НАУЧНИ ТРУДОВЕ
ТОМ LX
"ХРАНИТЕЛНА НАУКА, ТЕХНИКА И ТЕХНОЛОГИИ – 2013"
18-19 октомври 2013, Пловдив

SCIENTIFIC WORKS
VOLUME LX
„FOOD SCIENCE, ENGINEERING AND TECHNOLOGY 2013"
18-19 October 2013, Plovdiv

EXACT PARAMETER ESTIMATION WITHOUT PERSISTENT EXCITATION IN NONLINEAR ADAPTIVE CONTROL SYSTEMS

Rumen Mishkov, Stanislav Darmonski
Technical University Sofia, Branch Plovdiv

Abstract
The paper presents a new general approach for exact unknown parameter estimation in nonlinear adaptive control systems, without imposing the persistent excitation requirement. The proposed approach modifies the basic adaptive parameter estimator dynamics and is based on a generalization of the prediction error concept and the introduction of the stable data accumulation concept. The modified estimator dynamics is of least-squares type and the resulting closed loop adaptive system is asymptotically stable with respect to the tracking and parameter estimation errors. This property is achieved by controlling the rank of the data accumulation matrices. The advantage of the new approach is the exact parameter estimation achieved in one transient response without using the standard excitation techniques. The approach is applied to a DC motor driven inverted pendulum for illustration.

Keywords: *adaptive control, exact parameter estimation, Lyapunov stability, nonlinear systems.*

Introduction

The exact unknown parameter estimation in nonlinear adaptive control is an attractive problem, which does not have a general solution yet. The standard techniques for achieving exact estimation rely on sufficiently rich reference trajectories or persistent excitation of the control system and the recent results in this field [1, 2, 3, 5, 8] are not an exception. These requirements are in contradiction with the control goal determined by technological considerations and are therefore irrelevant. This paper considers the exact unknown parameter estimation task in nonlinear adaptive control without imposing any excitation or special trajectory requirements on the controlled nonlinear dynamics. The attention is focused on constructing a sufficiently rich information process, embedded in the adaptive controller by defining and manipulating different information channels. An information channel is a system variable which is indirectly connected with the unknown parameter estimation error. The main information channel is the well known prediction error and the paper provides a generalization of the prediction error concept. The data accumulation concept is introduced on the basis of this error which is the main tool for achieving exact unknown parameter estimation without persistency of excitation. The idea is to dynamically construct a full rank transformation matrix between the unknown parameter vector and a known suitably defined mapping vector. The data accumulation concept considers some results in [2]. The introduced generalization of the prediction error concept is based on the nonlinear swapping techniques found in [4]. The plant model and control design are taken from [6, 7]. As a result, exact unknown parameter estimation in one transient response without persistent excitation is achieved and illustrated by the simulation example.

Problem Statement

The nonlinear systems considered are of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) + \mathbf{G}(\mathbf{x},\mathbf{u})\boldsymbol{\theta}, \qquad (1a)$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}). \qquad (1b)$$

Here $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^r$, $\mathbf{y} \in \Re^m$, $\boldsymbol{\theta} \in \Re^p$ are the state, control, output, unknown parameters vectors, and $\mathbf{f}(\mathbf{x},\mathbf{u}): \Re^n \times \Re^r \to \Re^n$, $\mathbf{G}(\mathbf{x},\mathbf{u}): \Re^n \times \Re^r \to \Re^{n\times p}$, $\mathbf{h}(\mathbf{x}): \Re^n \to \Re^m$ are known nonlinear mappings.

Assumption 1: It is assumed that the control task is the tracking of a reference trajectory $\mathbf{y}_d(t)$, whose time derivatives are bounded, and a certain nonlinear adaptive control design approach has been applied to obtain the basic adaptive control and estimation laws

$$\mathbf{u}(t) = \boldsymbol{\alpha}(\mathbf{x}, \hat{\boldsymbol{\theta}}, \dot{\hat{\boldsymbol{\theta}}}, t), \qquad (2a)$$

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}_\theta \boldsymbol{\beta}(\mathbf{z}, \mathbf{x}, \hat{\boldsymbol{\theta}}, t). \qquad (2b)$$

It is further assumed that the closed loop nonlinear adaptive system dynamics in error coordinates $\mathbf{z} \in \Re^n$ is of the form

$$\dot{\mathbf{z}} = \mathbf{f}_z(\mathbf{z}, \hat{\boldsymbol{\theta}}, t) + \mathbf{G}_z(\mathbf{z}, \hat{\boldsymbol{\theta}}, t)\tilde{\boldsymbol{\theta}}, \qquad (3a)$$

$$\dot{\tilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}_\theta \boldsymbol{\beta}(\mathbf{z},\mathbf{x},\hat{\boldsymbol{\theta}},t), \tag{3b}$$

where the vector functions $\mathbf{f}_z \in \Re^{n\times 1}$, $\mathbf{G}_z \in \Re^{n\times p}$, and $\boldsymbol{\beta} \in \Re^{p\times 1}$ are locally Lipschitz with respect to $\mathbf{z}$, $\tilde{\boldsymbol{\theta}}$, uniformly in t, with the property $\mathbf{f}_z(\mathbf{0},\boldsymbol{\theta},t) = \mathbf{0}$, $\boldsymbol{\beta}(\mathbf{0},\mathbf{x},\hat{\boldsymbol{\theta}},t) = \mathbf{0}$. Moreover, there exists a known Lyapunov function of the form

$$V(\mathbf{z},\tilde{\boldsymbol{\theta}}) = (1/2)\mathbf{z}^T\mathbf{z} + (1/2)\tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}_\theta^{-1}\tilde{\boldsymbol{\theta}}, \tag{4}$$

whose total time derivative with respect to the closed loop adaptive system dynamics (3) is

$$\dot{V}(\mathbf{z},\tilde{\boldsymbol{\theta}}) = -\mathbf{z}^T\mathbf{C}\,\mathbf{z} + \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\beta} - \boldsymbol{\Gamma}_\theta^{-1}\dot{\hat{\boldsymbol{\theta}}}) = -\mathbf{z}^T\mathbf{C}\,\mathbf{z}, \tag{5}$$

where $\mathbf{C} = \mathrm{diag}(c_1,\ldots,c_n)$, $\boldsymbol{\Gamma}_\theta = \mathrm{diag}(\gamma_{\theta_1},\ldots,\gamma_{\theta_p})$ are positive definite design matrices and $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ is the unknown parameter estimation error. □

The main purpose of assumption 1 is to summarize the results from the application of a given nonlinear adaptive control design method. Assumption 1 ensures the global stability of the closed loop system (3), along with the asymptotic stability of $\mathbf{z}(t)$, and the Lyapunov stability of the parameter estimation error $\tilde{\boldsymbol{\theta}}(t)$. This is due to the negative semi-definiteness of the derivative $\dot{V}(\mathbf{z},\tilde{\boldsymbol{\theta}})$, which is not explicitly dependent on $\tilde{\boldsymbol{\theta}}(t)$. Hence, exact estimation of the unknown parameters cannot be provided. Moreover, most of the existing methods for nonlinear adaptive control deliberately eliminate the explicit dependence of the total derivative $\dot{V}$ on the unknown error $\tilde{\boldsymbol{\theta}}(t)$ via the adaptive estimation law. Negative definiteness of $\dot{V}$ on $\tilde{\boldsymbol{\theta}}$ has to be provided in order to achieve guaranteed exact parameter estimation. This is the main idea behind the exact estimation approach presented in this paper, which is realized by generalization of the prediction error concept and the concept of the stable data accumulation.

Prediction Error in x-Coordinates

The construction of an algebraic connection between the system trajectories $\mathbf{x}(t)$ and the unknown parameters $\boldsymbol{\theta}$ is the main idea of the prediction error concept. Based on this connection, an estimate $\hat{\mathbf{x}}$ depending on the estimated parameters $\hat{\boldsymbol{\theta}}$ can be defined. The error $\mathbf{e}_x = \mathbf{x} - \hat{\mathbf{x}}$ is called prediction error and can be used in the adaptation for providing indirect information about $\tilde{\boldsymbol{\theta}}(t)$. The next lemma presents a generalization of the prediction error concept for nonlinear systems of the form (1).

Lemma 1: Let the state estimate vector $\hat{\mathbf{x}} \in \Re^{n\times 1}$, the matrix $\mathbf{W}_x \in \Re^{n\times p}$ and the signal $\tilde{\boldsymbol{\varepsilon}} \in \Re^{n\times 1}$ be described by the vector-matrix differential equations

$$\dot{\hat{\mathbf{x}}} = -\boldsymbol{\Lambda}_x(\mathbf{x} - \hat{\mathbf{x}}) + \mathbf{f} + \mathbf{G}\hat{\boldsymbol{\theta}} + \mathbf{K}_x^{-1}\mathbf{W}_x\dot{\hat{\boldsymbol{\theta}}}, \tag{6a}$$

$$\dot{\mathbf{W}}_x = \boldsymbol{\Lambda}_x\mathbf{W}_x + \mathbf{K}_x\mathbf{G},\ \mathbf{W}_x(0) = \mathbf{0}, \tag{6b}$$

$$\dot{\tilde{\boldsymbol{\varepsilon}}} = \boldsymbol{\Lambda}_x\tilde{\boldsymbol{\varepsilon}},\ \tilde{\boldsymbol{\varepsilon}}(0) = \mathbf{e}_x(0), \tag{6c}$$

composed in view of the original nonlinear system dynamics (1), where $\mathbf{K}_x = \mathrm{diag}(k_{x_1},\ldots,k_{x_n}) > \mathbf{0}$ and $\boldsymbol{\Lambda}_x = \mathrm{diag}(\lambda_{x_1},\ldots,\lambda_{x_n}) < \mathbf{0}$ are design matrices. Then the relation $\mathbf{W}_x\tilde{\boldsymbol{\theta}} = \boldsymbol{\varsigma}_x$ with $\boldsymbol{\varsigma}_x = \mathbf{K}_x(\mathbf{e}_x - \tilde{\boldsymbol{\varepsilon}})$ holds.□

Data Accumulation Concept

The main idea behind the data accumulation concept is the dynamic construction of a coordinate transformation between the $\boldsymbol{\theta}$ parameter space and the space spanned by a suitably defined vector $\boldsymbol{\psi}(t) \in \Re^{p\times 1}$. The vector $\boldsymbol{\psi}(t)$ is a mapping of the original vector $\boldsymbol{\theta}$ in new coordinates defined as

$$\boldsymbol{\psi}(t) = \mathbf{Q}(t)\,\boldsymbol{\theta}, \tag{7}$$

where $\mathbf{Q}(t) \in \Re^{p\times p}$ is the coordinate transformation. If $\mathbf{Q}(t)$ has full rank then the transformation (7) is a diffeomorphism. The dynamics of $\mathbf{Q}(t)$ is chosen as

$$\dot{\mathbf{Q}} = -\boldsymbol{\Lambda}_r(\mathbf{Q}_r - \mathbf{Q})\mathbf{W}_x^T\mathbf{R}_x\mathbf{W}_x,\ \mathbf{Q}(0) = \mathbf{0}. \tag{8}$$

The key idea behind the definition (8) is to control the rank of the transformation matrix $\mathbf{Q}(t)$ via the reference design matrix $\mathbf{Q}_r$. The dynamics (8) can be interpreted as a stable data accumulation process, with the accumulated information matrix being $\mathbf{Q}(t)$. It can be shown that if the input $\mathbf{W}_x^T\mathbf{R}_x\mathbf{W}_x$ has full rank for a sufficient time period, then the matrix $\mathbf{Q}(t)$ will converge to its reference $\mathbf{Q}_r$. The convergence to $\mathbf{Q}_r$ is not necessary for achieving exact parameter estimation, but the full rank of the data matrix $\mathbf{Q}(t)$ is sufficient to guarantee exact parameter estimation, as it will be shown later. In this sense, the introduction of the reference $\mathbf{Q}_r$ in the dynamics (8) provides a way of controlling the rank of $\mathbf{Q}(t)$, and on the other hand stabilizes the data accumulation process, ensuring that $\mathbf{Q}(t)$ will remain bounded. Now, differentiating (7), considering the relation $\boldsymbol{\theta} = \tilde{\boldsymbol{\theta}} + \hat{\boldsymbol{\theta}}$ and lemma 1, we obtain the dynamics

$$\dot{\boldsymbol{\psi}} = -\boldsymbol{\Lambda}_r(\mathbf{Q}_r - \mathbf{Q})\mathbf{W}_x^T\mathbf{R}_x(\boldsymbol{\varsigma}_x + \mathbf{W}_x\hat{\boldsymbol{\theta}}),\ \boldsymbol{\psi}(0) = \mathbf{0}. \tag{9}$$

which is implementable. Rewriting (7) with respect to the estimated vector $\hat{\boldsymbol{\psi}}(t)$ gives

$$\hat{\boldsymbol{\psi}}(t) = \mathbf{Q}(t)\hat{\boldsymbol{\theta}} \tag{10}$$

and the mapping error

$$\tilde{\boldsymbol{\psi}}(t) = \boldsymbol{\psi}(t) - \hat{\boldsymbol{\psi}}(t) = \mathbf{Q}(t)\tilde{\boldsymbol{\theta}} \tag{11}$$

becomes known. Like the prediction error, the mapping error also provides indirect information about the estimation error. The main difference is that the rank of $\mathbf{Q}(t)$ is controllable, while the rank of $\mathbf{W}_x(t)$ is not. The data accumulation concept is summarized in the next lemma.

Lemma 2: Let the signals $\mathbf{Q}(t)$, $\boldsymbol{\psi}(t)$, and $\hat{\boldsymbol{\psi}}(t)$ are generated by the equations (8), (9) and (10), composed in correspondence with lemma 1, where $\mathbf{R}_x = \mathrm{diag}(r_{x_1},\ldots,r_{x_n}) > \mathbf{0}$, $\boldsymbol{\Lambda}_r = \mathrm{diag}(\lambda_{r_1},\ldots,\lambda_{r_p}) < \mathbf{0}$ are design matrices and $\mathbf{Q}_r$ is a constant reference matrix with full rank. Then all signals $\mathbf{Q}$, $\boldsymbol{\psi}$, and $\hat{\boldsymbol{\psi}}$ are globally bounded and the connection $\mathbf{Q}\tilde{\boldsymbol{\theta}} = \tilde{\boldsymbol{\psi}}$ holds, where $\tilde{\boldsymbol{\psi}} = \boldsymbol{\psi} - \hat{\boldsymbol{\psi}}$ is the mapping error. □

Modification for Exact Estimation

The idea behind the modification for exact estimation concept is to achieve negative definiteness of $\dot{V}$ with respect to both $\mathbf{z}(t)$ and $\tilde{\boldsymbol{\theta}}(t)$. Let the adaptive estimation law dynamics is modified into

$$\dot{\hat{\boldsymbol{\theta}}} = \dot{\hat{\boldsymbol{\theta}}}_\beta + \dot{\hat{\boldsymbol{\theta}}}_\mu .$$

The term $\dot{\hat{\boldsymbol{\theta}}}_\beta$ describes the basic adaptive estimator dynamics, which stems from the application of a given control design method. The term $\dot{\hat{\boldsymbol{\theta}}}_\mu$ describes the modification for exact estimation dynamics, which can be freely designed. In order to achieve explicit dependence of $\dot{V}$ on $\tilde{\boldsymbol{\theta}}(t)$ the dynamics $\dot{\hat{\boldsymbol{\theta}}}_\mu$ has to include indirect information about the estimation error into the estimation law. According to lemmas 1 and 2 such information is contained in the signals $\varsigma_x(t)$ and $\tilde{\boldsymbol{\psi}}(t)$, which are also available for feedback. Let us define the augmented vector

$$\tilde{\boldsymbol{\zeta}} = [\boldsymbol{\varsigma}_x^T, \tilde{\boldsymbol{\psi}}^T]^T, \ \tilde{\boldsymbol{\zeta}} \in \Re^{n+p\times 1}$$

which can be interpreted as the total prediction error. Then, according to the relations in lemmas 1 and 2

$$\tilde{\boldsymbol{\zeta}} = \mathbf{N}(t)\tilde{\boldsymbol{\theta}}, \tag{12}$$

where $\mathbf{N}(t) \in \Re^{n+p\times p}$ is defined as the block matrix

$$\mathbf{N}(t) = \begin{bmatrix} \mathbf{W}_x(t) \\ \mathbf{Q}(t) \end{bmatrix}. \tag{13}$$

The vector $\tilde{\zeta}$ can also be represented in the form $\tilde{\boldsymbol{\zeta}} = \boldsymbol{\zeta} - \hat{\boldsymbol{\zeta}}$, where $\boldsymbol{\zeta} = \mathbf{N}\boldsymbol{\theta}$ and $\hat{\boldsymbol{\zeta}} = \mathbf{N}\hat{\boldsymbol{\theta}}$. Now let us introduce the matrix

$$\mathbf{M}(t) = \mathbf{N}^T(t)\boldsymbol{\Gamma}_\zeta \mathbf{N}(t), \ \mathbf{M}(t) \in \Re^{p\times p}, \tag{14}$$

with the positive-definite block-weighting matrix

$$\boldsymbol{\Gamma}_\zeta = \begin{bmatrix} \boldsymbol{\Gamma}_x & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma}_\psi \end{bmatrix}, \ \boldsymbol{\Gamma}_\zeta \in \Re^{n+p\times n+p}$$

and $\boldsymbol{\Gamma}_x = \mathrm{diag}(\gamma_{x_1},\ldots,\gamma_{x_n})$, $\boldsymbol{\Gamma}_\psi = \mathrm{diag}(\gamma_{\psi_1},\ldots,\gamma_{\psi_p})$. The matrix $\mathbf{M}(t)$ is symmetric and at least positive semi-definite by definition. The modification dynamics $\dot{\hat{\boldsymbol{\theta}}}_\mu$ will be designed on the basis of the well known least-squares methodology. For this purpose let us define the following cost function

$$J = \frac{1}{2}\int_0^t \{[\boldsymbol{\zeta}^T(\tau) - \hat{\boldsymbol{\theta}}^T(t)\mathbf{N}^T(\tau)]\boldsymbol{\Gamma}_\zeta[\boldsymbol{\zeta}(\tau) - \mathbf{N}(\tau)\hat{\boldsymbol{\theta}}(t)]\}d\tau .$$

The estimates $\hat{\boldsymbol{\theta}}(t)$ should be updated so that the cost functional J maintains a minimum along the complete trajectory $\hat{\boldsymbol{\theta}}(t)$. This is equivalent to minimizing the weighted squared prediction error $\tilde{\boldsymbol{\zeta}}^T(t)\boldsymbol{\Gamma}_\zeta\tilde{\boldsymbol{\zeta}}(t)$. The mathematical description of the least-squares methodology is

$$\partial J / \partial\hat{\boldsymbol{\theta}}(t) = 0, \ \partial^2 J / \partial\hat{\boldsymbol{\theta}}^2(t) > 0 .$$

The partial derivative $\partial J / \partial\hat{\boldsymbol{\theta}}(t)$ can be evaluated as

$$\frac{\partial J}{\partial\hat{\boldsymbol{\theta}}(t)} = \int_0^t \mathbf{N}^T(\tau)\boldsymbol{\Gamma}_\zeta\mathbf{N}(\tau)d\tau\,\hat{\boldsymbol{\theta}}(t) - \int_0^t \mathbf{N}^T(\tau)\boldsymbol{\Gamma}_\zeta\boldsymbol{\zeta}(\tau)d\tau .$$

Let

$$\mathbf{P}^{-1}(t) = \int_0^t \mathbf{N}^T(\tau)\boldsymbol{\Gamma}_\zeta\mathbf{N}(\tau)d\tau . \tag{15}$$

Then, the extremum condition becomes

$$\mathbf{P}^{-1}(t)\hat{\boldsymbol{\theta}}(t) = \int_0^t \mathbf{N}^T(\tau)\boldsymbol{\Gamma}_\zeta\boldsymbol{\zeta}(\tau)d\tau . \tag{16}$$

The extremum can only be a minimum, because

$$\partial^2 J / \partial\hat{\boldsymbol{\theta}}^2(t) = \int_0^t \mathbf{N}^T(\tau)\boldsymbol{\Gamma}_\zeta\mathbf{N}(\tau)d\tau \geq 0 .$$

Now, differentiating (16) and considering that

$$\dot{\mathbf{P}}^{-1}(t) = \mathbf{N}^T(t)\boldsymbol{\Gamma}_\zeta\mathbf{N}(t) . \tag{17}$$

we obtain

$$\mathbf{P}^{-1}(t)\dot{\hat{\boldsymbol{\theta}}}(t) = \mathbf{N}^T\boldsymbol{\Gamma}_\zeta\boldsymbol{\zeta} - \mathbf{N}^T\boldsymbol{\Gamma}_\zeta\mathbf{N}\hat{\boldsymbol{\theta}} = \mathbf{N}^T\boldsymbol{\Gamma}_\zeta\tilde{\boldsymbol{\zeta}} .$$

The modification dynamics $\dot{\hat{\boldsymbol{\theta}}}_\mu$ is defined on the basis of the last equation as

$$\dot{\hat{\boldsymbol{\theta}}}_\mu = \mathbf{P}(t)\mathbf{N}^T\boldsymbol{\Gamma}_\zeta\tilde{\boldsymbol{\zeta}} = \mathbf{P}(t)\mathbf{M}(t)\tilde{\boldsymbol{\theta}} = \mathbf{P}(t)\boldsymbol{\mu}, \tag{18}$$

where (12) and (14) are considered and the vector

$$\boldsymbol{\mu} = \mathbf{N}^{\mathrm{T}}\boldsymbol{\Gamma}_{\zeta}\tilde{\boldsymbol{\zeta}} = \mathbf{M}(t)\tilde{\boldsymbol{\theta}}, \quad (19)$$

called the modifier is defined. The dynamics (18) is implementable, because $\tilde{\zeta}$ is available trough ς_x and $\tilde{\boldsymbol{\psi}}$. On the other hand, by using the property

$$\frac{d}{dt}[\mathbf{P}\mathbf{P}^{-1}] = \mathbf{0}$$

it can be shown that

$$\dot{\mathbf{P}} = -\mathbf{P}\mathbf{M}(t)\mathbf{P}. \quad (20)$$

With the results so far we are ready to state the next theorem, which describes the modification for exact estimation concept.

Theorem 1: Let the basic adaptive estimation law (2b) is modified to

$$\dot{\hat{\boldsymbol{\theta}}} = \dot{\hat{\boldsymbol{\theta}}}_{\beta} + \dot{\hat{\boldsymbol{\theta}}}_{\mu} = \mathbf{P}(t)(\boldsymbol{\beta} + \boldsymbol{\mu}), \quad (21)$$

with the modifier $\boldsymbol{\mu}$ defined as in (19) and the gain matrix $\mathbf{P}(t)$ dynamics described by (20), with $\mathbf{P}(0) = \mathrm{diag}(\gamma_{\theta_1}, \ldots, \gamma_{\theta_p}) > \mathbf{0}$. In addition let there exist a time instant $t_1 < \infty$ after which the accumulated data matrix $\mathbf{Q}(t)$ maintains full rank. Then the equilibrium $(\mathbf{z}, \tilde{\boldsymbol{\theta}}) = \mathbf{0}$ of the modified closed loop adaptive system

$$\dot{\mathbf{z}} = \mathbf{f}_z(\mathbf{z}, \hat{\boldsymbol{\theta}}, t) + \mathbf{G}_z(\mathbf{z}, \hat{\boldsymbol{\theta}}, t)\tilde{\boldsymbol{\theta}}, \quad (22a)$$

$$\dot{\tilde{\boldsymbol{\theta}}} = -\mathbf{P}(t)(\boldsymbol{\beta} + \mathbf{M}(t)\tilde{\boldsymbol{\theta}}), \quad (22b)$$

is globally uniformly asymptotically stable, i.e.

$$\lim_{t\to\infty} \mathbf{z}(t) = \mathbf{0}, \ \lim_{t\to\infty} \tilde{\boldsymbol{\theta}}(t) = \mathbf{0}. \quad (23)$$

and the exact estimation of the unknown parameters is guaranteed. □

Proof: The basic Lyapunov function candidate (4) can be rewritten with $\boldsymbol{\Gamma}_{\theta} = \mathbf{P}(t)$ as

$$V(\mathbf{z}, \tilde{\boldsymbol{\theta}}) = (1/2)\mathbf{z}^{\mathrm{T}}\mathbf{z} + (1/2)\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathbf{P}^{-1}(t)\tilde{\boldsymbol{\theta}}, \quad (24)$$

Considering (5) and the time-varying nature of $\mathbf{P}(t)$, along with the fact that the symmetry of $\mathbf{P}^{-1}(0)$ implies that $\mathbf{P}^{-1}(t)$ is symmetric for all t, the total derivative of (24) with respect to the modified closed loop adaptive system dynamics (22) will be

$$\dot{V}(\mathbf{z}, \tilde{\boldsymbol{\theta}}) = -\mathbf{z}^{\mathrm{T}}\mathbf{C}\mathbf{z} + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}(\boldsymbol{\beta} - \mathbf{P}^{-1}(t)\dot{\hat{\boldsymbol{\theta}}}) + (1/2)\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\dot{\mathbf{P}}^{-1}(t)\tilde{\boldsymbol{\theta}}.$$

The last expression is transformed into

$$\dot{V}(\mathbf{z}, \tilde{\boldsymbol{\theta}}) = -\mathbf{z}^{\mathrm{T}}\mathbf{C}\,\mathbf{z} - (1/2)\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\mathbf{M}(t)\tilde{\boldsymbol{\theta}} \le 0 \quad (25)$$

after taking into account (17), and the modified parameter estimator dynamics (21). The negative semi-definiteness of (25) proves the global uniform Lyapunov stability of the equilibrium $(\mathbf{z}, \tilde{\boldsymbol{\theta}}) = \mathbf{0}$. The convergence statements (23) follow from the assumption that $\mathrm{rank}\,\mathbf{Q}(t) = p$, $\forall t \ge t_1$. Indeed, the full rank of $\mathbf{Q}(t)$ implies that the matrix $\mathbf{N}(t)$, defined by (13), also has full rank and hence, the matrix $\mathbf{M}(t) = \mathbf{N}^{\mathrm{T}}\boldsymbol{\Gamma}_{\zeta}\mathbf{N}$ is positive-definite $\forall t \ge t_1$. Then the derivative (25) can be represented as

$$\dot{V}(\mathbf{z}, \tilde{\boldsymbol{\theta}}) \le -c_0 |\mathbf{z}|^2 - \frac{m_0}{2}|\tilde{\boldsymbol{\theta}}|^2 = W(\mathbf{z}, \tilde{\boldsymbol{\theta}}) < 0, \ \forall t \ge t_1,$$

where $c_0 = \lambda_{\min}(\mathbf{C})$ and $m_0 = \lambda_{\min}(\mathbf{M}(t))$ $\forall t \ge t_1$, and according to the LaSalle-Yoshizawa theorem [4], the modified closed loop adaptive system (22) will converge to the invariant set, where

$$\lim_{t\to\infty} W(\mathbf{z}(t), \tilde{\boldsymbol{\theta}}(t)) = 0.$$

Hence, it follows that (23) holds, which completes the proof. □

Remark: The standard techniques for exact estimation are a special case of the proposed approach, with $\mathbf{M}(t) = \mathbf{W}_x^{\mathrm{T}}\mathbf{W}_x$, and require that the matrix $\mathbf{W}_x(t)$ is "persistently exciting". This can only be achieved by including excitation terms in the control or the reference trajectories and practically results in positive-definiteness of $\mathbf{M}(t)$. However, the proposed approach does not need such stringent excitation requirements, because the positive-definiteness of $\mathbf{M}(t)$ is controlled with the help of the accumulated data matrix $\mathbf{Q}(t)$. Strictly speaking, in order for $\mathbf{Q}(t)$ to converge to a full rank matrix the input $\mathbf{W}_x^{\mathrm{T}}\mathbf{R}_x\mathbf{W}_x$ in the dynamics (8) has to be "sufficiently exciting". This excitation requirement however is greatly relaxed in comparison with the standard ones, and can be satisfied just after one transient response, as it is illustrated later with the simulation example.

Application of the Approach

The proposed approach is applied to a current-fed DC motor driven inverted pendulum nonlinear system, whose dynamic description is

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\theta_1 \sin x_1 - \theta_2 x_2 + \theta_3 u \end{aligned} \quad (26a)$$

$$y = x_1, \quad (26b)$$

where x_1 is the pendulum angular position [rad], x_2 is the angular velocity [rad/s], u is the motor armature current [A], and

$$\theta_1 = \frac{mgl}{(J + ml^2)}, \ \theta_2 = \frac{b}{(J + ml^2)}, \ \theta_3 = \frac{K_m}{(J + ml^2)}$$

are the unknown system parameters. The control and the basic parameter estimation laws (2) are designed via the adaptive backstepping approach, resulting in

$$u = \hat{\delta}_2 \alpha_2, \tag{27}$$

$$\alpha_2 = \ddot{y}_d + c_1(\dot{y}_d - x_2) + z_1 - c_2 z_2 + \hat{\theta}_1 \sin x_1 + \hat{\theta}_2 x_2,$$

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}_\theta \boldsymbol{\beta}(\mathbf{z}, \mathbf{x}), \ \dot{\hat{\delta}}_2 = -\gamma_{\delta_2} \operatorname{sign}(\theta_3) \alpha_2 z_2,$$

where $z_1 = y_d - x_1$, $z_2 = -c_1 z_1 + x_2 - \dot{y}_d$, and $\boldsymbol{\beta} = [-z_2 \sin x_1, -z_2 x_2, 0]^T$, $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3]^T$. Here $\hat{\delta}_2$ is an estimate of the unknown parameter $\delta_2 = 1/\theta_3$. Apparently, the control system does not estimate the unknown parameter θ_3. The basic closed loop system error dynamics reads

$$\dot{z}_1 = -c_1 z_1 - z_2, \tag{28a}$$

$$\dot{z}_2 = z_1 - c_2 z_2 - \tilde{\theta}_1 \sin x_1 - \tilde{\theta}_2 x_2 - \tilde{\delta}_2 \theta_3 \alpha_2. \tag{28b}$$

A Lyapunov function for the closed loop system is

$$V(\mathbf{z}, \tilde{\boldsymbol{\theta}}, \tilde{\delta}_2) = \frac{1}{2}\mathbf{z}^T \mathbf{z} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma}_\theta^{-1} \tilde{\boldsymbol{\theta}} + \frac{|\theta_3|}{2\gamma_{\delta_2}} \tilde{\delta}_2^2,$$

which is of the form (4), with $\mathbf{z} = [z_1, z_2]^T$. The total derivative of this function with respect to the closed loop system error and basic parameter estimator dynamics is

$$\dot{V}(\mathbf{z}) = -\mathbf{z}^T \mathbf{C} \mathbf{z}.$$

The above derivative is negative semi-definite with respect to the estimation error only and as a result exact estimation cannot be achieved.

The objective system (26) and the closed loop system (28) have to be presented in the general form (1)–(3), in order to apply the proposed methodology for exact parameter estimation. This is accomplished by considering the relations $u = \hat{\delta}_2 \alpha_2$, $\tilde{\delta}_2 = \delta_2 - \hat{\delta}_2$, $\theta_3 = \tilde{\theta}_3 + \hat{\theta}_3$, into the basic closed loop error dynamics (28) which gives the vector-functions

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} x_2 \\ 0 \end{bmatrix}; \ \mathbf{f}_z = \begin{bmatrix} -c_1 z_1 - z_2 \\ z_1 - c_2 z_2 + \alpha_2(\hat{\theta}_3 \hat{\delta}_2 - 1) \end{bmatrix};$$

$$\mathbf{G}(\mathbf{x}, \mathbf{u}) = \mathbf{G}_z = \begin{bmatrix} 0 & 0 & 0 \\ -\sin x_1 & -x_2 & u \end{bmatrix}.$$

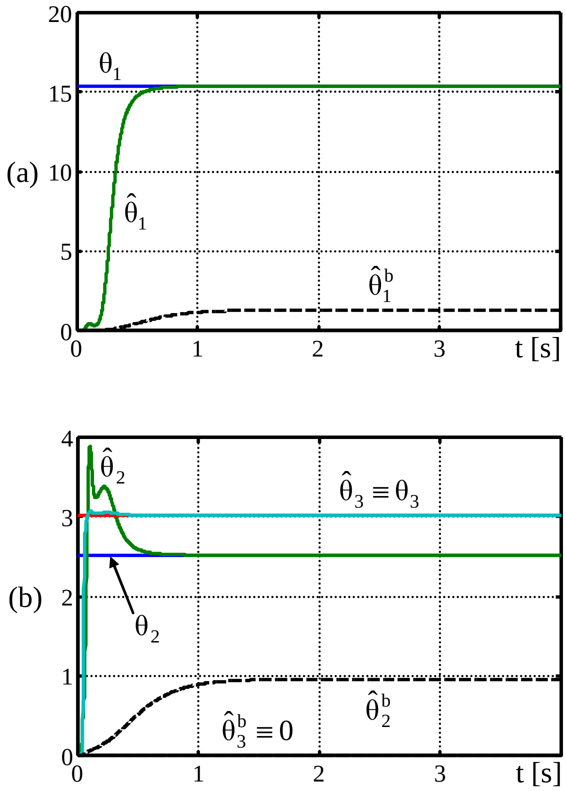
Thus, the modified closed loop control system consists of the control law (27), estimation law (21), gain dynamics (20) and filters (6), (8), (9) and (10).

Simulation and System Time Responses

The physical parameters used in the simulations are the pendulum mass $m = 0.5$ kg, pendulum length $l = 0.5$ m, moment of inertia $J = 0.0341$ $\mathrm{kg\,m^2}$, torque constant $K_m = 0.48$ $\mathrm{Nm/A}$, viscous friction coefficient $b = 0.4$ Nms and gravity constant $g = 9.81$ $\mathrm{m/s^2}$. The simulation is performed from zero initial conditions with design matrices

$\boldsymbol{\Lambda}_x = \operatorname{diag}(-10, -10)$, $\boldsymbol{\Lambda}_r = \operatorname{diag}(-10, -10, -10)$,
$\mathbf{K}_x = \operatorname{diag}(2, 2)$, $\mathbf{R}_x = \boldsymbol{\Gamma}_x = \operatorname{diag}(10, 10)$, $\gamma_{\delta_2} = 0.1$,
$\mathbf{C} = \operatorname{diag}(30, 30)$, $\mathbf{P}(0) = \operatorname{diag}(500, 100, 10)$,
$\boldsymbol{\Gamma}_\psi = \operatorname{diag}(100, 100, 10)$, $\mathbf{Q}_r = \operatorname{diag}(30, 20, 10)$.

The closed loop system time responses are shown in figures 1 and 2. The reference trajectory is generated via a second order linear reference model with double pole $\lambda = -3$. The desired pendulum positioning angle is chosen to be $v = \pi$ rad. This set point is the worst case scenario for the parameter estimator, because at this angular position all system signals are identically zero and thus all data channels are closed. Nevertheless, all original system model parameters are exactly estimated in one transient response only. In contrast, the closed loop adaptive system without the modification does not estimate



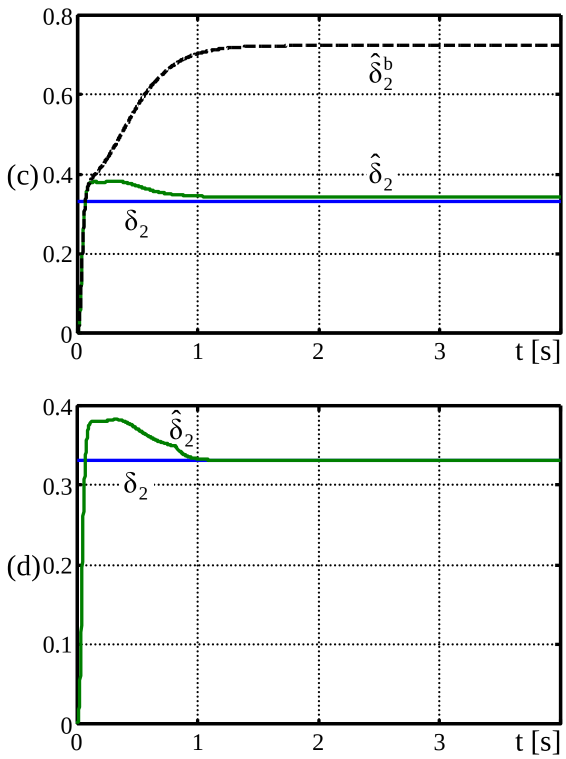


Figure 1: Parameter estimates responses (modified estimates —, unmodified estimates ---)

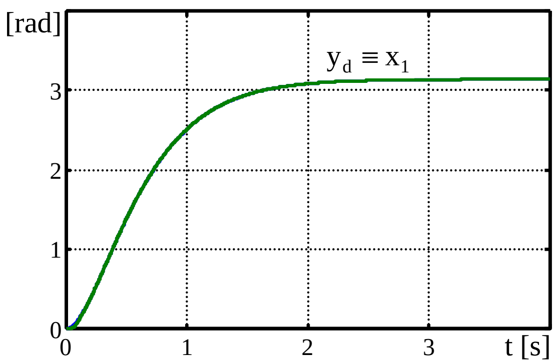


Figure 2: Trajectory tracking response

exactly any of the unknown parameters, seen by the unmodified estimates $\hat{\theta}_1^b$, $\hat{\theta}_2^b$, $\hat{\theta}_3^b$, $\hat{\delta}_2^b$. Hence, the proposed approach greatly improves the adaptive system performance. This is due to the fact that the matrix $\mathbf{Q}(t)$ maintains full rank as required by theorem 1, and as a result $\mathbf{M}(t)$ is a positive definite matrix. The only inaccurately estimated parameter is δ_2. This is because δ_2 is not a system model parameter and therefore it is not included for exact estimation by the proposed approach. Moreover, the steady state equation for $\tilde{\delta}_2$

$$0 = -\tilde{\delta}_2 \theta_1 \theta_3 \sin x_1$$

is satisfied for both $\tilde{\delta}_2 = 0$, $x_1 \neq \pm k\pi$ or $\tilde{\delta}_2 \neq 0$, $x_1 = \pm k\pi$, $k = 0,1,2,\ldots$. Therefore, in steady state δ_2 may not be exactly estimated when $x_1 = \pm k\pi$, but is guaranteed to be exactly estimated when $x_1 \neq \pm k\pi$. A possibility to estimate δ_2 exactly is to use the fact that the true value of δ_2 can be computed after the $\hat{\theta}_3$ transient response. Then the error $\tilde{\delta}_2$ is known and the δ_2 parameter estimator can be modified into

$$\dot{\hat{\delta}}_2 = -\gamma_{\delta_2} \operatorname{sign}(\theta_3) \alpha_2 z_2 + c_{\delta_2} (1/\hat{\theta}_3 - \hat{\delta}_2),\ c_{\delta_2} > 0 .$$

The result is given on figure 1d. This modification provides exact estimation of the parameter δ_2, regardless of the x_1 steady state value. In this way, all control system parameters are exactly estimated.

Conclusions

The paper has presented a new general approach for exact unknown parameter estimation in nonlinear adaptive control systems. The approach presumes that a known nonlinear adaptive control design method is applied for the objective nonlinear system. Then, the basic adaptive control system is modified and the modification is based on a generalization of the prediction error concept and introducing the concept of the stable data accumulation. This results in an asymptotically stable closed loop adaptive system with respect to both the tracking and the parameter estimation errors $\mathbf{z}(t)$ and $\tilde{\boldsymbol{\theta}}(t)$. The asymptotic stability is provided by control of the data accumulation dynamics to achieve full rank of the matrices $\mathbf{Q}(t)$ and $\mathbf{M}(t)$. The unmodified adaptive system cannot estimate the unknown parameters exactly. The major advantage of the new approach proposed is that exact parameter estimation is achieved by the closed loop adaptive system in one transient response only, without imposing the standard excitation techniques, even for systems that do not generate enough information naturally.

Acknowledgement

The support of Research Fund Project No. 132PD0013-19 from NIS in Technical University Sofia for this research work is gratefully acknowledged.

References

[1] Adetola, V., Guay, M., "Performance Improvement in Adaptive Control of Linearly Parameterized Nonlinear Systems", *IEEE Transactions on Automatic Control*, Vol. 55, No. 9, 2010, pp. 2182–2186.

[2] Adetola, V., Guay, M., "Finite-Time Parameter Estimation in Adaptive Control of Nonlinear Systems", *IEEE Transactions on Automatic Control*, Vol. 53, No. 3, 2008, pp. 807–811.
[3] Ciliz, M., "Combined Direct and Indirect Adaptive Control for a Class of Nonlinear Systems", *IET Control Theory & Applications*, Vol. 3, No. 1, 2009, pp. 151–159.
[4] Krstic, M., Kanellakopoulos, I., Kokotovic, P., "Nonlinear and Adaptive Control Design", *John Wiley and Sons Inc.*, 1995.
[5] Lin, J.-S., Kanellakopoulos, I., "Nonlinearities Enhance Parameter Convergence in Strict Feedback Systems", *IEEE Transactions on Automatic Control*, Vol. 44, No. 1, 1999, pp. 89–94
[6] Mishkov, R., Darmonski, S., "Adaptive Nonlinear Trajectory Tracking Control for DC Motor Driven Inverted Pendulum", *International Conference Automatics and Informatics'11*, 2011, pp. B-67–B-70.
[7] Mishkov, R., Darmonski, S., "Adaptive Tuning Functions System Design for Inverted Pendulum", *International Conference Engineering*, *Technologies and Systems TechSys*, Vol. 16, book 1, 2011, pp. 329–334.
[8] Na, J., Herrmann, G., Ren, X., Mahyuddin, M., Barber, P., "Robust Adaptive Finite-Time Parameter Estimation and Control of Nonlinear Systems", *2011 IEEE International Symposium on Intelligent Control (ISIC)*, 2011, pp. 1014–1019.