$$

Scaling the stabilizing function as $\alpha = \hat{\delta}\overline{\alpha}$ the above equation reduces to

$$\dot{e}_1 = e_2\theta_2 + \hat{\delta}(\dot{y}_r + \overline{\alpha})\theta_2 + \hat{z}_2 + \overline{\boldsymbol{\omega}}^T\boldsymbol{\theta} + \tilde{z}_2 - \dot{y}_r . \tag{12}$$

The task is to stabilize (12). We use the Lyapunov function

$$V_1 = \frac{1}{2}e_1^2 + \frac{|\theta_2|}{2\gamma_\delta}\tilde{\delta}^2 + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}} ,$$

whose derivative along the solution of (12) is

$$\begin{aligned}\dot{V}_1 &= e_1(\overline{\alpha} + \overline{\boldsymbol{\omega}}^T\hat{\boldsymbol{\theta}} + e_2\hat{\theta}_2 + \hat{z}_2 + \tilde{z}_2) - \gamma_\delta^{-1}\theta_2\tilde{\delta}(\mathrm{sign}(\theta_2)\dot{\hat{\delta}} + \\ &\gamma_\delta(\dot{y}_r + \overline{\alpha})e_1) - \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - (\boldsymbol{\omega} - \hat{\delta}(\dot{y}_r + \overline{\alpha})\mathbf{v}_2)e_1) ,\end{aligned} \tag{13}$$

where $\mathbf{v}_2 = [0, 1, 0]^T$. The $\hat{\delta}$ dynamics

$$\dot{\hat{\delta}} = -\gamma_\delta \mathrm{sign}(\theta_2)(\dot{y}_r + \overline{\alpha})e_1 ,\ \hat{\delta}_0 = \hat{\delta}(0) \tag{14}$$

$$\overline{\alpha}_1 = -c_1e_1 - d_1e_1 - \overline{\boldsymbol{\omega}}^T\hat{\boldsymbol{\theta}} - \hat{z}_2 ,$$

with $d_1 > 0$, $c_1 > 0$, where the damping term $-d_1e_1$ is added to suppress the destabilizing effect of the error $\tilde{z}_2$, reduces equation (13) to

$$\dot{V}_1 = -c_1e_1^2 + e_1e_2\hat{\theta}_2 - d_1e_1^2 + e_1\tilde{z}_2 - \tilde{\boldsymbol{\theta}}(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_1) ,$$

where

$$\boldsymbol{\tau}_1 = \boldsymbol{\omega} - \hat{\delta}(\dot{y}_r + \overline{\alpha})\mathbf{v}_2e_1$$

is the first tuning function. The sign indefinite term $e_1e_2\hat{\theta}$ will be eliminated with the next step. The last term in $\dot{V}_1$ can be eliminated via the parameter estimator dynamics $\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\tau}_1$, but at this stage it will be postponed to avoid overparametrization. The error e_2 dynamics is

$$\dot{e}_2 = -\lambda m_2 + u - \hat{\delta}\ddot{y}_r - \frac{\partial\alpha}{\partial\boldsymbol{\mu}}\dot{\boldsymbol{\mu}} - \frac{\partial\alpha}{\partial y}(\hat{z}_2 + \tilde{z}_2 + \boldsymbol{\omega}^T\hat{\theta} + \boldsymbol{\omega}^T\tilde{\theta}) \tag{15}$$

where $\boldsymbol{\mu} = [\hat{\delta}, y_r, m_1, m_3, \hat{z}_2, \hat{\boldsymbol{\theta}}^T]^T$. The stabilization of (15) is achieved via the recursive Lyapunov function

$$V_2 = \frac{1}{2}e_2^2 + V_1 ,$$

whose time derivative considering (15) is

$$\begin{aligned}\dot{V}_2 &= e_2(-\lambda m_2 + u - \hat{\delta}\ddot{y}_r - \frac{\partial\alpha}{\partial\boldsymbol{\mu}}\dot{\boldsymbol{\mu}} - \frac{\partial\alpha}{\partial y}(\hat{z}_2 + \boldsymbol{\omega}^T\hat{\theta}) - \frac{\partial\alpha}{\partial y}\tilde{z}_2) \\ &-c_1e_1^2 + e_1e_2\hat{\theta}_2 - d_1e_1^2 + e_1\tilde{z}_2 - \tilde{\boldsymbol{\theta}}(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_2),\end{aligned} \tag{16}$$

where $\boldsymbol{\tau}_2 = \boldsymbol{\tau}_1 - (\partial\alpha/\partial y)\boldsymbol{\omega}e_2$ is the second tuning function. The adaptive control design has to eliminate all sign indefinite terms, hence

$$\begin{aligned}u &= \lambda m_2 + \hat{\delta}\ddot{y}_r + \frac{\partial\alpha}{\partial\boldsymbol{\mu}}\dot{\boldsymbol{\mu}} + \frac{\partial\alpha}{\partial y}(\hat{z}_2 + \boldsymbol{\omega}^T\hat{\theta}) - e_1\hat{\theta}_2 - c_2e_2 \\ &-d_2\left(\frac{\partial\alpha}{\partial y}\right)^2 e_2 ,\end{aligned} \tag{17}$$

where $c_2 > 0$, $d_2 > 0$. The last term $-d_2(\partial\alpha/\partial y)^2e_2$ aims to damp the $(\partial\alpha/\partial y)\tilde{z}_2$ term. The completion of squares is made with the damping term. That is why, the sign indefinite terms including $\tilde{z}_2$ become sign definite. Replacing the adaptive control law (17) into the derivative (16) gives

$$\begin{aligned}\dot{V}_2 &= -c_1e_1^2 - c_2e_2^2 - d_1\left(e_1 - \frac{\tilde{z}_2}{2d_1}\right)^2 - d_2\left(\frac{\partial\alpha}{\partial y}e_2 + \frac{\tilde{z}_2}{2d_2}\right)^2 \\ &+\left(\frac{1}{4d_1} + \frac{1}{4d_2}\right)\tilde{z}_2^2 - \tilde{\boldsymbol{\theta}}(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_2).\end{aligned}$$

The stability of the closed loop system is analyzed via the Lyapunov function candidate

$$V = V_2 + V_o .$$

Its time derivative reads

$$\begin{aligned}\dot{V} &= -c_1e_1^2 - c_2e_2^2 - d_1\left(e_1 - \frac{\tilde{z}_2}{2d_1}\right)^2 - d_2\left(\frac{\partial\alpha}{\partial y}e_2 + \frac{\tilde{z}_2}{2d_2}\right)^2 \\ &+\left(\frac{1}{4d_1} + \frac{1}{4d_2}\right)\tilde{z}_2^2 - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} - \tilde{\boldsymbol{\theta}}(\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}} - \boldsymbol{\tau}_3),\end{aligned}$$

where $\boldsymbol{\tau}_3 = \boldsymbol{\tau}_2 + \boldsymbol{\tau}_0$ is the last tuning function. With the parameter estimator update law

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}\boldsymbol{\tau}_3 \tag{18}$$

the above derivative reduces to

$$\begin{aligned}\dot{V} &= -c_1e_1^2 - c_2e_2^2 - d_1\left(e_1 - \frac{\tilde{z}_2}{2d_1}\right)^2 - d_2\left(\frac{\partial\alpha}{\partial y}e_2 + \frac{\tilde{z}_2}{2d_2}\right)^2 \\ &+\left(\frac{1}{4d_1} + \frac{1}{4d_2}\right)\tilde{z}_2^2 - \tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}}.\end{aligned}$$

The stability of the whole system depends on the matrix $\mathbf{Q}$. If

$$\mathbf{Q} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$$

for $q_1 > 0$, $q_2 = \frac{1}{4d_1} + \frac{1}{4d_2} + c_3$, $c_3 > 0$, then

$$\begin{aligned}\dot{V} &= -c_1e_1^2 - c_2e_2^2 - d_1\left(e_1 - \frac{\tilde{z}_2}{2d_1}\right)^2 - d_2\left(\frac{\partial\alpha}{\partial y}e_2 + \frac{\tilde{z}_2}{2d_2}\right)^2 \\ &-q_1\tilde{z}_1^2 - c_3\tilde{z}_2^2\end{aligned} \tag{19}$$

will be negative semi-definite with respect to $\tilde{\boldsymbol{\theta}}$. The asymptotic stability of the tracking and state observer errors follows from the La-Salle-Yoshizawa convergence theorem, which completes the nonlinear adaptive control design.

APPLICATION OF THE APPROACH

The approach is applied to a permanent magnet synchronous motor (PMSM) driven inverted pendulum system. The PMSM in rotating dqo coordinates, controlled in current mode, has a state space system model in observer canonical form

$$\dot{x}_1 = x_2 - \theta_3 y$$
$$\dot{x}_2 = -\theta_1 \sin(y) + \theta_2 u \qquad (20a)$$
$$y = x_1, \qquad (20b)$$
where x_1 is the mechanical angle, x_2 is the rotor speed, u is the control current i_d. The model parameters are

$$\theta_1 = mgl/J,\ \theta_2 = \sqrt{3/2}\, n_p \psi_{pm}/J,\ \theta_3 = b/J,$$

where m and l are the pendulum mass and length, J – total moment of inertia, g – gravity acceleration, n_p – number of pole pairs, ψ_{pm} – magnets flux linkage magnitude and b is the viscose friction coefficient. The matrix $\mathbf{F}(y,u)$ for the system (20) is

$$\mathbf{F}(y,u) = \begin{bmatrix} 0 & 0 & -y \\ -\sin(y) & u & 0 \end{bmatrix}.$$

Hence, the filters (2) will be

$$\dot{m}_1 = -\lambda m_1 - \sin(y)$$
$$\dot{m}_2 = -\lambda m_2 + u$$
$$\dot{m}_3 = -\lambda (m_3 - y)$$

Then the vector $\boldsymbol{\omega}^T$ takes the form

$$\boldsymbol{\omega}^T = [m_1, m_2, m_3 - y].$$

The system in adaptive observer canonical form (3) reads

$$\dot{z}_1 = z_2 + m_1\theta_1 + m_2\theta_2 + (m_3 - y)\theta_3$$
$$\dot{z}_2 = b_2(m_1\theta_1 + m_2\theta_2 + (m_3 - y)\theta_3)/b_1$$
$$y = z_1$$

with adaptive nonlinear state observer (4)

$$\dot{\hat{z}}_1 = \hat{z}_2 + m_1\hat{\theta}_1 + m_2\hat{\theta}_2 + (m_3 - y)\hat{\theta}_3 + n_1(y - \hat{y})$$
$$\dot{\hat{z}}_2 = b_2(m_1\hat{\theta}_1 + m_2\hat{\theta}_2 + (m_3 - y)\hat{\theta}_3)/b_1 + n_2(y - \hat{y})$$
$$\hat{y} = \hat{z}_1,$$

observer error (5)

$$\dot{\tilde{z}}_1 = \tilde{z}_2 + m_1\tilde{\theta}_1 + m_2\tilde{\theta}_2 + (m_3 - y)\tilde{\theta}_3 + n_1(y - \hat{y})$$
$$\dot{\tilde{z}}_2 = b_2(m_1\tilde{\theta}_1 + m_2\tilde{\theta}_2 + (m_3 - y)\tilde{\theta}_3)/b_1 + n_2(y - \hat{y}).$$

and adaptive parameter estimator dynamics (9) with observer tuning function $\boldsymbol{\tau}_0$

$$\boldsymbol{\tau}_0 = b_1^{-1}[m_1, m_2, m_3 - y]^T (y - \hat{y}).$$

Following the procedure described in the previous section the next tuning functions are computed

$$\boldsymbol{\tau}_1 = \begin{bmatrix} e_1 m_1 \\ e_1 e_2 \\ e_1(m_3 - y) \end{bmatrix},\ \boldsymbol{\tau}_2 = \begin{bmatrix} m_1\,(e_1 + e_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3)) \\ e_2(e_1 + m_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3)) \\ (m_3 - y)(e_1 + e_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3)) \end{bmatrix},$$

$$\boldsymbol{\tau}_3 = \frac{1}{2b_1}\begin{bmatrix} m_1(\tilde{z}_1 + b_1(e_1 + e_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3))) \\ m_2\tilde{z}_1 + b_1 e_2(e_1 + m_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3)) \\ (m_3 - y)(\tilde{z}_1 + b_1(e_1 + e_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3))) \end{bmatrix}.$$

The tuning functions $\boldsymbol{\tau}_k$ $k = 0,1,2,3$ define the final parameter estimator dynamics (18), (14)

$$\dot{\hat{\theta}}_1 = (2b_1)^{-1}\gamma_1 m_1(\tilde{z}_1 + b_1(e_1 + e_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3)))$$
$$\dot{\hat{\theta}}_2 = (2b_1)^{-1}\gamma_2(m_2\tilde{z}_1 + b_1 e_2(e_1 + m_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3)))$$
$$\dot{\hat{\theta}}_3 = (2b_1)^{-1}\gamma_3(m_3 - y)(\tilde{z}_1 + b_1(e_1 + e_2\hat{\delta}(c_1 + d_1 - \hat{\theta}_3)))$$
$$\dot{\hat{\delta}} = \mathrm{sign}(\theta_2)\gamma_\delta e_1(e_1(c_1 + d_1) - \dot{y}_r + \hat{z}_2 + m_1\hat{\theta}_1 + (m_3 - y)\hat{\theta}_3).$$

The complete adaptive control law is

$$u = -\mathrm{sign}(\theta_2)(e_1\gamma_\delta((c_1 + d_1)e_1 - \dot{y}_r + \hat{z}_2 + m_1\hat{\theta}_1 + (m_3 - y)\hat{\theta}_3)^2$$
$$-(2b_1)^{-1}\tilde{z}_1(m_1^2\gamma_1 + (m_3 - y)^2\gamma_3)\hat{\delta} + 2b_2 m_2(\hat{\delta}\hat{\theta}_2 - 1) - c_2 e_2 +$$
$$2^{-1}((2\ddot{y}_r - 2n_2\tilde{z}_1 + 2(c_1 + d_1)(\dot{y}_r - \hat{z}_2) - e_1 m_1^2\gamma_1 - e_1(m_3 -$$
$$y)^2\gamma_3)\hat{\delta} - 2e_1\hat{\theta}_2 + \hat{\delta}(-(c_1 + d_1)(e_2(2(c_1 + d_1)d_2 + m_1^2\gamma_1 + (m_3$$
$$-y)^2\gamma_3)\hat{\delta} + 2(m_1\hat{\theta}_1 + m_2\hat{\theta}_2)) + (2\hat{z}_2 + e_2(m_1^2\gamma_1 + (m_3 - y)^2\gamma_3)$$
$$\hat{\delta} + 2(c_1 + d_1)(-m_3 + y + 2d_2 e_2\hat{\delta}) + 2m_1\hat{\theta}_1 + 2m_2\hat{\theta}_2)\hat{\theta}_3 - 2(-m_3$$
$$+y + d_2 e_2\hat{\delta})\hat{\theta}_3^2)) + \hat{\delta}\hat{\theta}_1 \sin(y).$$

As it was proved in the previous section the Lyapunov function derivative (19) is strictly negative definite with respect to the state observer and tracking errors and semidefinite with respect to the parameter estimation errors.

SIMULATION AND SYSTEM TIME RESPONSES

The PMSM synchronous servomotor used is Lenze MDSKS071-03 with the following parameters: nominal power – $P_N = 2$ kW, nominal torque – $T_N = 5.7$ Nm, nominal AC voltage – $U_N = 330$ V, nominal current – $I_N = 4.2$ A, torque constant – $k_T = 1.37$ Nm/A, stator resistance – $R_s = 3.4\ \Omega$, moment of inertia – $J = 6\times 10^{-4}$ kg.m^2, stator inductance – $L_s = 10.6$ mH, number of pole pairs – $n_p = 3$. The pendulum attached to the shaft is determined by the mass $m = 0.5$ kg, link length $l = 0.5$ m, and the gravity constant $g = 9.81$ m s^{-2}. The closed-loop adaptive system including the nonlinear adaptive state and parameter observers with adaptive tuning functions control is dynamically simulated at initial conditions $\mathbf{x}_0 = [0, 0]^T$, $\mathbf{z}_0 = [0, 0]^T$, $\hat{\mathbf{z}}_0 = [0.2, 0]^T$, $\hat{\boldsymbol{\theta}}_0 = [0, 0, 0]^T$, $\hat{\delta}_0 = 0$, $\mathbf{m}_0 = [0, 0, 0]^T$. The reference trajectory is the state vector of a second order linear system with double real pole $p_{12} = -5$. The adaptive tuning functions control law is defined by the parameters $c_1 = 100$, $c_2 = 1$, $d_1 = d_2 = 10$. The observer gain matrix is $\mathbf{N} = [100, 2500]^T$ which sets a double pole -50 for the obsever dynamics. The parameter estimator gains are $\gamma_1 = \gamma_2 = 10000$, $\gamma_3 = 15$, $\gamma_\delta = 10$. Figure 1 displays the evolution of the closed loop adaptive system

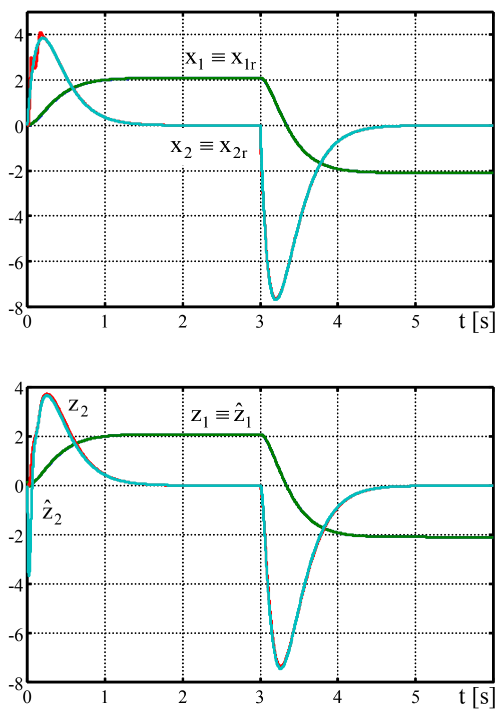

Figure 1: Adaptive system and state observer errors

tracking and adaptive state observer errors. The dynamics of the estimated parameters is depicted on figure 2. The control input u which is the motor torque current in rotating dq

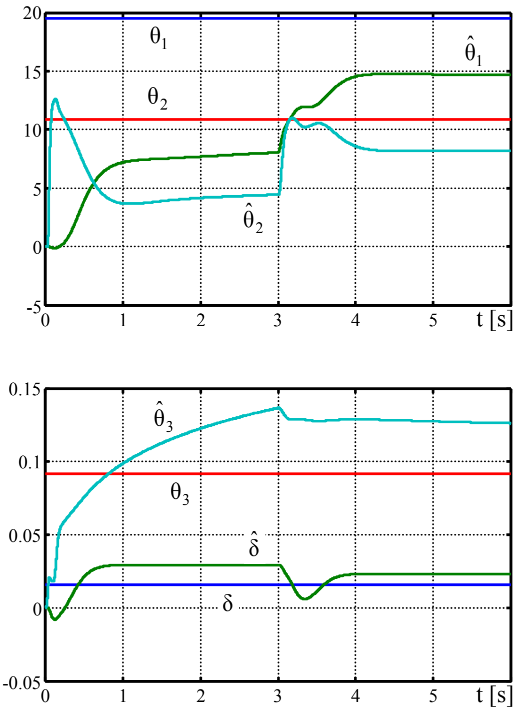

Figure 2: Parameter observer dynamics

coordinates i_d and the currents in original abc coordinates are given on figure 3. The i_q current is asymptotically stabilized

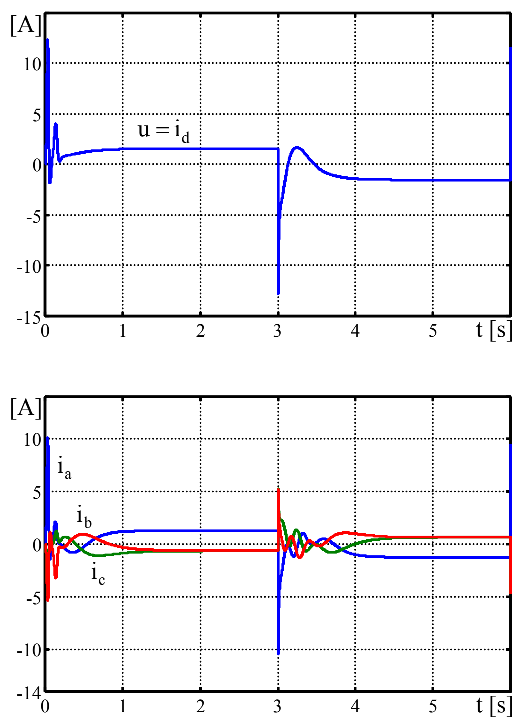

Figure 3: Control currents dynamics

at zero. The tracking of the reference trajectory is achieved by the closed loop adaptive system with the prescribed overdamped performance specification. The parameter estimates are only Lyapunov stable while the state observer is asymptotically stable.

CONCLUSIONS

The paper investigates the task for adaptive tracking control of a synchronous motor driven inverted pendulum with state and parameter adaptive observers. The adaptive tracking control is designed by the application of the adaptive tuning functions approach. The adaptive state observer is designed by transformation in adaptive observer canonical form and provides asymptotically stable state tracking. The adaptive parameter estimator is designed via the direct Lyapunov method and ensures only Lyapunov stability of the parameter estimates. The closed loop nonlinear adaptive tracking control system with embedded adaptive nonlinear state and parameter observers achieve the prescribed overdamped tracking performance specifications by asymptotically stable state tracking and Lyapunov stable parameter estimation.

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