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Research Article

Dynamics of Vehicle-Pedestrian Impact. Cauchy Problem and Finite Element Method

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Abstract

Technology advancement has contributed not only to the reduction of road related accidents but to the improvement of road safety, increasing pedestrian safety significantly. Pedestrian lower injury risk is mainly achieved through the effect of the vehicle's front-end profile, the use of specific shock absorbing materials and so on. The presented model in such a way could predict with sufficient accuracy the complex pedestrian's behaviour and movement according to the vehicle's front-end profile so as to improve safety measures. The complexity of the study is related to the fact that the two objects, the automobile and the pedestrian, are completely different and do not have the same characteristic features, e.g. there is significant difference in their masses, which is in the range of more than 10 times, especially in an auto-pedestrian accident.This paper presents a methodology for identifying a front-impact car accident and car-pedestrian lateral impact, modelling the relative motion of the victim's body with respect to the automobile, taking into account the possible alternation of the different phases of motion. The whole complex mechanism of motion is divided into separate phases and at the beginning of each one a task is solved when the pedestrian gets hit by a specific part of the car's body.

Keywords: automobile, pedestrian, impact, dynamics, finite elements, Abaqus, SolidWorks

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1. Introduction

Globally, statistical analysis reports that the share of pedestrian motor vehicle accidents is between 30-35% of the total number of events. This means that this type of collision is decisive and significant for pedestrian safety on the one hand, and on the other hand, for the complex dynamic analysis of the impact processes. The events in which the car comes into contact with its front part with the pedestrians are mainly 70-75% of the total number of such type of collisions. Thus, the focus of this paper is going to be on the dynamic model of the relative motion of the two bodies, the automobile and the pedestrian, in a head-on car crash.

The study was conducted in two aspects: firstly, a dynamic study of the relative motion between the two bodies, the car and the pedestrian, was performed, based on fundamental principles of impact and applied mechanics, namely the law of momentum and impulse-momentum change theorem (Daily, Shigemura [1], Schmidt, Haight, Szabo, Welcher [2], Jiang, Grzebieta [3]). The Cauchy problem or the so-called boundary value problem was solved, where some of the initial conditions were known, while post-impact velocities of the centers of mass of the two bodies had been sought so as to satisfy the criteria for the known final positions, contact zones and traumatic injuries from the car accident. Since the number of unknown variables was greater than the number of equations, Newton's law of impact was applied as well.

Secondly, a finite element study using the specialized software product Abaqus/Explicit is shown. At the heart of this study is the impact process of two deformable bodies in which the contact surfaces have little sliding interaction. The implementation of the method allows to predict and analyse the deformation zone at the front of the body shell.

An advantage of these methods is the fact that the vehicle's front-end profile is known, from which it is possible to compare the pedestrian's body contact areas known from the traumatic injuries due to the frontal car crash. The complexity of the study is that the two objects, the car and the pedestrian, are placed at a disadvantage, namely there is a significant difference in mass, which is within 10 times range at vehicle contact with pedestrian (Teng, Liang, Hsu, Tai [4], Higuchi, Akiyama [5], Higuchi, Akiyama [6], Linder, Clark, Douglas, Fildes, Yang, Sparke [7]). Modern research on vehicle-pedestrian impact is based on experimental tests in specialized laboratories, where the main goal for car manufacturers is to significantly increase safety impact. In some cases, this is achieved by passive protection systems, in which the pedestrian's body falls on the front of the car, and during the relative motion, protective airbags are activated. Here, safety impact process is not going to be looked at, but the possibility of predicting the phases of relative motion, once the profiles of the two bodies are known, thus suggesting tips to modern car manufacturers, with sufficient accuracy, what safety measures to be taken and how to improve pedestrian safety.

2. Mechanical-mathematical model of vehicle-pedestrian impact

The impact for each of the two bodies is characterized by the change of momentum/momentum theorem/for the time of impact, which has the form

$$m.\vec{u} - m.\vec{V} = \vec{S}. \tag{1}$$

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Here, $\vec{u}$ is post-impact velocity of the given body center of mass; $\vec{V}$ – prior-to-impact velocity of the given body center of mass; $\vec{S}$ - crash pulse.

According to the theorem, the velocity of the given body center of mass varies in direction, magnitude and position, and its change is determined by the expression

$$\vec{u} - \vec{V} = \overrightarrow{\Delta V}. \tag{2}$$

The impulse-momentum change theorem of the mechanical system of the pedestrian in relation to the center of mass during its relative motion around it $d\vec{K}_C^r$ has the type of

$$\frac{d\vec{K}_C^r}{dt} = \vec{M}_C^{(e)}, \tag{3}$$

where $\vec{M}_C^{(e)}$ is the crash impulse of impact forces with respect to the pedestrian center of mass.

The given expression characterizes the acquired rotation of the pedestrian after the impact and according to the impact theory the pedestrian motion, close to planar motion, acquires the form of

$$I_c(\omega_1 - \omega_0) = dS, \tag{4}$$

where I_c is pedestrian mass moment of inertia with respect to their central horizontal axis parallel to the plane of motion; ω_1 – post-impact angular velocity; ω_0 – pre-impact angular velocity, negligibly smaller compared to post-impact angular velocity; d - shoulder of the crash pulse.

This article presents a model of the relative motion of a pedestrian's body with respect to the car, taking into account the possible alternation of the different phases of moton (Karapetkov [8, 9], Karapetkov, Uzunov [10], Karapetkov, Dimitrov, Uzunov, Dechkova [11 , 12], Uzunov, Dechkova, Dimitrov, Uzunov [13], Karapetkov, [14]). The whole complex mechanism of motion is divided into separate phases. At the beginning of each one, a task of hitting the pedestrian's body by a specific spot of the car's body is solved, applying the rules for changing the quantity of motion and the impulse-momentum change as well as the theory of impact. The movement of the victim's body after each local impact by the car's body is described by differential equations of motion, which after integration determine its law of motion. Mechanical-mathematical modelling takes into account the vehicle specific geometry and configuration of the front of the car, the pedestrian height and mass.

Mechanical-mathematic modelling includes the following mathematical expressions:

The change in quantity of motion theorem

$$\begin{aligned} & m_1u_1 - m_1V_1 = -\cos\varphi_s S; \\ & m_2u_{2x} - m_2V_{2x} = \cos\varphi_s S; \\ & m_2u_{2y} - m_2V_{2y} = \sin\varphi_s S; \end{aligned} \tag{5}$$

where $m_{1,2}$ - masses of the car and the pedestrian, respectively; $u_{1,2}$ - post-impact speed of the center of mass of the car and the pedestrian; $V_{1,2}$ - pre-impact speed of the center of mass of the car and the pedestrian; S - crash pulse, defined as equivalent to a system of parallel impact forces and applied at the point of contact; φ_s - angle between the crash pulse and the abscissa axis.

- impulse-momentum change theorem for the pedestrian's body;

$$m_2 i^2(\omega_1 - \omega_0) = (x_P - x_C)\sin\varphi_s S - (y_P - y_C)\cos\varphi_s S, \tag{6}$$

where $\omega_{1,0}$ - angular velocities of the body before and after the impact, respectively; i - inertial radius of the body relative to its central axis.

- coefficient of restitution;

$$k = \pm\frac{(\vec{u}_{P2} - \vec{u}_{P1})\vec{e}}{(\vec{V}_{P1} - \vec{V}_{P2})\vec{e}} \tag{7}$$

where $\vec{e}$ - single vector of the crash pulse vector.

Speed distribution law of the contact point;

$$\begin{aligned} & \vec{u}_{P2} = \vec{u}_2 + \vec{\omega}_1 \times \vec{\rho} \\ & \vec{V}_{P2} = \vec{V}_2 + \vec{\omega}_0 \times \vec{\rho}\ , \end{aligned} \tag{8}$$

where $\vec{\rho}$ - is the radius vector of the contact point relative to the center of mass of the body.

Equation of the relationship between the coefficient of restitution and the kinematic parameters;

$$\begin{aligned} & \cos\varphi_n u_{2x} - \cos\varphi_n u_1 + \sin\varphi_n u_{2y} \\ & \qquad + \left(\sin\varphi_n \rho_x - \cos\varphi_n \rho_y\right)\omega_1 = V_{\omega n}; \\ & V_{\omega n} = (\pm k)\left[\left(V_1 - V_{2x} + \omega_0\rho_y\right)\cos\varphi_n - \left(V_{2y} + \omega_0\rho_x\right)\sin\varphi_n\right]; \\ & \rho_x = x_P - x_C; \quad \rho_y = y_P - y_C, \end{aligned} \tag{9}$$

where φ_n - angle between the normal to the impact surface and the abscissa.

The algebraic system of five equations contains five unknowns – post-impact car speed, the projections of the speed of the center of mass of the body after the impact, its angular velocity and the crash pulse. The system is solved in Matlab environment.

- Differential equations of body motion in air;

$$m_2\ddot{x}_C = m_2\, j; \qquad m_2\ddot{y}_C = -m_2\, g; \qquad m_2 i^2\ddot{\varphi} = 0. \tag{10}$$

Law of motion of the body's center of mass in its relative motion with respect to the car;

$$\begin{aligned} & x_C = (u_{2x} - u_1)t + \frac{jt^2}{2} + x_{C0}; \\ & y_C = u_{2y}t - \frac{gt^2}{2} + y_{C0}, \end{aligned} \tag{11}$$

where x_{C0}, y_{C0} - initial coordinates of the mass center.

Law of rotation. Change of rotation angle of the body

$$\varphi = \omega_1 t + \varphi_0, \tag{12}$$

where φ_0 is the starting angle.

It goes without saying, such an impact on the car leaves characteristic deformation marks, indents and abrasion marks. In more severe cases, the deformation is significant, and not infrequently there is a characteristic breakage of the car's windshield, a spider crack that gives the appearance of spider web. The contact zones are divided into different phases of the impact. Initially, the car comes into contact in the area of its front bumper with the lower limbs causing leg injury. Under the action of the generated impact force and in accordance with the impilse-momentum change theorem, the body acquires a significant rotation around the transverse axis of the car and falls with the torso on the hood, windshield or ceiling. This is the second contact, from which follows the second phase of relative motion. The problem to be solved in the individual phases is the Cauchy problem, in which the geometric parameters, masses and mass moments of inertia, the place of each contact, prior-to-impact velocities of the centers of mass of the car and the pedestrian are considered to be known. By iterating the initial velocity at the initial contact and the time of motion of the body during the relative displacement, solutions are sought that satisfy the final position in the first phase. The initial conditions of the second phase are obtained after re-solving the task of the impact.

The typical expert task in a pedestrian collision with the help of mechanical-mathematical modelling and computer simulation is to restore the relative motion of the body during contact with the car, taking into account its deformations and injuries of the victim. Thus, the speed of the vehicle at the time of impact is determined with sufficient accuracy and the location of the impact is determined.

According to the dynamics of the impact process, there are three phases of impact - the first is the initial contact with the front part of the car, in the area of the front bumper. This is followed by a hood and windshield hitting, and then a fall on the ground. The car deformation is typical and in such cases is characteristic so that it determines the contact between the elasto-plastic metal surface and the soft tissues and bones of the body with the car surface. Therefore, the behaviour of the pedestrian after the initial contact is directly related to the profile of the car in a head-on collision for him. The pedestrian's body is represented by its midline, with dots marking its center of mass, the top of the head and the places of impact. The computer simulation was performed for the configuration shown on the front of the Audi A4. The gross vehicle weight is 1650 kg; pedestrian mass - $m_2 = 82\ kg$; height - 1,71 m; coefficient of restitution $k = 0{,}01$.

In a variety of studies, it has been proven that after the initial contact with the bumper, the body moves relative to the car due to its inertia and turns sharply. According to the impulse-momentum change theorem, the body rotates around its horizontal central axis as the head approaches the car and the legs move away, respectively, in a counterclockwise direction, from a right-side view of the car. In this way it reaches the front hood surface, gets hit and slides on it. Then the victim strikes the windshield, gets hit in the head, and bounces off the car, so as to fall on the ground, sliding down and rolling down. This means that pedestrian safety depends primarily on the elastic characteristics of the front bumper at initial contact and the elastic and plastic characteristics of the front hood surface of the car. Furthermore, the time of the pedestrian's relative movements until he falls on the front cover is essential, taking into consideration the possibility of having passive safety devices to reduce injuries.

The dynamic model of the relative body movements and the effect of impact force direction as well as real-time detection serve as an indicator and opportunity to create passive models to protect pedestrians' lives and health in the event of a front-impact car accident. The main safety indicator for a frontal impact for the vehicle and a side impact for the pedestrian is impact force direction relative to the normal surface impact. (Fig. 1).

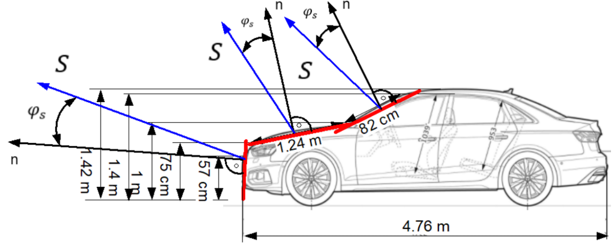


Fig. 1. Safety indicator for a frontal impact for the vehicle and a side impact for the pedestrian is impact force direction relative to the normal surface impact

Under the action of the impact force equal to the given contact surface, the body acquires a certain direction of motion, and the dynamic task is divided into three stages. Through successive iterations for the short interval in the given section, the direction of the body's movement is achieved.

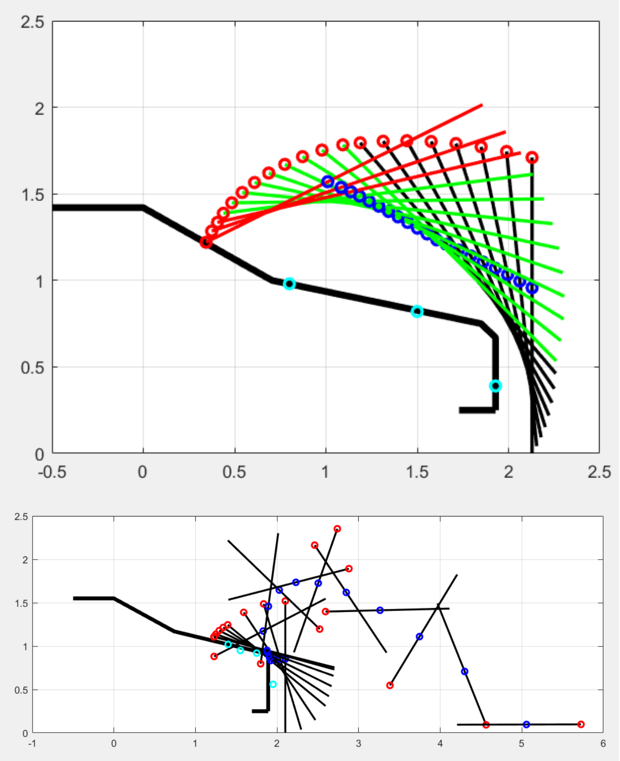


Fig. 2. Model of relative motion between automobile and pedestrian and model of relative movement between car and pedestrian in high front.

The numerical experiment was performed at a car impact speed of $u_1 = 13{,}9\ m/s = 50{,}0\ km/h$. According to the data from the computer simulation the following results were obtained:

- Post-impact vehicle speed:	$u_1 = 13{,}5\ m/s = 48{,}6\ km/h$;
- Duration of phases:	$t_1 = 0{,}065\ s;\ t_1 = 0{,}080\ s;\ t_1 = 0{,}002\ s$;
- Total distance of the vehicle up to the moment of impact with the windshield:	$s_1 = 0{,}88 + 1{,}08 + 0{,}03 = 1{,}99$ m

3. Dynamic model of vehicle- pedestrian impact by finite element method

3.1. Creating 3D models SolidWorks environment

It is common knowledge from the theory of geometric modelling that free-form surfaces are used to describe the silhouette of a 3D geometric object. In computer 3D modelling, they are described by CAD systems with their poles, curvature and a number of areas. One of the main characteristics is the surface degree, presented analytically as a polynomial with variable gradients and values. The order of the polynomial is greater than its degree, and is equal to the number of coefficients, not the greatest surface degree.

The surfaces are united in one, the resulting surface, through nodal lines (intersections). The number of nodes determines the influence of the poles on the two joined surfaces as well as the smoothness of the transition.

3.1.1. CAD model of a car and pedestrian in SolidWorks environment

The main goal of the present modelling is to recreate a real-life situation of a traffic road accident (car collision) between a car and a pedestrian (Dechkova, [15]). Characteristics of real life participants in the accident, determined by their make and model, were used for the construction of the prototypes of the objects. Geometric modelling takes place in SolidWorks environment. The models of the studied car and pedestrian presented in figures 3 and 4 are assembled from several separate parts. The respective movable and immovable connections between them are defined. For each detail, in addition to its geometric dimensions, material characteristics are defined as well. Based on these input data, mass characteristics of the modelled object are automatically determined in the CAD system.

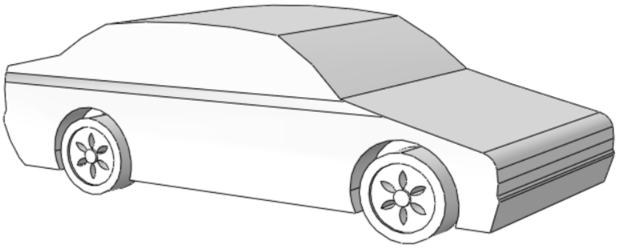


Fig. 3. 3D model of Audi car created in SolidWorks.

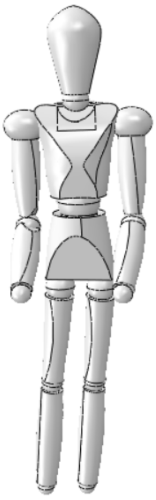


Fig. 4. 3D model of a pedestrian created in SolidWorks.

3.2. Creating a finite element model in Abaqus/CAE environment

The models created in SolidWorks are imported into the Abaqus dynamic analysis software system. The Abaqus/CAE system exports a sketch in standard STEP file format, recording all geometric data of a layer of the model.

The Abaqus/Explicit package allows the study of the finite element method of elastic systems whose elements perform mutual movements. It is based on an unambiguously formed integration scheme. This feature largely determines its suitability in the study of vehicle accidents.

The dynamics of the processes occurring in a given crash event, as well as the complex interconnection between the quantities, suggest taking into account the changes in the coordinates of the centers of mass of the models over time. Last but not least, the effect of dynamic external loads, when obtaining the intermediate calculation results must be considered. Moreover, it is known that the dynamics in the change of the speed of the modelled objects, when they consist of heterogeneous materials, introduces additional uncertainties in the implementation of the calculation procedures. For this reason, when modelling a collision between a car and a pedestrian at variable speeds of the centers of mass, the finite element method (FEM) is applicable.

A real model of collision between a car and a pedestrian from finite elements was created using the Abaqus/Explicit software (Fig. 5). Second-order elements are used in the finite element model of the objects, as they use a lower network density than the first-order elements. The tetrahedral element of the second row has ten nodes (four angular and six internal) and each of them has three degrees of freedom. The edges and surfaces of the second-order elements take on curvilinear shapes after deformation. This justifies the selected modelling approach, namely by applying the methods and means of curvilinear geometry.

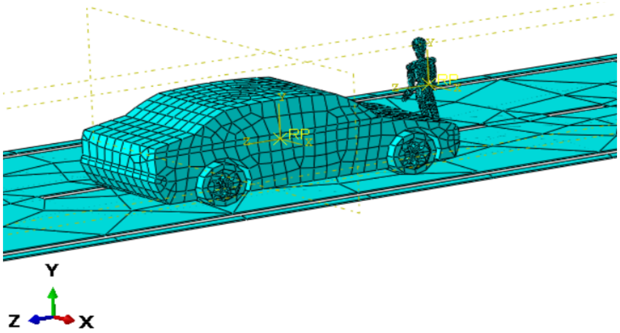


Fig. 5. Finite element modelling of a car and a pedestrian in Abaqus.

3.3. Dynamic analysis in Abaqus environment

The main stage in the modelling of a car and a pedestrian is the connection of the elements in a complex model. In the present study, the term complex model should be understood as a set of models of those involved in the impact, car and pedestrian. Fig. 6 shows a spatial model consisting of a car and a pedestrian. A fixed coordinate system X, Y, Z is given, which is invariably connected coordinate system x, y, z. The created 3D models are positioned on the basis of a selected fixed Cartesian coordinate system with a common origin of the coordinates z_C and x_C. The y_C coordinate is chosen according to the geometric arrangement of the mass center of the given object.

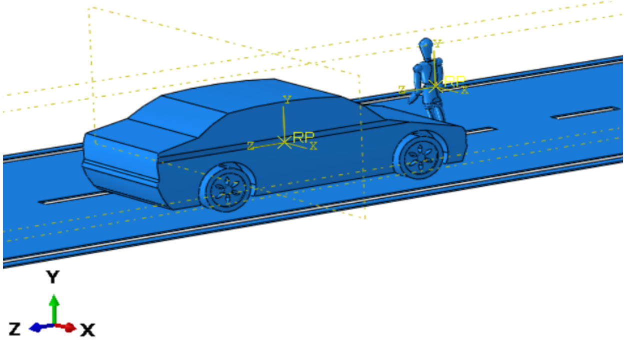


Fig. 6. Spatial model of an Audi car and a pedestrian in Abaqus.

Materials and their properties have been selected from the Abaqus database. When selecting a material, it is assumed that the deformation depends linearly on the stress. For this reason, the chosen steel-30XGSA isotropic material is considered to be suitable. It has a region of plasticity, characterized by the fact that significant deformations are accompanied by small changes in stress. The created models have material density of 355 N/mm^2 with isotropic hardening to strength of 490 N/mm^2 at a plastic stress of 0.025.

It is assumed that the car and the pedestrian are solids and this will not cause inaccuracies in obtaining the initial results. The objects are modelled with standard housing elements (Abaqus elements S3R and S4R) with thickness of 0.01 mm. The selected contact conditions between the tires and the ground allow friction between the individual parts to be taken into account during their deformation. In this case, a coefficient of friction $\mu = 0.05$ is set.

The loads are applied using the so-called steps. For nonlinear problems, each load step consists of several iterations. Each load step can also have its own type of analysis, boundary conditions and output. In this case we set a step with 21 iterations.

The next step in the analysis is to set the boundary conditions and load the mechanical system. A boundary condition is defined - fixing certain surfaces of the created models. Loading is realized using volumetric gravity forces, Gravity type (own weight is taken into account). Under this type of load, the magnitude of the self weight forces is calculated by Abaqus for each individual element, considering the density of the material.

In this case, the speed of the vehicle at the time of impact is 50 km/h and is applied in the longitudinal direction along the z axis. Accordingly, the speed of the pedestrian is 5 km/h, which is applied in the transverse direction along the x-axis.

Fig. 7 shows the positions in the impact phase between a car and a pedestrian at a speed of 50 km/h.

Investigation determines the ability to predict with sufficient accuracy the behavior of the passenger and the car after the impact. It is evident that impact safety mainly depends on two important factors - the automobile speed of the center of mass at the time of impact and the contour of the front of the car. The construction of modern cars has a certain aerodynamic shape, which greatly benefits the inequality between the two objects - car and pedestrian. There are structures that definitely affect the patterns of pedestrian's body movement at great extent and increase the risk of fatalities. This unquestionably requires new constructive decisions. The fact is that these structures can rely on the function of passive protection systems, such as airbags. The system should provide a high-speed gas generator with an activation period shorter than the impact time between the first and third phases.

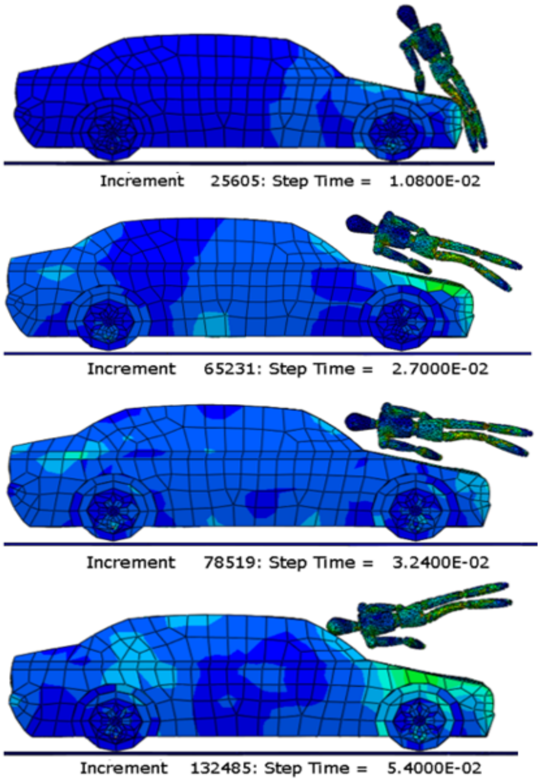
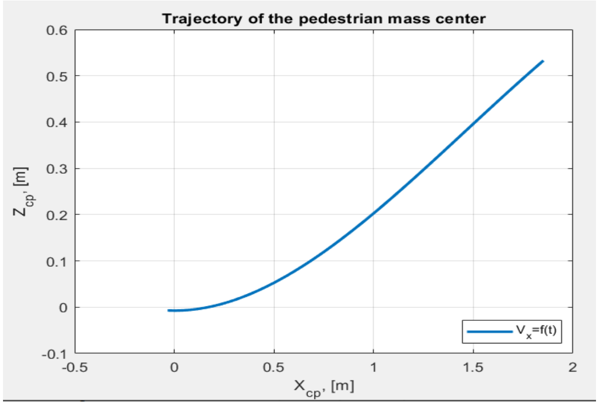
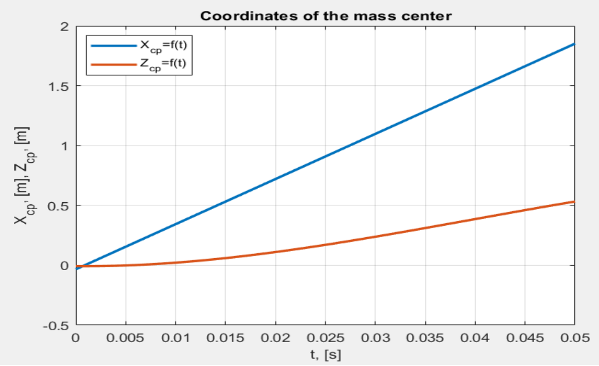


Fig. 7. Simulation model of a vehicle - pedestrian collision, at a speed of 50 km/h.

Fig. 8 shows the results of the dynamic study by finite element method using the software product Abaqus/Explicit.



Trajectory of the pedestrian center of mass



Coordinates of the pedestrian mass center

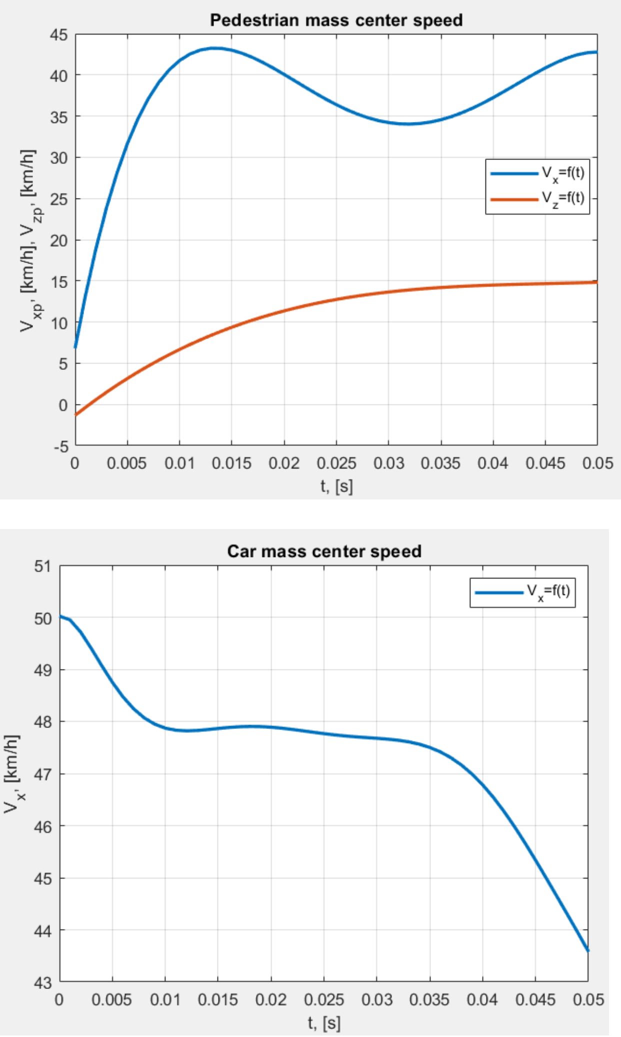


Fig. 8. Change in speed of the center of mass of the car and the pedestrian in Abaqus/Explicit, an explicit dynamic finite element solver-program.

4. Example

Figures 9-18 show sequential positions of an actual pedestrian hit by a car. After the initial contact in the area of the front bumper, the body rolls into the air, strikes the lower half of the windshield with the head and right shoulder, and continues its movement with increasing rotation. In this way, it separates from the car and performs a somersault in the air. The body then falls onto the front hood with the head pointing forward. The relative motion continues forward, with the victim separated from the vehicle and falling onto the roadway headfirst and downward.

Numerical calculation was performed based on the data of the cited incident at a vehicle speed of 72 km/h. The car at the moment of the impact and after it moves with a braking delay of $j = 7\ m/s^2$. (The smoke from the friction between the tires and the asphalt can be seen on the video clip/. The mass of the car is $m_1 = 1200\ kg$, and that of the pedestrian is $m_2 = 70\ kg$. The height of the pedestrian is $l = 1{,}70\ m$, the height of the center of mass is $h_c = 0{,}95\ m$, and its inertial radius is $i = 0{,}49\ m$. The application point of the crash pulse in the given configuration of the front part of the car is taken at a height of $y_p = 0{,}4\ m$. The coefficient of restitution is assumed to be $k = 0{,}01$.

In computer modelling, several characteristic phases of impact and relative movement of the pedestrian's body are observed. After the first impact, the body reaches the windshield, in which position the task of the impact is solved. Then follows the next phase, where the body strongly rotates and performs somersaults in the air. At the end of this phase, the body reaches the front hood and the hull of the car hits it. From this moment on, what follows is movement until the moment when the body falls on the road surface.

Fig. 9. Initial contact between the vehicle and the pedestrian.

Fig. 10. Relative motion before hitting the windshield.

Fig. 11. Impact of the body on the windshield.

Fig. 12. Start of air roll of the body /somersault.

Fig. 13. Separation of the body from the vehicle.

Fig. 14. Body roll in the air /somersault.

Fig. 15. Body roll in the air before falling onto the hood.

Fig. 16. Body fall on the front hood.

Fig. 17. Body fall on the road surface.

Fig. 18. Body slide on the lane.

The results for each phase are shown below.

4.1. Phase 1. Impact on initial contact

Fig. 19 shows the relative movement of the body up to the moment of impact with the front windshield. The pedestrian's body is represented by its midline, with dots marking its center of mass, the top of its head, and the impact locations in the passenger compartment.

Relative generalized velocities of the body after impact:

$$V_{Cx} = -10{,}32\,m/s = -37{,}16\,km/h;\ \ V_{Cy} = \ 0{,}80\,m/s = 2{,}87\,km/h;\ \ \omega_1 = 19{,}88\,s^{-1}.$$

Vehicle speed after impact:

$$u_1 = 19{,}43\,m/s = 69{,}95\,km/h.$$

Crash pulse:

$$S = \ 685{,}7\,N.s.$$

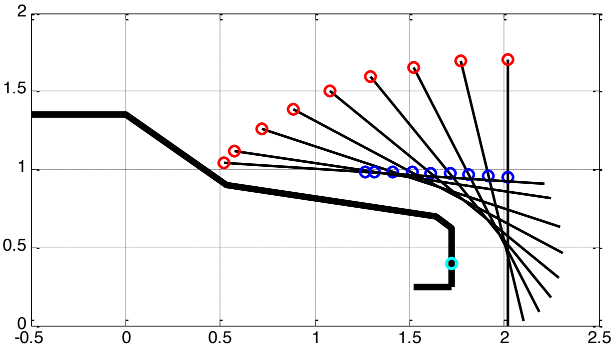


Fig. 19. Relative motion of the body until the moment of impact with the front windshield.

4.2. Phase 2. Free movement of the body until the moment of hitting the windshield

Phase time:

$$t = 0{,}075\,s$$

Generalized parameters at the time of impact on the windshield:

$$x_c = 1{,}26\,m;\ y_c = 0{,}98\,m;\ \varphi = 175{,}4^o.$$

Relative generalized velocities of the body at the moment of impact with the windshield:

$$0{,}0611 \div 0{,}2199$$

$$V_{Cx} = -9{,}80\frac{m}{s} = -35{,}27\frac{km}{h};$$

$$V_{Cy} = \ 0{,}060\frac{m}{s} = \ 0{,}22\frac{km}{h};$$

$$\omega = 19{,}88\,s - 1\ .$$

Coordinates of the contact point when hitting the windshield: $x_P = 0{,}5\,m\,;\ y_P = 0{,}95\,m$.

4.3. Phase 3. Impact on the front windshield

Relative generalized velocities of the body after impact:

$$V_{Cx} = -3{,}71\frac{m}{s} = -13{,}37\frac{km}{h};$$

$$V_{Cy} = \ 5{,}78\frac{m}{s} = \ 20{,}84\frac{km}{h};$$

$$\omega_1 = 2{,}51\ s^{-1}.$$

Vehicle speed after impact:

$$u_1 = 18{,}55\,m/s = 66{,}77\,km/h.$$

Crash pulse:

$$S = \ 607{,}4\,N.s.$$

4.4. Phase 4. Body roll in the air after hitting the windshield

Fig. 20 shows the next relative movement up to the time of the front hood impact.

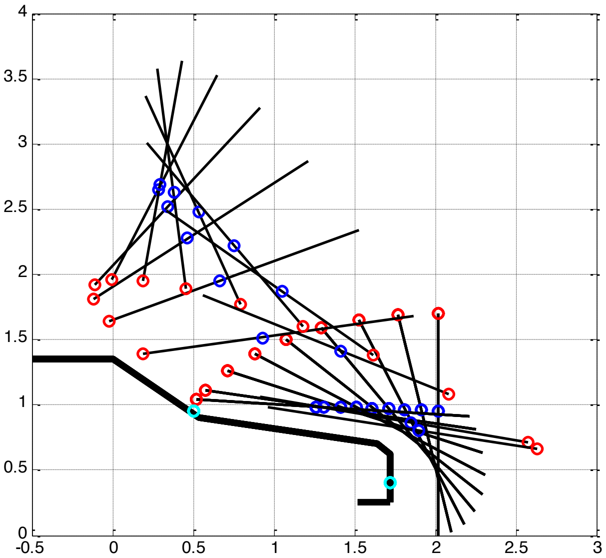


Fig. 20. Relative motion up to the time of impact on the front hood.

Phase time:

$$t = 1{,}21\ s.$$

Generalized parameters at the moment of the front hood impact:

$$x_c = 1{,}90\,m;\ y_c = 0{,}81\,m;\ \varphi = 349{,}2^o.$$

Relative generalized body velocities at the moment of impact with the front hood:

$$-6{,}0822 \div 21.8958$$

$$V_{Cx} = 4{,}76\,{}^{m}/_{s} = 17{,}12\,km/h\,;$$

$$V_{Cy} = -\,6{,}08\,m/s = -21{,}90\frac{km}{h};$$

$$\omega = 2{,}51\,s^{-1}.$$

Coordinates of the contact point at the front hood impact:

$$x_P = 1{,}6\,m\,;\ \ y_P = 0{,}7\,m.$$

4.5. Phase 5. Impact on the front hood

Fig. 21 shows the subsequent relative movement of the body until the moment of impact with the road surface.

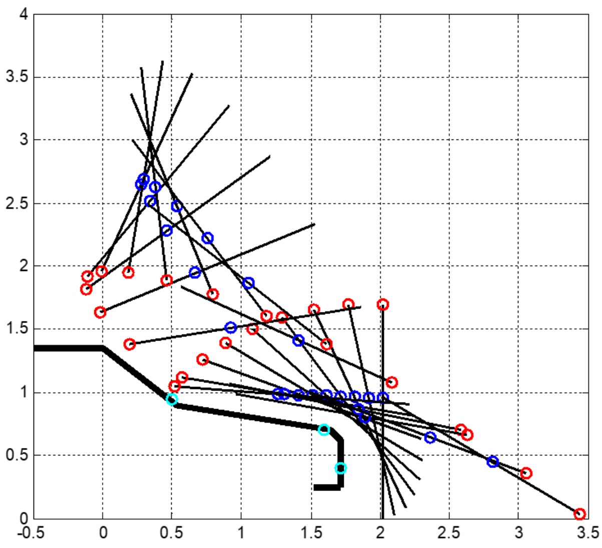


Fig. 21. Relative motion of the body until the moment of impact on the road surface.

Relative generalized velocities of the body after impact:

$$V_{Cx} = 5{,}59\frac{m}{s} = 20{,}11\frac{km}{h};$$

$$V_{Cy} = -1{,}65\frac{m}{s} = -5{,}93\frac{km}{h};$$

$$\omega_1 = -2{,}60\,s^{-1}.$$

Vehicle speed after impact:

$$u_1 = 10{,}03\,m/s = 36{,}10\,km/h.$$

Crash pulse:

$$S = \ 337{,}7\,N.s.$$

4.6. Phase 6. Relative movement of the body after the impact on the front hood until the moment of falling on the road surface

Phase time:

$$t = 0{,}15\,s.$$

Generalised parameters at the time of ground impact:

$$x_c = 2{,}81\,m;\ y_c = 0{,}45\,m;\ \varphi = 326{,}9^o$$

Relative generalized velocities of the body at the moment of impact with the ground:

$$-3.1190\ \ \div -11.2284$$

$$V_{Cx} = 6{,}63\frac{m}{s} = 23{,}90\frac{km}{h};$$

$$V_{Cy} = -3{,}12\,m/s = -11{,}23\,km/h;\ = \omega - 2{,}60\,s - 1.$$

At the moment of impact with the ground the car speed is

$$V_1 = 8{,}98\,m/s = 32{,}32\,km/h$$

The body's center of mass hits the ground with an absolute velocity whose projections are

$$V_{Cax} = 15{,}62\frac{m}{s} = 56{,}22\frac{km}{h};$$

$$V_{Cay}\ = \ -3{,}12\,m/s = \ -11{,}23\,km/h.$$

The absolute velocity of the body center of mass at the moment of impact with the ground is

$$V_{Ca} = 15{,}92\,m/s = 57{,}32\,km/h.$$

The given numerical example represents a real identification of the given accident, where there is an almost complete match of the relative body movement in the modeling and the real accident.

Fig. 22 shows an example of the relative motion of a pedestrian's body at a speed of 65 km/h. The difference between the two examples can be clearly observed.

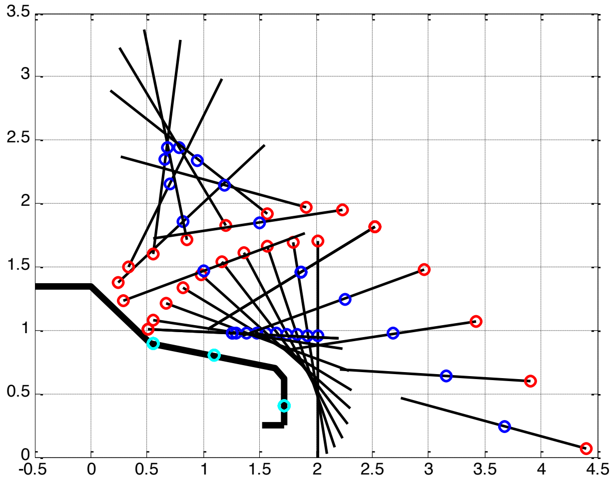


Fig. 22. Relative movement of the body in the impact with the car at speed of 65 km/h.

At a vehicle speed of 40 km/h, six phases are distinguished /Fig. 3.21/. The first phase is of the first contact, after which the body reaches the windshield at its lower end. Here, the head hits it, then free motion follows. The legs are not raised high and the body in its relative movement begins to be projected forward in relation to the car. After falling on the hood with the upper part of the hull, the impact task is solved, after which the sliding phase occurs. The body slides along the front hood, moving forward with the legs and falls with the legs forward and down. It can be seen that at this speed the contact point of the body with the vehicle hull is at the beginning of the windshield.

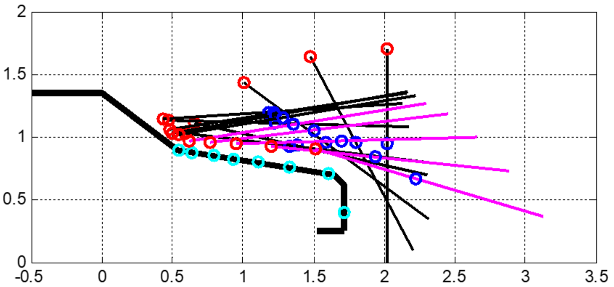


Fig. 23. Relative movement of the body in the impact with the car at speed of 40 km/h.

Fig. 24 shows the relative motion of a body hit by a car at a speed of 80 km/h, with the car moving uniformly. In this case, the relative motion of the body up to the moment of hitting the windshield is almost identical, but given the uniform motion of the car, the body moves backwards relative to it in its relative motion. In reality, the pedestrian rolls over the vehicle and falls behind it.

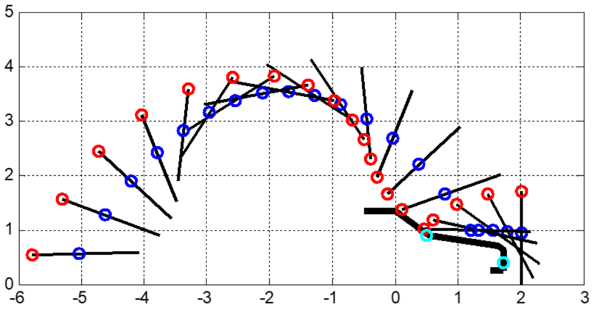


Fig. 24. Relative motion of the body during vehicle uniform motion and speed impact of 80 km/h.

Implementing the proposed mechanical and mathematical modelling of the pedestrian impact as well as the automated algorithm accurately estimate the car speed at the moment of impact. Taking into account the injuries of the pedestrian, the local impacts of the body in the passenger compartment could be determined, while its relative motion can be deduced by the deformations on the car.

Application can be found in judicial system when investigating a similar type of accident with complex dynamics of relative body movement, as well as in identifying the phases of an impact between a car and a pedestrian with analysis of traumatic injuries.

5. Conclusion

1. A dynamic analysis of the impact phase between the vehicle and the pedestrian has been performed by the finite element method using the software product Abaqus/Explicit.
2. Cauchy's problem on the study of the dynamics of the impact process in a vehicle - pedestrian collision has been solved, as the initial kinematic quantities at the moment of impact satisfy the phase of body movement until the moment of impact with the windscreen.
3. The effect of the configuration and contour of the front part of the car has been determined in order to influence the passive safety systems on the behavior of the person in frontal impact.
4. The time for relative motion of a pedestrian in a head-on collision with a car until the moment of contact with the windscreen has been determined by two independent methods. The study has been based on the Cauchy problem or the so-called boundary value problem.

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