Identification of Lighting Clouds Presented by Electric Dipole

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***Abstract*—Identification of a moving lightning cloud is being carried out. The lightning cloud is seen as an electric dipole. The electromagnetic field distribution in moving environments is considered. For determination of the electromagnetic field, electromagnetic model and relativistic approach are used. A four-dimensional potential is applied. The magnetic vector potential and the scalar electric potential are the elements of four-dimensional potential. The Maxwell tensor is used to determine the vectors of the electromagnetic field. The electric field vector components of the electric dipole are obtained. The quality of dipole charge, its coordinates and velocity are determined by solving the inverse problem. To solve the inverse problem, the obtained analytical expressions for the components of the electric field, created by a moving electric dipole, are used.**

Keywords— electric dipole, lightning clouds, identification, inverse problem, electromagnetic field

I. Introduction

Lightning discharges generate radio wave pulses and blue-white light. Interference can disrupt the operation of electronic equipment used for navigation and cause catastrophes. To protect electrical devices from lightning, it is necessary to identify the electrical charges that are accumulated in the clouds [1]. Charged cloud can be presented approximately by concentrated charge in case of small cloud. Identifying a moving concentrated charge is possible and a relatively easy task. [1]. Therefore, in most cases, the cloud charge is represented as a concentrated charge. Most lightning, which causes not only disturbances but also serious power outages, is caused by lightning between the cloud and the earth.

Lightning can cause serious damage and even destroy objects. The most investigated lightning bolts are cloud-to-ground lightning bolts. This type of lightning is also best explained by the other types of lightings; although intercloud and intracloud lightning occur more frequently. The appearance of lightning is very difficult to predict, due to the fact that the reasons that lead to its occurrence have not yet received sufficiently accurate explanations, although they have been the subject of serious research for more than a century. Most lightning occurs over land and less often over water, most often in the tropics [2] where convection is significant.

Lightning is usually produced by cumulonimbus clouds, which have bases that are typically 1–2 km above the ground and tops up to 15 km in height. Sometimes the size of these clouds is huge and they accumulate distributed charges. These lightning clouds need to be represented by electric dipoles, multipoles, etc. [3]. Identification of a moving lightning cloud, which can be presented by electric dipole, is considered.

For an electrostatic discharge, several conditions are necessary that must be met. First, there must be a high critical voltage between two objects or between the studied object and the ground. The environment must have a high resistivity to prevent easy equalization of the charges. The earth's atmosphere has a high electrical insulation and does not allow the free movement of charges and the formation of charged regions of opposite polarity.

The mechanism of accumulation of charges that can cause lightning is not yet sufficiently studied. For example, a study [4] shows that breakdown is possible when the resulting electric field is greater than the dielectric strength of air at high humidity (which is approximately around 3000 KV/m). Then an electrical discharge occurs, called a strike. The movement and propagation of electric charges around and inside the lightning cloud is an ever-changing process. The structure of the dipole is dominant. The negative charge is at the bottom of the cloud and the positive charge is at the top. Depending on the geographical location of the main charge is the place where the temperature is from -5 to -17° C [4].

The model of charge formation in a lightning cloud presented in [5] is used. The formation of charges in a lightning cloud is represented graphically in Fig. 1.

In order for lightning to occur, it is necessary to generate an electric field in a relatively small volume of the cloud with an intensity sufficient to generate an electric discharge (approximately 1 MV/m) and in a significant part of the cloud to form a field of medium intensity sufficient to maintain of the started discharge (approximately 0.1 - 0.2 MV/m).

II. Used Appoach

The case of identification of moving lightning cloud, presented by electrical dipole is considered. The algorithm described in previous work [1] is used. For determination of the electromagnetic field, electromagnetic relativistic approach is used. A four-dimensional magnetic potential is applied to find the component of electric intensity vector.

Einstein's special theory of relativity considers physical phenomena that evolve at a constant velocity. In this theory, the space coordinates x, y, z of the three-dimensional space, and time t form a four-dimensional vector [1].

Coordinates of charges of electric dipole can be taken around of the center of gravity of a lighting cloud with the total value of the electric charge q and distance between charges of dipole l. The dipole excites an electric field in and around the cloud. A model of the charges in a lightning cloud is presented in Fig.1.

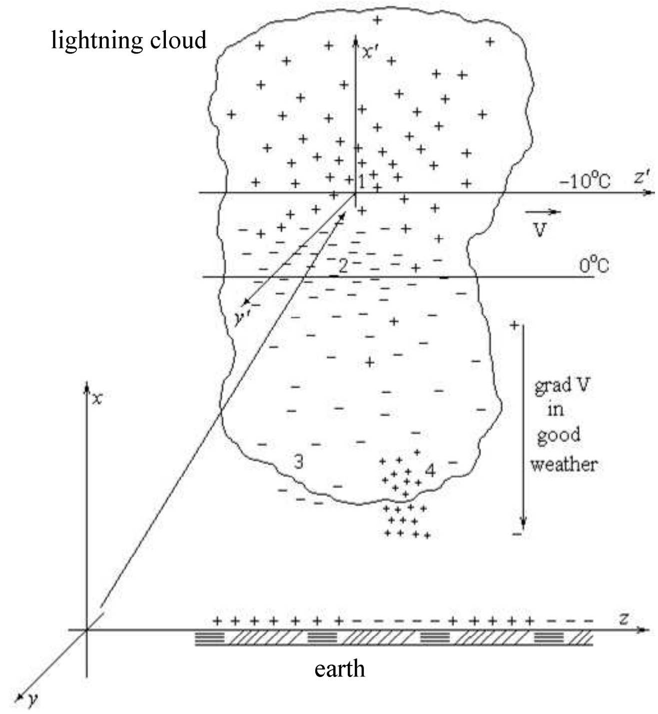

Fig. 1. Model of formation of electric charges in a lightning cloud

A model of an electric dipole in a lightning cloud in Fig.2 is presented. A lighting cloud is connected with moving coordinate system. $x'y'z'$. It moves in a plane, parallel to YOZ at a speed $v \approx const.$

The method of mirror images is also used [1]. The atmosphere (the upper semi space) has electric permeability ε_1, and magnetic permeability μ_1. The earth (down semi space) has electric permeability - ε_2 and magnetic permeabilityμ_2, respectively. A case in which $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$, and $\varepsilon_2 = \varepsilon_0 \varepsilon_r$, $\mu_2 = \mu_0$, (where $\varepsilon_0 = 8{,}8510^{-12}\ F/_m$ and $\mu_0 = 4\pi 10^{-7}\ H/_m$ are characteristics of the free space) is considered [1].

This model makes it possible to apply the basic dependences on the electromagnetic potentials in Minkowski space [1, 5]. The elements of used four-dimensional magnetic potential are magnetic vector potential $\overrightarrow{A_\mu}$ and scalar electric potential V_ε [1, 5].

At the solution of inverse problem, the quantity of charge, the moment of dipole, its coordinates and velocity of movement are searched. In case of moving a dipole along the axis z only, for example, the unknown quantities can be reduced on 6.

For the inverse problem of identification the unknowns are:

- The charge of dipole q;
- The distance between charges of dipole l;
- The height, above the Earth's surface $x = h$ and the coordinates of the point y and z, where the lighting cloud is located;
- The velocity of movement in axis z -v_z.

III. AN ELECTRIC FIELD

A. *Determination of Four-Dimensional Magnetic Potential* $\overrightarrow{\Psi_\mu}$

The electromagnetic field vectors are the elements of dual Maxwell's tensor $F_{jk}^{(\mu)}$ [1].

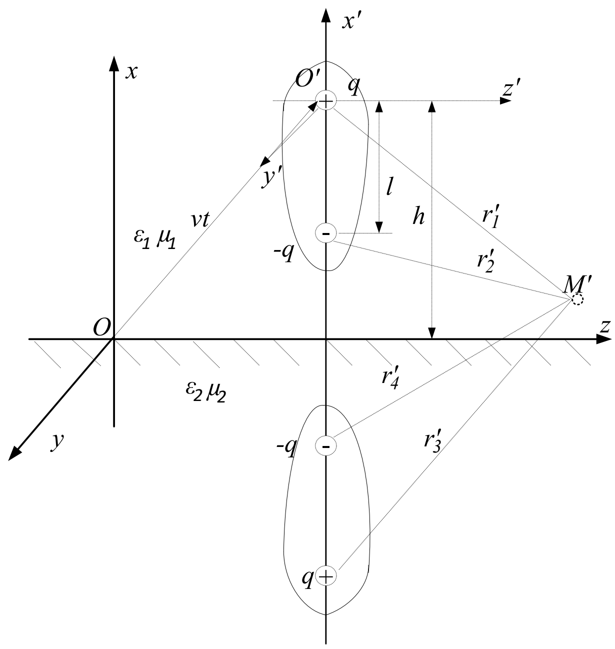

Fig. 2. Model of an electric dipole in a lightning cloud

First, the four-dimensional magnetic potential $\overrightarrow{\Psi'_\mu}$ is finding in moving coordinate system X'Y'Z' [1].

$$\vec{\Psi}'_\mu = \left\{ \begin{array}{c} \Psi'_{\mu 1} = A'_{\mu x} = 0 \\ \Psi'_{\mu 2} = A'_{\mu y} = 0 \\ \Psi'_{\mu 3} = A'_{\mu z} = 0 \\ \Psi'_{\mu 4} = \frac{j}{c} V'_\varepsilon = \\ = \frac{j}{c} \frac{q}{4\pi\varepsilon_0} \left[\left(\frac{1}{r'_1} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2} \frac{1}{r'_3} \right) - \left(\frac{1}{r'_2} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2} \frac{1}{r'_4} \right) \right] \end{array} \right\} \quad (1)$$

where:

$$r_1 = \sqrt{[x_1 - (x - v_x t)]^2 + [y_1 - (y - v_y t)]^2 + [z_1 - (z - v_z t)]^2};$$

$$r_3 = \sqrt{[x_2 - (x - v_x t)]^2 + [y_2 - (y - v_y t)]^2 + [z_2 - (z - v_z t)]^2};$$

$$r_2 = \sqrt{[x_1 - l - (x - v_x t)]^2 + [y_1 - (y - v_y t)]^2 + [z_1 - (z - v_z t)]^2};$$

$$r_4 = \sqrt{[x_2 - l - (x - v_x t)]^2 + [y_2 - (y - v_y t)]^2 + [z_2 - (z - v_z t)]^2}.$$

c is light speed in free space.

For the electromagnetic potentials and coordinates Lorentz transformations are used [5].

$$A'_{\mu x'} = A_{\mu x}\ ,\ A'_{\mu y'} = A_{\mu y}\ ,$$

$$A'_{\mu z'} = \alpha \left(A_{\mu z} - \frac{v}{c^2} V_\varepsilon \right)\ ,\ V'_\varepsilon = \alpha \left(V_\epsilon - v A_{\mu z} \right) \quad (2)$$

where α is relative factor, $\alpha = \frac{1}{\sqrt{1 - v^2/c^2}}$

In case of moving electric dipole, the four-dimensional magnetic potential $\overrightarrow{\Psi_\mu}$ in stationary coordinate system *XYZ*, is [1]:

$$\vec{\Psi}_\mu = \begin{cases} \Psi_{\mu 1} = A_{\mu x} = 0 \\ \Psi_{\mu 2} = A_{\mu y} = 0 \\ \Psi_{\mu 3} = A_{\mu z} = \alpha \mu_0 v \frac{q}{4\pi} \begin{bmatrix} \left(\frac{1}{r_1} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{1}{r_3}\right) \\ -\left(\frac{1}{r_2} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{1}{r_4}\right) \end{bmatrix} \\ \Psi_{\mu 4} = \frac{j}{c} V_\varepsilon = \frac{j}{c}\frac{q}{4\pi\varepsilon_0} \begin{bmatrix} \left(\frac{1}{r_1} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{1}{r_3}\right) \\ -\left(\frac{1}{r_2} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{1}{r_4}\right) \end{bmatrix} \end{cases} \quad (3)$$

where:

$$r_1 = \sqrt{[x_1 - (x - v_x t)]^2 + [y_1 - (y - v_y t)]^2 + [z_1 - (z - v_z t)]^2} ;$$

$$r_3 = \sqrt{[x_2 - (x - v_x t)]^2 + [y_2 - (y - v_y t)]^2 + [z_2 - (z - v_z t)]^2}.$$

$$r_2 = \sqrt{[x_1 - l - (x - v_x t)]^2 + [y_1 - (y - v_y t)]^2 + [z_1 - (z - v_z t)]^2} ;$$

$$r_4 = \sqrt{[x_2 - l - (x - v_x t)]^2 + [y_2 - (y - v_y t)]^2 + [z_2 - (z - v_z t)]^2}.$$

When the dipole speed is relative small, the relativity factor α acquires small values and at rest $\alpha \approx 1$ [1].

Magnetic vector-potential $\vec{A}_\mu$ can be observed in a stationary coordinate system only. The moving electric dipole creates a magnetic vector-potential $A_{\mu z}$, directed along the z axis(in the direction of movement). The magnetic field excites extra electric field in the stationary observer, respectively [1].

B. Determination of the electric field

The electric field components are elements of Maxwell tensor $\vec{F}_{jk}^{(\mu)}$ and they are found using the tensor dependence [1]

$$F_{jk} = \frac{\partial \Psi_k}{\partial x_j} - \frac{\partial \Psi_j}{\partial x_k} \quad (4)$$

Maxwell tensor $F_{jk}^{(\mu)}$ is tensor, which has 16 components and only six of them are independent, In this case $F_{jk}^{(\mu)}$ is an anti-symmetric tensor [1].

$$\left\{F_{jk}^{(\mu)}\right\} = \\ = \begin{Bmatrix} F_{11} = 0 & F_{12} = B_z & F_{13} = -B_y & F_{14} = -\frac{j}{c}E_x \\ F_{21} = -B_z & F_{22} = 0 & F_{23} = B_x & F_{24} = -\frac{j}{c}E_y \\ F_{31} = B_y & F_{32} = -B_x & F_{33} = 0 & F_{34} = -\frac{j}{c}E_z \\ F_{41} = \frac{j}{c}E_x & F_{42} = \frac{j}{c}E_y & F_{43} = \frac{j}{c}E_z & F_{44} = 0 \end{Bmatrix} \quad (5)$$

In stationary coordinate system, where the observer is located, in case of moving only in one direction (only on the z axis), for the electric field intensity components is obtained (6).

These results for the electric field will be used for identification of the moving electrical dipole about solution of the inverse problem.

$$\vec{E} = \begin{cases} E_x = -\frac{q}{4\pi\varepsilon_0} \begin{bmatrix} \frac{x_1 - x}{R_1^3} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{x_1 - l + x}{R_3^3} - \\ -\frac{x_1 - x}{R_2^3} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{x_1 - l + x}{R_4^3} \end{bmatrix} \\ E_y = -\frac{q}{4\pi\varepsilon_0} \begin{bmatrix} \frac{y_1 - y}{R_1^3} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{y_1 + y}{R_3^3} - \\ -\frac{y_1 - y}{R_2^3} + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{y_1 + y}{R_4^3} \end{bmatrix} \\ E_z = -\frac{q}{4\pi\varepsilon_0} \begin{bmatrix} \frac{z_1 - (z - v_z t)}{R_1^3} + \\ + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_0 + \varepsilon_2}\frac{z_1 + (z - v_z t)}{R_3^3} - \\ -\frac{z_1 - (z - v_z t)}{R_2^3} + \\ + \frac{\varepsilon_0 - \varepsilon_2}{\varepsilon_2 + \varepsilon_2}\frac{z_1 + (z - v_z t)}{R_4^3} \end{bmatrix} \end{cases} \quad (6)$$

where:

$$R_1 = \sqrt{[x_1 - x]^2 + [y_1 - y]^2 + [z_1 - (z - v_z t)]^2} ;$$

$$R_3 = \sqrt{[x_2 - x]^2 + [y_2 - y]^2 + [z_2 - (z - v_z t)]^2}.$$

$$R_2 = \sqrt{[x_1 - l - x]^2 + [y_1 - y]^2 + [z_1 - (z - v_z t)]^2} ;$$

$$R_4 = \sqrt{[x_2 - l - x]^2 + [y_2 - y]^2 + [z_2 - (z - v_z t)]^2}.$$

IV. SOLUTION OF THE INVERSE PROBLEM

The solution of inverse problem follow the algorithm described in previous work [1]. In case of identification of electric dipole, the same algorithm is applied.

The number of unknown values is determined limited to 6, in case of moving only in one direction along z-axis , to be able to reliably solve the inverse problem. The average dielectric constant of the lower half-space ε_2 as an unknown value can be included also, but this would further complicate the solution of the problem [1].

The measurements the electric field intensity are performed at *m* numbers of different points in the space for a selected time *t* [1]. The components of electric field E_x, E_y and E_z, which have the critical values and these values can lead to the occurrence of lightning, are observed [1].

The measured values, which match to the observed field component, are replaced in the equations of system (6) [1]. These values of the unknown $q , l, x_1, y_1, z_1, v_z$ are searched, that satisfy the equivalence [1].

A system of equations, whose rank is equal to the number *p* of the unknowns, is compiled [1]. In the left side of (6) the measurement values of electrical strength E_x, E_y, E_z in 6 point are substituted.

V. NUMERICAL RESULTS

The values of the quantities involved in the solution of the inverse problem are the following. The coordinates of observer (in point witch the measurement of electrical strength are made) – (in the center of coordinate system) $x = 0 , y = 0 , z = 0$; time of location $t = 2\ s$; electrical permeability of atmosphere (upper semi-space) $\varepsilon = \varepsilon_0$; electrical permeability of Earth (lower semi-space) - $\varepsilon = \varepsilon_r \varepsilon_0$, $\varepsilon_r = 10$ (in case of humid environment).

Like most of inverse problems this inverse problem is incorrect. Not always, the solution does respond of the necessary and sufficient conditions for correctness [1]. The solution must be unique and sustainable. For as much as, the condition of uniqueness can't be fulfilled, due to the nonlinearity of the system (6). In this case, however, there is a unique solution to the inverse problem. Because there are non-unique solutions, it is necessary to use the regularization procedure. The regularization procedure can be fulfilled according to different optimization procedures [1, 6]. It this case, an optimization criterion is [1]

$$\min_{s} \sum_{k=1}^{p} \left(E_{js}^{(k)} - \hat{E}_{js}^{(k)}\right)^2 ;\; j = 1,2,3 ;\; s = 1,2,3, \ldots, n \quad (7)$$

$\{E_j\} = \{E_1, E_2, E_3\} = \{E_x, E_y, E_z\}$ are component of electric field, crated of electric dipole , p is the number of unknown quantities and the rank of the equation system, $E_{js}^{(k)}$ are calculated values and $\hat{E}_{js}^{(k)}$ are measured values, respectively [1].

Because there are many ambiguous solutions, is necessary to be performed parallel calculations and observations, the number of which is s. The correct solution is this solution, whereby criterion (7) is satisfied. The common maximum number of computation and measurements is defined by product $m = p \times n$. The upper limit of m is defined by the possibilities of high level hardware and measurement devices.

The solution of inverse problem is made numerically [1]. The software package Mathematica is used.

The results of inverse problem solution by electric dipole identification at enumerate up data are:

-Value of charge $q = 10.03\,C$;

-Distance between charges of dipole $l = 7799\ m$;

-Velocity of movement - $v = 50\ m/s$;

-Height over the Earth surface $h = 9802\ m$;

-The coordinate $y = 0\ , m$;

-The coordinate $z = 2098\ , m$.

Finding the actually and unique solution makes sense and it can be realistically applicable, if the electrical dipole (lightning cloud) will create a critical electrical intensity, through which a lightning discharge can happened. In order to obtain a unique solution, it is necessary to make measurements at several points simultaneously. On this case, more points of measurement are needed, than in the representation of a charged cloud with a single concentrated charge [1].

Conclusion

The electric charge of a lightning cloud is modeled by means of an electric dipole.

The possibilities of four-dimensional potential for determination of electrical field, created by moving exciter – electrical dipole are shown.

An inverse problem is solved. The results of an identification of moving lightning cloud, modelled by an electrical dipole are - the amount of the electric dipole charges, distance between charges in dipole, the height above the earth's surface, the distances between point of identification and the lightning cloud in the both directions of the surface of the Earth and velocity of movement.

The occurrence of a high-intensity electric field depends on the accumulation of sufficiently large electric charges in the cloud. Therefore, the task of identifying moving electric charges that can cause an electric discharge in the form of lightning is essential.

Proposed approach can be applied for solving of analogy inverse problem in cases of identification of moving charged objects with different velocity like unidentified objects, presented by electrical dipole or electrical multipoles.

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