Fractional Control of a Model of the Current-Decoupled AC Motor

B. Grasiani, N. Nikolova

***Abstract* – This paper proposes an approach to oppose the parameters disturbances in control system. This control principle is applied in the present work to the Current-Decoupled AC Motor of a technological module.**

***Index Terms* – fractal control, robust stability margin, robust performance margin, robust performance, robust stability.**

I. Introduction

It is known in [4÷10] systems of control by non-integer order based on generalized fractional calculus. The use of algorithms based on generalized fractional calculus is recommended as efficient when the dominants parametric disturbance are in objects and one of the general exigency for the performance of control system is a stability and robust stability then.

In general, the fractal control systems are integer order dynamic systems. They approximate to rational functions in a limited frequency range the properties of hypothetical irrational fractal dynamic systems. For the approximation range, coinciding with the operating range of the control systems of the specific object, they have the properties and characteristics of the fractal dynamic systems, which join them into the class of robust control systems. One of the possible applications of the method for synthesis of fractal control systems of simplified model of the current-decoupled induction motor as part of the synergetic operation of the robotic system developed in the project КП-06-Н37/17 is presented in this paper.

Model of the control object

The mathematical model (1) of the current-decoupled AC induction motor known in the literature [2] can be reduced to a second-order system when the motor is operating in its constant-torque field [1÷3].

It should be noted that the current-controlled PWM (Pulse-width modulation) inverter is also included in this simplified equivalent model. Therefore, its input is the desired torque current command.

$$W_O(p)=\frac{y(p)}{u(p)}=k_T\,\frac{k_e}{\tau_e\,p+1}\,\frac{k_m}{\tau_m\,p+1}\,. \qquad (1)$$

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B. Grasiani is with the Industrial Automation Department, FA, Technical University of Sofia, 1000 Sofia, Bulgaria (e-mail: bgrasiani@tu-sofia.bg).

N. Nikolova is with the Industrial Automation Department, FA, Technical University of Sofia, 1000 Sofia, Bulgaria (e-mail: ninan@tu-sofia.bg).

where: $k_T = T_e\, i_{mr}$ – torque constant; T_e [N.m] – moment created by the motor; i_{mr}, [A] – equivalent stator-based magnetizing current taking a component for magnetic flux leakage; k_e and τ_e are the corresponding electrical time constant and gain of the current loop controller. k_m and τ_m are the equivalent mechanical time constant and gain of the loaded motor.

This simplified model (1) is used to illustrate the application of algorithms to synthesis of controller of non-integer order in speed control (revolutions per minute) of an AC motor.

Fractal control system

Control system with feedback is shown in fig. 1, where: $R_{D_{NE}}(p)$ (2) is an approximating fractal differentiator (D controller), $W_o(p)$ (1) is the object of control (speed (revolutions per minute) of an AC motor), $u(p)$ - output signal of the controller set to the nominal process parameters, $\varepsilon(p)$ – error. The method of polynomial recursive approximation [4,5,6,10] is used in the synthesis of the controller, which follows the stages of the analytical synthesis (3 ÷ 8). The method must be convergent to the algorithm, in order the exigencies (9) ÷ (11) to be satisfied.

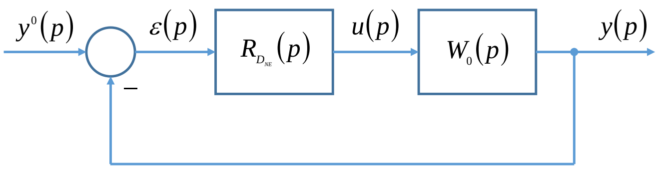

Fig. 1

$$\mathrm{R}_{D_{NE}}=\left(D^{\beta}\right)_{app}=\left(\frac{1+p\left(\omega_{bD}\right)^{-1}}{1+p\left(\omega_{hD}\right)^{-1}}\right)^{\beta}\prod_{j=1}^{N}\left(\frac{1+p\left(\omega_{Dj}\right)^{-1}}{1+p\left(\omega'_{Dji}\right)^{-1}}\right),$$
$$\forall\,\omega\left(\bar{\ell}_a,\bar{\ell}_m\right)\in\left[\omega_{DA}\,,\ \omega_{DB}\right],\ \left\{0<\beta<1;\ 1<\beta<2\right\}; \qquad (2)$$

$$n' = 2\left(1-(\pi)^{-1}PM_m^{nom}\right) \qquad (3)$$

$$N\geq 5\,;\ \omega_u > 250\ \ \omega_c;$$
$$\beta = n-n' = n - 2\,(\pi)^{-1}arc\ sin\left(GM_m^{nom}\right)^{-1} \qquad (4)$$

$$\omega_{DA}=0.1\,\omega_u\ ,\ \lambda=\left(\omega_h\,\omega_b^{-1}\right)^{(0.1\,+\,\beta)} \qquad (5)$$

$$\omega_{DB}=10\,\omega_u\ ;\ \eta=\left(\left(\omega_h\ \omega_b^{-1}\right)^{N-1}\right)^{(0.9\,-\,\lambda)} \qquad (6)$$

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$$\omega_b = 0.2\ \omega_A;\ \omega'_{i+1} = (\lambda\ \eta)^i .\ \eta^{0,5}\ \omega_b \qquad (7)$$

$$\omega_h = 1{,}2\ \omega_B;\ \omega_{i+1} = (\lambda\ \eta)^i .\ \lambda .\eta^{0,5}\ \omega_b \qquad (8)$$

$$0.5(\omega'_A - \omega_B) < (\omega_u - \omega_c)^{-1}; \qquad (9)$$

$$\omega_u > \omega_c;\omega_u \geq 250\ \omega_c;\ \omega_A > \omega_c;\ \omega_B >> \omega_c; \qquad (10)$$

$$\omega_b > \omega_c;\omega_b < \omega_A;\ \omega_h > \omega_B; \quad (\lambda\eta)_{opt} = 3.98;\ (\omega_h/\omega_b)_{opt} = 250 \div 600 \qquad (11)$$

Where:

D^β	-	fractal operator (original, irrational function);
D^β_{app}	-	approximating operator (approximation of the original, rational function);
i, j	-	counter of the components of the approximating polynomial (integers numbers);
M, N	-	number of the participating lead-lag in the approximating polynomial (integers numbers);
GM_m^{nom}, PM_m^{nom}	-	estimated values of stability: gain margin and phase margin of the synthesized nominal system;
$(\omega_i)^{-1}, (\omega'_i)^{-1}$	-	time constants of the participating lead-lag in the approximating polynomial (real, positive numbers);
$\omega_u,\ (\omega_u)^{-1}$	-	unit frequency and basic time constant of the fractal controller;
n'	-	order of the object model;
$\hat{W}_0*,\ e^{-\tau* p}$	-	rational and irrational components in the nominal model W_0* of object W_0;
ω_b, ω_h	-	lowest and highest frequency of approximation;
ω_A, ω_B	-	lower and upper frequency of the approximation range;
λ, η	-	recursive factors (recursion indicators);
$\bar{\ell}_a; \bar{\ell}_m$	-	additive and multiplicative disturbance.

II. Numeric Example

Current-Decoupled AC Servo Motor (1) is represented by its nominal model of object transfer function (12) and transfer function of the model in the upper limit of the disturbance (13).

$$W_O*(p) = 1\frac{0.5}{(0.004p+1)}\frac{200}{(p+1)} \qquad (12)$$

$$W_O^{■}(p) = \frac{250}{(0.008p+1)}\frac{1}{(5p+1)} \qquad (13)$$

The criteria set in the synthesis of a controller (20), need to be criteria for performance: settling time (14) and overshoot 0% (15), robust stability and robust performance (16) of the synthesized system, for the set of the uncertainty model Π . (17) and (18) are the functions of sensitivity and additional sensitivity, and ν (19) is an integral disturbance. The result of the synthesis of the controller according to the method of polynomial recursive approximation (3) ÷ (11) is presented by (20).

$$t_S \cong 4, \sec \qquad (14)$$

$$\sigma = 0\% \qquad (15)$$

$$\Pi = \left\{ \begin{array}{ll} \Delta G : \left| \dfrac{G(j\omega) - G*(j\omega)}{G*(j\omega)} \right| \leq \bar{\ell}_m(\omega) = G^{[]} & \\ \left\| \eta\ \bar{\ell}_m \right\|_\infty = \sup_\omega \left|\eta \bar{\ell}_m\right| < 1 ; & \bar{\ell}_m(\omega) < |\eta|^{-1}, \forall\omega \\ \left\| e\nu \right\|_\infty = \sup_\omega |e\nu| < 1 ; & \left|\eta\ \bar{\ell}_m\right| + |e\nu| < 1, \forall\omega \end{array} \right\} \qquad (16)$$

$$e(\omega) = (1 + R*(\omega) G*(\omega))^{-1} \equiv \equiv \Phi_{y^0\varepsilon}(\omega),\ (e(\omega) = 1 - \eta(\omega)) \qquad (17)$$

$$\eta(\omega) = R*(\omega)\ G*(\omega)\ (1 + R*(\omega) G*(\omega))^{-1} \equiv \equiv \Phi_{y^0 y}(\omega),\ (\eta(\omega) = 1 - e(\omega)) \qquad (18)$$

$$\upsilon = \upsilon(\nu, \xi, f) \qquad (19)$$

$$R = 0.0066 \times \left(\begin{array}{c} \dfrac{0.08p+1}{0.08p} \times \dfrac{0.001141217\ p+1}{0.0000057236\ 9\ p+1} \times \\ \times \dfrac{0.00004659\ \ p+1}{0.0000002336\ 7\ p+1} \times \\ \times \dfrac{0.000001903\ \ p+1}{0.0000000009\ 5395\ p+1} \end{array} \right) \qquad (20)$$

The synthesized system (fig. 1) is modeled and simulated in nominal parametric mode $(W_O(p) = W_O*)$ and in the upper limit of the disturbance mode $(W_O(p) = W_O^{■})$. The time and frequency characteristics are shown as follows:

- the step response of the closed loop system in nominal $h_{W*}(t)$ and disturbed $h_{W■}(t)$ in the upper limit mode – fig. 2.
- the frequency characteristics of the open loop system $W(j\omega)$ in nominal and disturbed parametric mode (in the upper limit of the disturbance mode) – fig. 3 and fig. 4.

The analysis of the performance in nominal parametric mode confirms that the system satisfies criteria (14) and (15) (fig. 2).

The analysis of the performance in the disturbed at the upper limit parametric mode of the system (fig. 2 – fig. 4) confirms that it keeps stability, reflecting the settling times and illustrating the gain margin and phase margin.

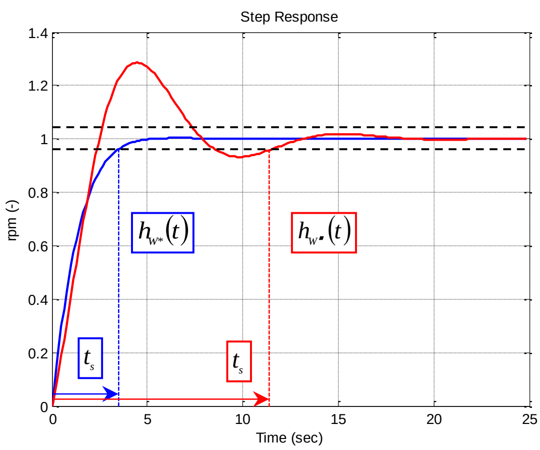

Fig. 2

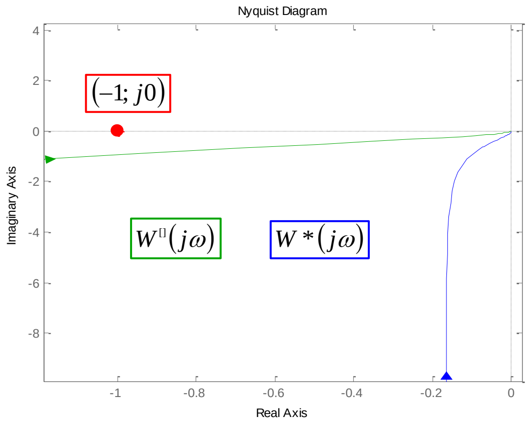

Fig. 3

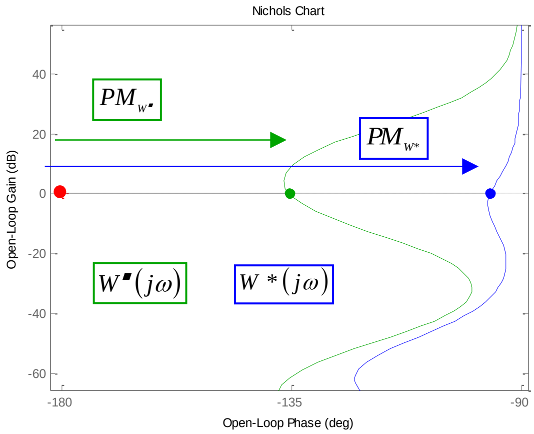

Fig. 4

III. Robust Analysis of the Synthesized Fractal Control System

The influence of the uncertainty on the performance of the synthesized system in a pre-specified range of reparameterization and/or restructuring in the model of the controlled object (12)-(13) will be studied in this part of the paper.

By definition, the analyzed system has robust stability and robust performance, if it satisfies exigencies (16) for the whole frequency range. For the open loop system in the Nyquist space, the parameterization/restructuring in the control object can be represented by a circle π (21) with circumference π^0 (22), with radius r^0 (23) and with centers at the points ω_i. In this case, the exigencies for robust stability and robust performance can be expressed in the characteristics of an open loop system with (24) and (25).

$$\pi(j\omega)\in W(j\omega),\ (\forall\omega,\ \omega\in[0,\infty)) \tag{21}$$

$$\pi^0(j\omega)=\begin{cases}\mathrm{Re}^0(\omega_i)=\mathrm{Re}^*(\omega_i)+r(\omega_i)\cos\Omega,\ (\Omega\in[0,\infty))\\ \mathrm{Im}^0(\omega_i)=\mathrm{Im}^*(\omega_i)+r(\omega_i)\sin\Omega,\ (\Omega\in[0,\infty))\end{cases} \tag{22}$$

$$r^0(\omega_i)=|\ell_a(\omega_i)R(\omega_i)|=|\ell_m(\omega_i)R(\omega_i)W_0*(\omega_i)| \tag{23}$$

The results of the frequency analysis are visualized as follows:

– 2D Nyquist-robust analysis (24) – (25) according to the characteristics of the open loop systems - in Fig. 5;

– the robust analysis according to the characteristics of the closed loop systems (17) [2,5] in fig. 6 and fig. 7.

The results (fig. 5 – fig. 7) prove analytically that in the conditions of reparameterization/restructuring (12) ÷ (13) the synthesized system (fig. 1) satisfies the exigencies (16), (24) and (25) for robust stability, and for robust performance.

$$|1+R(\omega)W_0*(\omega)|>r^0(\omega),\forall\omega \tag{24}$$

$$|1+W_0*(\omega)R(\omega)|>|W_0*(\omega)R(\omega)|\overline{\ell}_m(\omega),\forall\omega \tag{25}$$

According to the characteristics of the open loop systems can be determined: • the margin of robust stability $k_{M\,SOL}(\omega)$ (26) and • the margin of robust performance $k_{M\,POL}(\omega)$ (27). The margin of robust stability is shown in fig. 8 and fig. 10, and the margin of robust performance - in fig. 9 and fig. 10.

$$k_{MSOL}(\omega)=r^0(\omega)|1+R(j\omega)W_0*(j\omega)|^{-1}\le 1,\ (\forall\omega,\omega\in[0,\infty)) \tag{26}$$

$$k_{M\,POL}(\omega)=\frac{|1+R(j\omega)W_0*(j\omega)|-r^0(\omega)}{|1+R(j\omega)W_0^{*}(j\omega)|}\le 1,\ (\forall\omega,\omega\in[0,\infty)) \tag{27}$$

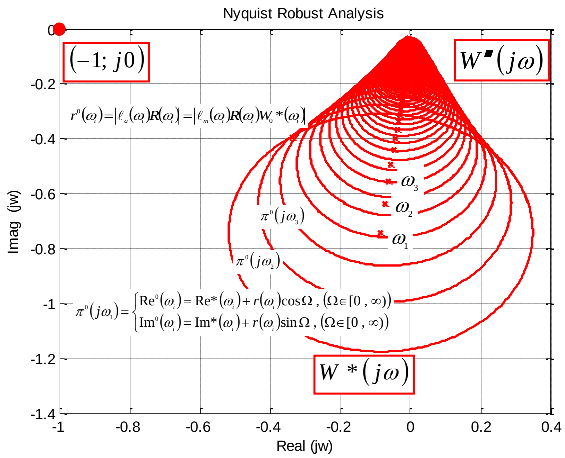

Fig. 5

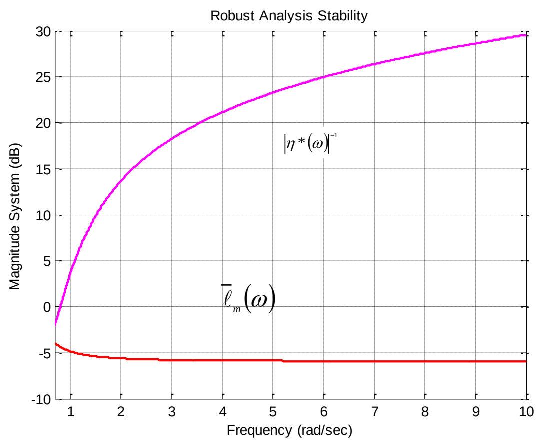

Fig. 6

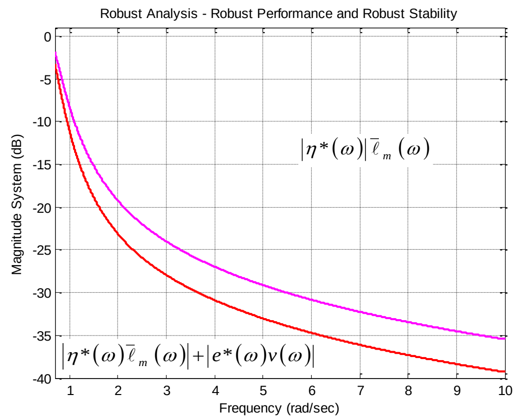

Fig. 7

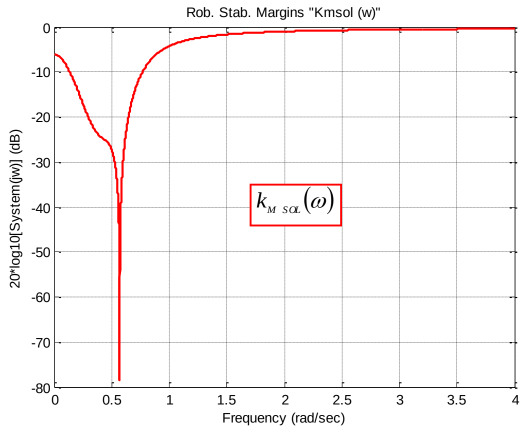

Fig. 8

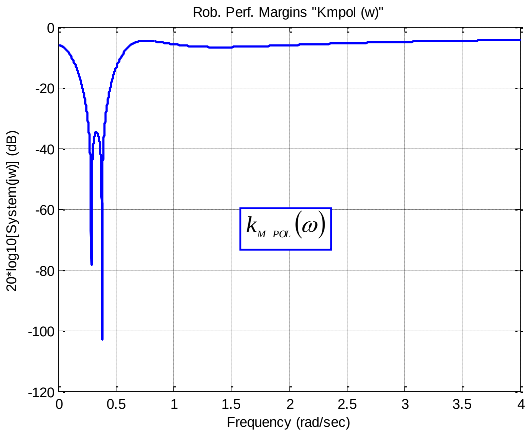

Fig. 9

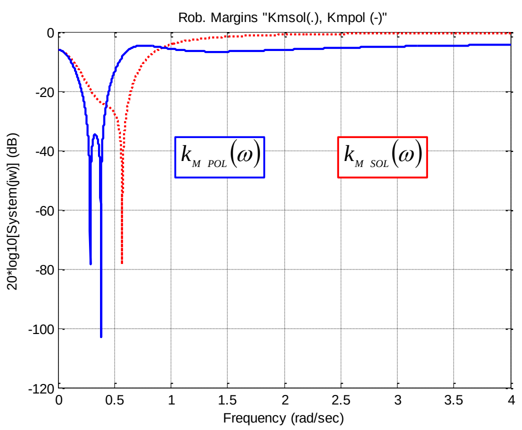

Fig. 10

IV. Conclusion

This paper proposes an approach of application of fractal algorithms in the speed control of the current-decoupled AC induction motor. There is a controller adjusted by the method of polynomial recursive approximation [4,5,6,10], to a mathematical model known in the literature [1÷3] of the current-decoupled AC induction motor. The synthesized system is modeled and simulated and there are proving stability, quick response, step response without overshoot, robust stability, robust performance, robust stability margin and robust performance margin.

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