NONLINEAR ADAPTIVE CONTROL AND EXACT PARAMETER ESTIMATION

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Abstract: The paper presents a new general approach for exact unknown parameter estimation in nonlinear adaptive control systems based on generalization of the prediction error concept and introducing the concept of stable data accumulation. The closed-loop adaptive system is asymptotically stable with respect to the tracking and parameter estimation errors. This is achieved by controlling the rank of the data accumulation matrices. The advantage of the new approach is the exact parameter estimation achieved in one transient response without standard excitation techniques. The approach is applied to a DC motor driven inverted pendulum for illustration.

Key words: nonlinear systems, adaptive control, exact parameter estimation, Lyapunov stability.

INTRODUCTION

The exact unknown parameter estimation in nonlinear adaptive control is an attractive problem, which does not have a general solution yet. The standard techniques for achieving exact estimation rely on sufficiently rich reference trajectories or persistent excitation of the control system and the recent results in this field [1, 2, 3, 7] are not an exception. These requirements are in contradiction with the control goal determined by technological considerations and are therefore irrelevant. This paper considers the exact unknown parameter estimation problem in nonlinear adaptive control without imposing any excitation or special trajectory requirements on the controlled nonlinear dynamics. The attention is focused on constructing a sufficiently rich information process, embedded in the adaptive control, by defining and manipulating different information channels. The information channel is a system variable which is indirectly connected with the unknown parameter estimation error. The main information channel is the well known prediction error and the paper provides a generalization of the prediction error concept. The data accumulation concept is introduced on the basis of this error which is the main tool for achieving exact unknown parameter estimation without persistency of excitation. The idea is to accumulate the estimation error information, coming from various information channels in a controlled manner, and as a result to construct a full rank transform matrix, between the unknown parameter vector and a known, suitably defined mapping vector. The data accumulation concept considers some results in [2]. The introduced generalization of the prediction error concept is based on the nonlinear swapping techniques found in [4]. The plant model and control design are taken from [5, 6]. As a result, exact unknown parameter estimation in one transient response without persistent excitation is achieved and illustrated by the simulation example.

PROBLEM STATEMENT

The multi-variable nonlinear systems considered are

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\mathbf{u}) + \mathbf{G}(\mathbf{x},\mathbf{u})\boldsymbol{\theta}, \tag{1a}$$

$$\mathbf{y} = \mathbf{h}(\mathbf{x}), \tag{1b}$$

where $\mathbf{x} \in \Re^n$, $\mathbf{u} \in \Re^r$, $\mathbf{y} \in \Re^m$, $\boldsymbol{\theta} \in \Re^p$ are the state, control, output, unknown parameters vectors, and $\mathbf{f}:\Re^n \times \Re^r \to \Re^n$, $\mathbf{G}:\Re^n \times \Re^r \to \Re^{n\times p}$, $\mathbf{h}:\Re^n \to \Re^m$ are known nonlinear mappings.

Assumption 1: The main control task is the tracking of a reference trajectory $\mathbf{y}_d(t)$ whose time derivatives are bounded. It is further assumed that a certain nonlinear adaptive control design approach has been applied to obtain the bounded adaptive control law and a basic dynamic estimation law

$$\mathbf{u}(t) = \boldsymbol{\alpha}(\mathbf{x},\hat{\boldsymbol{\theta}},\dot{\hat{\boldsymbol{\theta}}},t), \tag{2a}$$

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}_\theta \boldsymbol{\beta}(\mathbf{z},\mathbf{x},\hat{\boldsymbol{\theta}},t). \tag{2b}$$

The closed-loop nonlinear adaptive system dynamics in error coordinates $\mathbf{z} \in \Re^n$ is of the form

$$\dot{\mathbf{z}} = \mathbf{f}_z(\mathbf{z},\hat{\boldsymbol{\theta}},t) + \mathbf{G}_z(\mathbf{z},\hat{\boldsymbol{\theta}},t)\tilde{\boldsymbol{\theta}}, \tag{3a}$$

$$\dot{\tilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}_\theta \boldsymbol{\beta}(\mathbf{z},\mathbf{x},\hat{\boldsymbol{\theta}},t). \tag{3b}$$

The vector functions $\mathbf{f}_z \in \Re^{n\times 1}$, $\mathbf{G}_z \in \Re^{n\times p}$, and $\boldsymbol{\beta} \in \Re^{p\times 1}$ are locally Lipschitz with respect to $\mathbf{z}$, $\tilde{\boldsymbol{\theta}}$, uniformly in t, with the property $\mathbf{f}_z(\mathbf{0},\boldsymbol{\theta},t) = \mathbf{0}$, $\boldsymbol{\beta}(\mathbf{0},\mathbf{x},\hat{\boldsymbol{\theta}},t) = \mathbf{0}$. Moreover, there exists a known Lyapunov function of the form

$$V(\mathbf{z},\tilde{\boldsymbol{\theta}}) = \mathbf{z}^T\mathbf{z} + \tilde{\boldsymbol{\theta}}^T\boldsymbol{\Gamma}_\theta^{-1}\tilde{\boldsymbol{\theta}}, \tag{4}$$

whose total time derivative, with respect to the closed-loop adaptive system dynamics (3) is

$$\dot{V}(\mathbf{z},\tilde{\boldsymbol{\theta}}) = -\mathbf{z}^T\mathbf{C}\mathbf{z} + \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\beta} - \boldsymbol{\Gamma}_\theta^{-1}\dot{\hat{\boldsymbol{\theta}}}) = -\mathbf{z}^T\mathbf{C}\mathbf{z}, \tag{5}$$

where $\mathbf{C} = \mathrm{diag}(c_1, c_2, \ldots, c_n)$, $\boldsymbol{\Gamma}_\theta = \mathrm{diag}(\gamma_{\theta_1}, \gamma_{\theta_2}, \ldots, \gamma_{\theta_p})$ are positive definite design matrices, $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ is the unknown parameter estimation error. □

Assumption 1 ensures the global stability of the closed loop adaptive system (3), the asymptotic stability of $\mathbf{z}(t)$, and the Lyapunov stability of the parameter estimation error $\tilde{\boldsymbol{\theta}}(t)$. This is due to the negative semi-definiteness of the derivative $\dot{V}(\mathbf{z},\tilde{\boldsymbol{\theta}})$, which is not explicitly dependent on $\tilde{\boldsymbol{\theta}}(t)$. Hence, exact estimation of the unknown parameters cannot be provided via such an approach. Moreover, most of the existing methods for nonlinear adaptive control deliberately eliminate the explicit dependence of the total derivative $\dot{V}$ on $\tilde{\boldsymbol{\theta}}(t)$ via the adaptive estimation law. Negative definiteness of $\dot{V}$ on $\tilde{\boldsymbol{\theta}}$ has to be provided in order to achieve exact parameter estimation. This can be accomplished if the following holds

$\boldsymbol{\mu} = \mathbf{M}\tilde{\boldsymbol{\theta}} = -(\boldsymbol{\beta} - \boldsymbol{\Gamma}_\theta^{-1}\dot{\hat{\boldsymbol{\theta}}})$,

where $\boldsymbol{\mu}$ is the new key vector, which will modify the basic adaptive estimation law dynamics. Then, for the closed loop adaptive system with the modified adaptive estimator

$$\dot{\mathbf{z}} = \mathbf{f}_z(\mathbf{z}, \hat{\boldsymbol{\theta}}, t) + \mathbf{G}_z(\mathbf{z}, \hat{\boldsymbol{\theta}}, t)\tilde{\boldsymbol{\theta}} \tag{6a}$$

$$\dot{\tilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}_\theta(\boldsymbol{\beta} + \boldsymbol{\mu}) \tag{6b}$$

the Lyapunov function derivative will be

$$\dot{V}(\mathbf{z}, \tilde{\boldsymbol{\theta}}) = -\mathbf{z}^T\mathbf{C}\mathbf{z} - \tilde{\boldsymbol{\theta}}^T\mathbf{M}\tilde{\boldsymbol{\theta}}. \tag{7}$$

If the matrix $\mathbf{M} \in \Re^{p\times p}$ is positive definite, then the derivative $\dot{V}(\mathbf{z}, \tilde{\boldsymbol{\theta}})$ will be negative definite and the error $\tilde{\boldsymbol{\theta}}(t)$ will converge to zero according to the LaSalle-Yoshizawa theorem [4], i.e. exact unknown parameter estimation will be guaranteed. In this sense, the exact estimation problem reduces to the problem of designing the modifier $\boldsymbol{\mu}$ in (6) to be independent on $\tilde{\boldsymbol{\theta}}(t)$ and make the matrix $\mathbf{M}$ positive definite. The major advantage of this approach is that exact parameter estimation can be achieved in one transient response without imposing the standard excitation techniques, even for systems that do not generate enough information naturally. This is made possible by generalization of the prediction error concept and introducing the concept of the stable data accumulation.

PREDICTION ERROR IN x COORDINATES

The task for construction of an algebraic connection between the system trajectories $\mathbf{x}(t)$ and the unknown parameter vector $\boldsymbol{\theta}$ is central in the prediction error concept. Based on this algebraic connection, an estimate $\hat{\mathbf{x}}$ depending on the estimated parameters $\hat{\boldsymbol{\theta}}$, can be defined. The x coordinates error $\mathbf{e}_x = \mathbf{x} - \hat{\mathbf{x}}$ is called prediction error and is used in the adaptive process. This error provides indirect information about the estimation error and can be used for modifying the basic adaptive estimation law dynamics. The following lemma presents a generalization of the prediction error concept for nonlinear systems of the form (1).

Lemma 1: Let the state estimate vector $\hat{\mathbf{x}} \in \Re^{n\times 1}$, the matrix $\mathbf{W}_x \in \Re^{n\times p}$ and the signal $\tilde{\boldsymbol{\varepsilon}} \in \Re^{n\times 1}$ are described by the vector-matrix differential equations

$$\dot{\hat{\mathbf{x}}} = -\boldsymbol{\Lambda}_x(\mathbf{x} - \hat{\mathbf{x}}) + \mathbf{f} + \mathbf{G}\hat{\boldsymbol{\theta}} + \mathbf{K}_x^{-1}\mathbf{W}_x\dot{\hat{\boldsymbol{\theta}}},$$

$$\dot{\mathbf{W}}_x = \boldsymbol{\Lambda}_x\mathbf{W}_x + \mathbf{K}_x\mathbf{G}, \ \mathbf{W}_x(0) = \mathbf{0},$$

$$\dot{\tilde{\boldsymbol{\varepsilon}}} = \boldsymbol{\Lambda}_x\tilde{\boldsymbol{\varepsilon}}, \ \tilde{\boldsymbol{\varepsilon}}(0) = \mathbf{e}_x(0),$$

composed in correspondence with the original nonlinear system dynamics (1), where $\mathbf{K}_x = \mathrm{diag}(k_{x_1}, k_{x_2}, \ldots, k_{x_n})$ and $\boldsymbol{\Lambda}_x = \mathrm{diag}(\lambda_{x_1}, \lambda_{x_2}, \ldots, \lambda_{x_n})$ are positive definite and negative definite design matrices, respectively. Then, all signals are globally bounded and the connection $\mathbf{W}_x\tilde{\boldsymbol{\theta}} = \varsigma_x$ with $\varsigma_x = \mathbf{K}_x(\mathbf{e}_x - \tilde{\boldsymbol{\varepsilon}})$ holds. □

The generalization of the prediction error concept in lemma 1 is achieved by a special form of the nonlinear adaptive observer, introduction of the gain matrix $\mathbf{K}_x$, and considering the asymptotic stability of $\tilde{\boldsymbol{\varepsilon}}$.

DATA ACCUMULATION CONCEPT

The main idea behind the data accumulation concept is the dynamic construction of a coordinate transformation between the $\boldsymbol{\theta}$ parameter space and the space spanned by a suitably defined vector $\boldsymbol{\psi}(t) \in \Re^{p\times 1}$. The vector $\boldsymbol{\psi}(t)$ is a mapping of the original unknown vector $\boldsymbol{\theta}$ in new coordinates defined as

$$\boldsymbol{\psi}(t) = \mathbf{Q}(t)\boldsymbol{\theta}, \tag{8}$$

where the matrix $\mathbf{Q}(t) \in \Re^{p\times p}$ is the coordinate transformation. If $\mathbf{Q}(t)$ has full rank then the transformation (8) is a diffeomorphism. The dynamics of $\mathbf{Q}(t)$ is chosen to be

$$\dot{\mathbf{Q}} = -\boldsymbol{\Lambda}_r(\mathbf{Q}_r - \mathbf{Q})\mathbf{W}_x^T\mathbf{R}_x\mathbf{W}_x, \ \mathbf{Q}(0) = \mathbf{0}. \tag{9}$$

The key idea behind the definition (9) is to control the rank of the transformation matrix $\mathbf{Q}(t)$ via the reference design matrix $\mathbf{Q}_r$. The dynamics (9) can be interpreted as a stable data accumulation process, with the accumulated information matrix $\mathbf{Q}(t)$. It can be shown that if the input $\mathbf{W}_x^T\mathbf{R}_x\mathbf{W}_x$ has full rank for a sufficient time period, then the matrix $\mathbf{Q}(t)$ will converge to its reference $\mathbf{Q}_r$. The convergence to $\mathbf{Q}_r$ is not necessary for achieving exact parameter estimation, but the full rank of the data matrix $\mathbf{Q}(t)$ is sufficient to guarantee exact parameter estimation. In this sense, the introduction of the reference matrix $\mathbf{Q}_r$ in the dynamics (9) provides a way of controlling the rank of $\mathbf{Q}(t)$, and on the other hand stabilizes the data accumulation process and ensures that $\mathbf{Q}(t)$ will remain bounded. Now, differentiating (8), considering the relation $\boldsymbol{\theta} = \tilde{\boldsymbol{\theta}} + \hat{\boldsymbol{\theta}}$ and lemma 1, we obtain the dynamics

$$\dot{\boldsymbol{\psi}} = -\boldsymbol{\Lambda}_r(\mathbf{Q}_r - \mathbf{Q})\mathbf{W}_x^T\mathbf{R}_x(\varsigma_x + \mathbf{W}_x\hat{\boldsymbol{\theta}}), \tag{10}$$

which is completely computable. Rewriting (8) with respect to the estimates gives

$$\hat{\boldsymbol{\psi}}(t) = \mathbf{Q}(t)\hat{\boldsymbol{\theta}} \tag{11}$$

and the estimation error

$$\tilde{\boldsymbol{\psi}}(t) = \boldsymbol{\psi}(t) - \hat{\boldsymbol{\psi}}(t) = \mathbf{Q}(t)\tilde{\boldsymbol{\theta}} \tag{12}$$

becomes known. If the basic adaptive estimation law is modified to guarantee convergence of the error $\tilde{\boldsymbol{\psi}}(t)$ to zero and the data matrix $\mathbf{Q}(t)$ has full rank, then according to (12) the estimation error $\tilde{\boldsymbol{\theta}}(t)$ will converge to zero. Thus, the data accumulation concept is summarized in the following lemma.

Lemma 2: Let the signals $\mathbf{Q} \in \Re^{p\times p}$, $\boldsymbol{\psi} \in \Re^{p\times 1}$, and $\hat{\boldsymbol{\psi}} \in \Re^{p\times 1}$ are described by the following vector-matrix equations

$$\dot{\mathbf{Q}} = -\boldsymbol{\Lambda}_r(\mathbf{Q}_r - \mathbf{Q})\mathbf{W}_x^T\mathbf{R}_x\mathbf{W}_x, \ \mathbf{Q}(0) = \mathbf{0},$$

$$\dot{\boldsymbol{\psi}} = -\boldsymbol{\Lambda}_r(\mathbf{Q}_r - \mathbf{Q})\mathbf{W}_x^T\mathbf{R}_x(\varsigma_x + \mathbf{W}_x\hat{\boldsymbol{\theta}}), \ \boldsymbol{\psi}(0) = \mathbf{0},$$

$$\hat{\boldsymbol{\psi}} = \mathbf{Q}\hat{\boldsymbol{\theta}},$$

composed in correspondence with lemma 1, where $\mathbf{R}_x = \mathrm{diag}(r_{x_1}, r_{x_2}, \ldots, r_{x_n})$, $\boldsymbol{\Lambda}_r = \mathrm{diag}(\lambda_{r_1}, \lambda_{r_2}, \ldots, \lambda_{r_p})$ are positive and negative definite design matrices, respectively, and $\mathbf{Q}_r$ is a constant reference matrix with full rank. All signals are globally bounded and the connection $\mathbf{Q}\tilde{\boldsymbol{\theta}} = \tilde{\boldsymbol{\psi}}$ holds, where $\tilde{\boldsymbol{\psi}} = \boldsymbol{\psi} - \hat{\boldsymbol{\psi}}$ is the mapping error. □

MODIFICATION FOR EXACT ESTIMATION

The modification for exact estimation is based on the results in lemmas 1 and 2 and is summarized in the following theorem.

Theorem 1: Let the basic adaptive estimation law (2b) is modified into

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}_\theta(\boldsymbol{\beta} + \boldsymbol{\mu}), \tag{13}$$

with the modifier $\boldsymbol{\mu}$ defined as

$$\boldsymbol{\mu} = \mathbf{M}\tilde{\boldsymbol{\theta}} = \mathbf{W}_x^T\boldsymbol{\Gamma}_x\varsigma_x + \mathbf{Q}^T\boldsymbol{\Gamma}_\psi\tilde{\boldsymbol{\psi}}, \tag{14a}$$

$$\mathbf{\Gamma}_x = \mathrm{diag}(\gamma_{x_1},\dots,\gamma_{x_n}),\ \mathbf{\Gamma}_\psi = \mathrm{diag}(\gamma_{\psi_1},\dots,\gamma_{\psi_p}), \tag{14b}$$

being positive definite weighting matrices. Then the following asymptotic properties hold

$$\lim_{t\to\infty}\mathbf{z}(t)=\mathbf{0},\ \lim_{t\to\infty}\varsigma_x(t)=\mathbf{0},\ \lim_{t\to\infty}\tilde{\boldsymbol{\psi}}(t)=\mathbf{0}. \tag{15}$$

Moreover, when the accumulated data matrix $\mathbf{Q}(t)$ has full rank the symmetric matrix

$$\mathbf{M} = \mathbf{W}_x^T\mathbf{\Gamma}_x\mathbf{W}_x + \mathbf{Q}^T\mathbf{\Gamma}_\psi\mathbf{Q}, \tag{16}$$

also has full rank and it is positive definite. Then, the estimation error will be asymptotically stable or $\lim_{t\to\infty}\tilde{\boldsymbol{\theta}}(t)=\mathbf{0}$. □

Proof: The total derivative $\dot{V}(\mathbf{z},\tilde{\boldsymbol{\theta}})$, shown in (7), reads

$$\dot{V}(\mathbf{z},\tilde{\boldsymbol{\theta}}) = -\mathbf{z}^T\mathbf{C}\mathbf{z} - \tilde{\boldsymbol{\theta}}^T\mathbf{M}\tilde{\boldsymbol{\theta}}.$$

It can be expressed also as $\dot{V}(\mathbf{z},\varsigma_x,\tilde{\boldsymbol{\psi}})$ by considering the relations (16), $\mathbf{W}_x\tilde{\boldsymbol{\theta}}=\varsigma_x$, and $\mathbf{Q}\tilde{\boldsymbol{\theta}}=\tilde{\boldsymbol{\psi}}$ as follows

$$\dot{V}(\mathbf{z},\varsigma_x,\tilde{\boldsymbol{\psi}}) = -\mathbf{z}^T\mathbf{C}\mathbf{z} - \varsigma_x^T\mathbf{\Gamma}_x\varsigma_x - \tilde{\boldsymbol{\psi}}^T\mathbf{\Gamma}_\psi\tilde{\boldsymbol{\psi}}.$$

This derivative is obviously negative definite, which proves the asymptotic stability of $\mathbf{z}(t)$, $\varsigma_x(t)$, and $\tilde{\boldsymbol{\psi}}(t)$. In the general case, the matrix $\mathbf{M} = \mathbf{W}_x^T\mathbf{\Gamma}_x\mathbf{W}_x + \mathbf{Q}^T\mathbf{\Gamma}_\psi\mathbf{Q}$ is positive semidefinite, because the first term is positive semidefinite and $\mathbf{Q}$ does not have full rank, which implies only Lyapunov stability with respect to $\tilde{\boldsymbol{\theta}}$. However, in most of the nonlinear original systems, it is possible to achieve full rank of $\mathbf{Q}$ via its dynamic control, according to (9). If this is achieved, then the major result obtained is that the matrix $\mathbf{M}$ becomes positive definite and the derivative $\dot{V}(\mathbf{z},\tilde{\boldsymbol{\theta}})$ yields negative definiteness with respect to $\tilde{\boldsymbol{\theta}}$ also. This provides asymptotic stability of $\tilde{\boldsymbol{\theta}}$, i.e. exact parameter estimation. □

APPLICATION OF THE APPROACH

The proposed approach is applied to a DC motor driven inverted pendulum nonlinear system. The general original model (1) is described in this case by the equations

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\theta_1\sin x_1 - \theta_2 x_2 + \theta_3 u \tag{17a}$$

$$y = x_1 \tag{17b}$$

where x_1 is the pendulum position $[\mathrm{rad}]$, x_2 is the angular velocity $[\mathrm{rad/s}]$, u is the motor armature current $[\mathrm{A}]$ and $\theta_1 = mgl/(J+ml^2)$, $\theta_2 = b/(J+ml^2)$, $\theta_3 = K_m/(J+ml^2)$ are the unknown system parameters. Equations (17) describe the current-fed dynamics of the DC motor driven inverted pendulum, with the motor armature current as the control input. The adaptive backstepping approach is applied for the design of the control and the basic parameter estimation laws (2), resulting in the following control system equations with tracking errors

$$z_1 = y_d - x_1,\ z_2 = -c_1z_1 + x_2 - \dot{y}_d,$$

and control law u with the stabilizing function α_2

$$u = \hat{\delta}_2\alpha_2,\ \alpha_2 = \ddot{y}_d + c_1(\dot{y}_d - x_2) + z_1 - c_2z_2 + \hat{\theta}_1\sin x_1 + \hat{\theta}_2 x_2.$$

Here $\hat{\delta}_2$ is an estimate of $\delta_2 = 1/\theta_3$. The basic parameter estimator dynamics is (2b) and

$$\dot{\hat{\delta}}_2 = -\gamma_{\delta_2}\mathrm{sign}(\theta_3)\alpha_2 z_2,$$

where $\boldsymbol{\beta} = [-z_2\sin x_1, -z_2x_2, 0]^T$, $\boldsymbol{\theta} = [\theta_1,\theta_2,\theta_3]^T$. Apparently, the control system does not estimate the unknown parameter θ_3. The basic closed-loop system error dynamics reads

$$\dot{z}_1 = -c_1z_1 - z_2, \tag{18a}$$

$$\dot{z}_2 = z_1 - c_2z_2 - \tilde{\theta}_1\sin x_1 - \tilde{\theta}_2 x_2 - \tilde{\delta}_2\theta_3\alpha_2. \tag{18b}$$

A Lyapunov function for the closed-loop system (18) is

$$V(\mathbf{z},\tilde{\boldsymbol{\theta}},\tilde{\delta}_2) = \frac{1}{2}\mathbf{z}^T\mathbf{z} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T\mathbf{\Gamma}_\theta^{-1}\tilde{\boldsymbol{\theta}} + \frac{|\theta_3|}{2\gamma_{\delta_2}}\tilde{\delta}_2^2,$$

where $\mathbf{z} = [z_1, z_2]^T$. The total derivative of this function, with respect to the closed-loop system error and basic parameter estimator dynamics is

$$\dot{V}(\mathbf{z}) = -\mathbf{z}^T\mathbf{C}\mathbf{z}.$$

The above derivative is negative semi-definite with respect to the estimation error and as a result exact estimation cannot be achieved. The objective system (17) and the closed-loop system (18) have to be presented in the general form (1)–(3), in order to apply the proposed methodology for exact parameter estimation. This is accomplished by considering the relations $\tilde{\delta}_2 = \delta_2 - \hat{\delta}_2$, $\theta_3 = \tilde{\theta}_3 + \hat{\theta}_3$, $u = \hat{\delta}_2\alpha_2$ into the basic closed-loop error dynamics (18)

$$\mathbf{f}(\mathbf{x},\mathbf{u}) = \begin{bmatrix} x_2 \\ 0 \end{bmatrix};\ \mathbf{G}(\mathbf{x},\mathbf{u}) = \begin{bmatrix} 0 & 0 & 0 \\ -\sin x_1 & -x_2 & u \end{bmatrix},$$

$$\mathbf{f}_z = \begin{bmatrix} -c_1z_1 - z_2 \\ z_1 - c_2z_2 + \alpha_2(\hat{\theta}_3\hat{\delta}_2 - 1) \end{bmatrix};\ \mathbf{G}_z = \begin{bmatrix} 0 & 0 & 0 \\ -\sin x_1 & -x_2 & u \end{bmatrix}.$$

The resulting Lyapunov function derivative, for the modified closed-loop adaptive system (6) takes the general form (7) with modified parameter estimator (13), which completely define the closed-loop adaptive system equations.

SIMULATION AND SYSTEM TIME RESPONSES

The physical parameters used in the simulations are the pendulum mass $m = 0.5$ kg, pendulum length $l = 0.5$ m, motor moment of inertia $J = 0.0341$ $\mathrm{kg\,m^2}$, torque constant $K_m = 0.48$ $\mathrm{Nm/A}$, viscous friction coefficient $b = 0.4$ Nms and gravity constant $g = 9.81$ $\mathrm{m/s^2}$. The simulation is performed from zero initial conditions with design matrices

$\mathbf{\Lambda}_x = \mathrm{diag}(-10,-10)$, $\mathbf{\Lambda}_r = \mathrm{diag}(-10,-10,-10)$,

$\mathbf{K}_x = \mathrm{diag}(2,2)$, $\mathbf{R}_x = \mathbf{\Gamma}_x = \mathrm{diag}(10,10)$,

$\mathbf{C} = \mathrm{diag}(30,30)$ $\mathbf{\Gamma}_\theta = \mathrm{diag}(5,1,0.2)$, $\gamma_{\delta_2} = 0.1$,

$\mathbf{\Gamma}_\psi = \mathrm{diag}(200,175,75)$, $\mathbf{Q}_r = \mathrm{diag}(10,10,10)$.

The closed-loop system time responses are shown in figure 1, (a)–(d). The reference trajectory is generated via a second order linear reference model with double pole $\lambda = -3$. The desired pendulum positioning angle is chosen to be $v = \pi$ rad. This set point is the worst case scenario for the parameter estimator, because in this angular position all system signals are identically zero and thus all data channels are closed. Nevertheless, all original system model parameters are exactly estimated in one transient response only. In contrast, the closed-loop adaptive system without the modification doesn't estimate exactly any of the unknown parameters. Hence, the proposed approach greatly improves the adaptive system performance. This is due to the fact that the matrix $\mathbf{Q}$ maintains full rank during the control system operation and as a result $\mathbf{M}$ is a positive definite matrix. The only inaccurately estimated parameter is δ_2, because it is not a system model parameter and therefore is not included for exact estimation by the approach. Moreover, the steady state equation for $\tilde{\delta}_2$

$$0 = -\tilde{\delta}_2\theta_1\theta_3\sin x_1$$

is satisfied for both $\tilde{\delta}_2 = 0$, $x_1 \neq \pm k\pi$ or $\tilde{\delta}_2 \neq 0$, $x_1 = \pm k\pi$, $k = 0,1,2,\dots$. Therefore, in steady state δ_2 may not be exactly

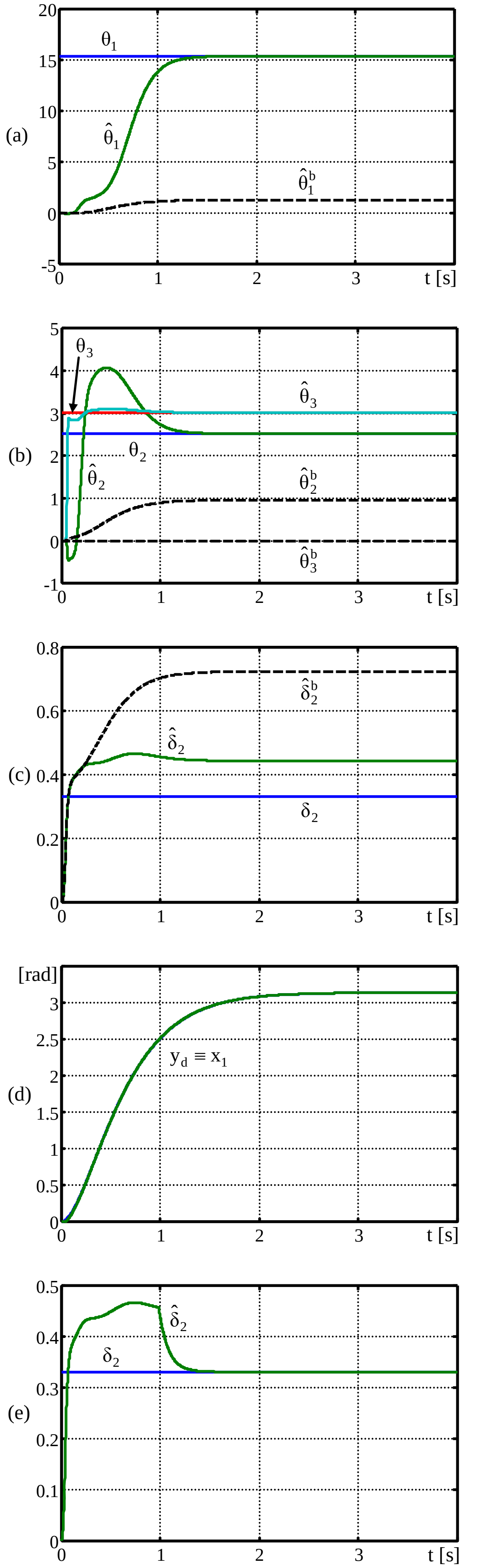

Figure 1: Trajectory tracking and parameter estimates responses (solid line – modified, dashed line – unmodified)

estimated when $x_1 = \pm k\pi$, but is guaranteed to be exactly estimated when $x_1 \neq \pm k\pi$. A possibility to estimate δ_2 exactly is to use the fact that the unknown parameter $\theta_3 = 1/\delta_2$ is guaranteed to be exactly estimated, i.e. the true value of δ_2 can be computed after the $\hat{\theta}_3$ transient response has settled. Then the estimation error $\tilde{\delta}_2$ is known and the δ_2 parameter estimator can be modified into

$$\dot{\hat{\delta}}_2 = -\gamma_{\delta_2}\text{sign}(\theta_3)\alpha_2 z_2 + c_{\delta_2}(1/\hat{\theta}_3 - \hat{\delta}_2).$$

The result from this modification is given on figure 1 (e). This modification provides exact estimation of the parameter δ_2, regardless of the x_1 steady state value. In this way, all system parameters are exactly estimated.

CONCLUSION

The paper has presented a new general approach for exact unknown parameter estimation in nonlinear adaptive control systems. The approach presumes that a known nonlinear adaptive control design method is applied for the objective nonlinear system. Then, the basic adaptive control system is modified by generalization of the prediction error concept and introducing the concept of the stable data accumulation. This results in an asymptotically stable closed-loop adaptive system with respect to both the tracking and parameter estimation errors $\mathbf{z}(t)$ and $\tilde{\boldsymbol{\theta}}(t)$. The asymptotic stability is provided by control of the data accumulation dynamics to achieve full rank of the matrices $\mathbf{Q}(t)$ and $\mathbf{M}(t)$. The unmodified adaptive system cannot estimate the unknown parameters exactly. The major advantage of the new approach proposed is that exact parameter estimation is achieved by the closed-loop adaptive system in one transient response only without imposing the standard excitation techniques, even for systems that do not generate enough information naturally.

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