SIL Simulation of Model-Free Method for Improving of Time Varying Dynamic Measurements

Miroslava Baraharska, Tsonyo Slavov, Ivan Markovsky

***Abstract*—In this paper, a SIL simulation of developed model-free method for time varying dynamic measurements in control system is presented. As an example, the dynamic mass-measurement process is examined. The method is based on the on-line estimation of time varying parameters of linear regressive model by recursive least square method with constant trace of covariance matrix. The model order selection is performed by Akaike's information criteria. The performance of method with respect to the variance of measurement noise is empirically tested by simulation experiments. For the aim of comparison, the Kalman filter for estimation of unknown measurement is designed. The simulation results show the advantage of model-free method**

***Index Terms* — Dynamic measurements, Model-free method for dynamic measurements, Kalman filter, SIL simulation**

I. Introduction

Speed and accuracy of control variable measurement in control systems are some of the limitations that require further examination. To overcome these limitations, data measurements can be considered as dynamic processes and sensors as dynamic systems. With this approach, the problem of improving dynamic measurements can be formulated as unknown input signal estimation of a dynamic system. In this paper we observe one very common example in real applications – the process of mass measurement. In literature, many methods for improving measurements dynamics have been suggested. Depending on the quantity of known prior information, they are divided in two types – methods, which are based on a model of the measurement process and methods, which are not based on the model (model-free methods). The first type are usually based on compensators [1] or Kalman filtering [2]. Usually a compensator with inverted dynamics of the sensor is designed. The main idea is that an estimation of the unknown input signal can be obtained as convolution of the impulse response of the compensator and the measured transient response of the sensor [1]. Methods, based on compensators with recursive estimation of parameters can be found in [3], methods, based on IIR and FIR filtering in [4,5]. The second type of methods do not use model of the measurement process and the missing information is acquired in real time. These methods are usually based on neural networks, identification, adaptive filtration etc. [6]. In [7] it is suggested and in [8] is studied a method for improving the dynamic measurements, based on identification with the standard recursive least squares method. A comparison of this method and a standard Kalman filer is presented in [2]. We must note that the methods, which do not use a model are more realistic, since it is rarely the case that an accurate enough model of the measurement process exists. Furthermore, in the case of mass measurement the model parameters depend on the mass of the measured substance or form, i.e. it depends of the unknown quantity. Most of the existing methods based on identification estimate constant quantity measurements. In control systems the measured quantity is time-varying. This motivates the authors to extend the method proposed in [7], for the case of time-varying quantity.

In this paper, a method for improved dynamic measurements of time-varying quantity, without using a model of the measurement process, is suggested. This method is based on real time identification of a non-stationary dynamic system with a linear regressive model. Initially we choose structure parameters of the model based on Akaike information criterion. Modification of the recursive least squares is used. This modification provides constant trace of the estimated parameters covariance matrix, which keeps sensitivity of the method to changes in the measured quantity. The method is examined with different levels of measurement noise. The effect of the model order on the accuracy and speed of estimation has also been examined. A comparison to the method based on standard Kalman filter has been made. Obtained results show the advantage of the suggested method, as the dynamic measurement of time-varying mass is improved. In order to test applicability of the suggested algorithm of model free method for improved dynamic measurements, it is implemented in Schneider Electric PLC M251 and SIL simulation is performed. The results from comparison between Simulink simulation and SIL simulation are presented.

II. Improved Time-Varying Quantity Measurement Method, Based on Recursive Identification

Let us denote the DC-gain of the sensor with W, therefore the sensor output signal will be as follows

$$y = W\bar{u} + y_{transient} + v, \tag{1}$$

where $\bar{u}$ is the unknown value of the measured quantity, $y_{transient}$ is the sensor transient response, which indicates its dynamics and can be presented as the output of an autonomous system and v is white Gaussian noise, modelling the stochastic measurement error [2].

In order to solve the problem estimation of unknown input signal through identification, we have to obtain an regression model from (1). The autonomous system is

https://doi.org/10.47978/TUS.2021.71.02.007

modelled with the model

$$y_{transient}(k) = -a_1\Delta y(k) - a_2\Delta y(k-1) - \dots \\ -a_n\Delta y(k-n) + e(k), \quad (2)$$

where $a_i, i = 1, 2, ..., n$ are the model parameters and

$$\Delta y(k) = y(k) - y(k-1) \quad (3)$$

$e(k)$ is residual error of the model in the form of zero mean white Gaussian noise, which displays the inconsistency of the chosen model and the examined process, as well as the impact of immeasurable factors to the observations. We must note that in model (2) is used the computable signal $\Delta y(k)$ instead of previous values of the immeasurable signal $y_{transient}(k)$. Hence the estimation problem can be solved with a linear estimator and the DC-gain $G\bar{u}$ is eliminated from the model of autonomous system. From (2) we obtain the measurement model

$$y(k) = W\bar{u} - a_1\Delta y(k) - a_2\Delta y(k-1) - \dots \\ -a_n\Delta y(k-n) + e(k), \quad (4)$$

Noting that

$$\varphi(k) = \begin{bmatrix} W & -\Delta y(k) & -\Delta y(k-1) & \dots & -\Delta y(k-n) \end{bmatrix}^T \\ \theta = \begin{bmatrix} \bar{u} & a_1 & a_2 & \dots & a_n \end{bmatrix}^T, \quad (5)$$

equation (4) can be transformed into the linear regression model

$$y(k) = \varphi^T(k)\theta + e(k) \quad (6)$$

The model (6) for the whole measurement interval $k,\ k+1,\ k+2,\ ...,\ k+N^*$ transforms into

$$\underbrace{\begin{bmatrix} y(k) \\ y(k+1) \\ \dots \\ y(k+N^*) \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} W & \Delta y(k) & \dots & \Delta y(k-n) \\ W & \Delta y(k+1) & \dots & \Delta y(k-n+1) \\ \dots & \dots & \dots & \dots \\ W & \Delta y(k+N^*) & \dots & \Delta y(k+N^*-n) \end{bmatrix}}_{\Phi} \underbrace{\begin{bmatrix} \bar{u} \\ a_1 \\ \vdots \\ a_n \end{bmatrix}}_{\theta} + \underbrace{\begin{bmatrix} e(k) \\ e(k+1) \\ \dots \\ e(k+N^*) \end{bmatrix}}_{E}. \quad (7)$$

Ideally, if there is no measurement noise and the model structure is accurate, the residual error $E(k)$ from equation (7) will be eliminated and the unknown parameters θ will be obtained after $n+1$ measurements of the output signal through the expression

$$\theta = \Phi^{-1}Y \quad (8)$$

In practice, measurement noise is always present and there is difference between the model of sensor dynamics and the actual sensor dynamics. Therefore, in order to obtain the parameters in (7) least squares method is used, where the squared residual error is minimized for the whole observation interval

$$J = E^T E \quad (9)$$

It is well known that in order to obtain good filtering of the random component in the model, it is necessary to carry out much more measurements than the model order, e.g. $N^* = n+1$. Therefore parameter estimates are obtained with the least squares method by

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y \ . \quad (10)$$

It is seen that estimates exist if the matrix $\Phi^T\Phi$ is nonsingular. This is ensured if there is no linear combination in the regressors of the matrix, i.e. in the model there are no excessive regressors and no data, obtained from an experiment with constantly exciting signal of order no less than $n+1$. In this adaptation, the least squares method process accumulates data N^* in one computation and thus it is not suitable for improving measurement dynamics, since it is not appropriate for real time analysis. When estimating in real time, similar to the standard least squares method is the recursive least squares method, which is described by equations

$$G(k) = \frac{P(k-1)\varphi(k)}{\varphi^T(k)P(k-1)\varphi(k)+1} \\ \hat{\theta}(k) = \hat{\theta}(k-1) + G(k)[y(k) - \varphi^T(k)\hat{\theta}(k-1)],\ \hat{\theta}(0) = \theta_0, \quad (11) \\ P(k) = P(k-1) - G(k)\varphi^T(k)P(k-1), P(0) = \alpha I, \alpha > 0$$

where $P(k)$ is the covariance estimations matrix and $G(k)$ is a DC-gain vector. The algorithm (11) is suitable for a stationary system parameter estimation.

When estimating time-varying quantities, it is suggested to implement a modifications of the recursive least squares with time-varying forget factor which provides constant trace of the covariance matrix $P(k)$ [9, 10, 11]. The equation for updating the covariance matrix in the recursive least squares algorithm with constant forgetting factor is as follows

$$P(k) = \frac{1}{\lambda}\left[P(k-1) - G(k)\varphi^T(k)P(k-1)\right],\ P(0) = P_0 \quad (12)$$

Considering the trace of matrix $P(k)$ and multiplying (12) by λ we obtain

$$\lambda tr\left[P(k)\right] = tr\left[P(k-1) - G(k)\varphi^T(k)P(k-1)\right] \\ \lambda tr\left[P(k)\right] = tr\left[P(k-1)\right] - tr\left[G(k)\varphi^T(k)P(k-1)\right]. \quad (13)$$

Noting that $tr\left[P(k)\right] = tr\left[P(k-1)\right]$ from (13) we obtain

$$\lambda = 1 - \frac{tr\left[G(k)\varphi^T(k)P(k-1)\right]}{tr\left[P(k-1)\right]}.. \quad (14)$$

After placing $G(k)$ from (11) into (14) we obtain

$$\lambda = 1 - \frac{1}{tr\left[P(k-1)\right]}\frac{tr\left[P(k-1)\varphi(k)\varphi^T(k)P(k-1)\right]}{\varphi^T(k)P(k-1)\varphi(k)+1}. \quad (15)$$

The product in the numerator of the second addend (15) is a scalar, which brings us to the final form of the forgetting factor, providing constant covariance matrix trace

$$\lambda = 1 - \frac{1}{tr\left[P(k-1)\right]}\frac{\varphi^T(k)P(k-1)P(k-1)\varphi(k)}{\varphi^T(k)P(k-1)\varphi(k)+1}. \tag{16}$$

Finally, to estimate the unknown quantity we suggest to use the algorithm (11), with model (5), (6), and for updating of the covariance matrix equation (12) to be used instead of covariance matrix equation from (11). The forgetting factor λ is determined by equation (16). Algorithm starts with zero initial conditions for estimates and initial value of the covariance matrix $P(0)=P_0$. Since the covariance matrix trace is constant, higher values of P_0 will provide better sensitivity to changes in the measured quantity, but also bigger estimates covariation. The opposite – lower values of P_0 will provide better results for the estimates, but slower tracking when changes in the process occur. Later on the impact of different parameters as model order and measurement noise to the modified recursive least squares method of variance will be examined. Fig. 1 presents a scheme of the measurement process and estimation of the unknown quantity through the modified recursive least squares

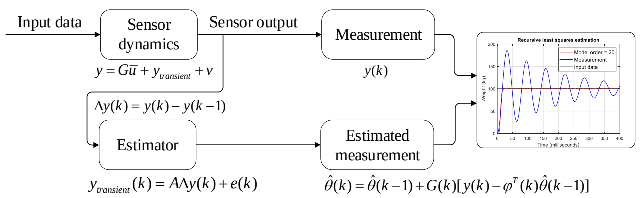

Fig.1. Scheme of the measurement process and estimation of the unknown quantity through the modified recursive least squares.

III. Kalman Filter for Improving Dynamic Measurements

For purpose of comparison, a Kalman filter for improving dynamic measurements of mass measurement is designed [2]. The measurement process dynamics is described by

$$(M+m)\frac{d^2y}{dt^2} = -cy - d\frac{dy}{dt} - Mg, \tag{17}$$

where $g = 9.81 m/s^2$ is the gravitational constant, $c=1$ is coefficient of elasticity, $d=1$ is the damping coefficient, $m=1kg$ is the platform mass, where the measured object is placed and M is the unknown object mass. The Kalman filter estimates the mass M, based on the model (17). Model (17) is presented in state-space form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + v \end{aligned}, \tag{18}$$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{c}{M+m} & -\frac{d}{M+m} \end{bmatrix}, B = \begin{bmatrix} 0 \\ -\frac{g}{M+m} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0 \tag{19}$$

where x is the vector of states, y is the sensor output and u is the unknown object mass. From (18) it is obvious that the measurement dynamics depends on the measured quantity, which makes a standard optimal Kalman filter design complicated. When forming matrices A and B for the design of standard Kalman filter, an average value of $M = 150kg$ will be used. The process v is zero-mean white Gaussian noise with covariance V_v. We use v to model the random sensor error. In order to estimate the unknown input signal, model (18) is extended with an additional state $x_u = u$. For the extended model we obtain

$$\begin{aligned} \dot{x}_u &= \eta \\ \dot{x} &= Ax + Bx_u \\ y &= Cx + v \end{aligned}, \tag{19}$$

where η is white Gaussian noise with covariance V_η. If we input the vector of states in (19) $\bar{x} = [x_1 \quad x_2 \quad x_u]^T$ we obtain

$$\begin{aligned} \dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}\eta \\ y &= \bar{C}\bar{x} + v \end{aligned}, \tag{20}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{c}{M+m} & -\frac{d}{M+m} & -\frac{g}{M+m} \\ 0 & 0 & 1 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

We use (20) to design Kalman filter with noise covariance $V_\eta = 10$, $V_v = 0.1$. Therefore, we obtain the state estimate $\hat{\bar{x}}$ from

$$\dot{\hat{\bar{x}}} = (\bar{A} - K_f\bar{C})\hat{\bar{x}} + K_f y, \tag{21}$$

where the Kalman filter coefficient K_f is defined by

$$K_f = D_e\bar{C}^T V_v^{-1}, \tag{22}$$

and the error covariance D_e is the positive definite solution of algebraic Riccati equation

$$\bar{A}D_e + D_e\bar{A}^T - D_e\bar{C}^T V_v^{-1}\bar{C}D_e + \bar{B}V_n\bar{B}^T = 0. \tag{23}$$

IV. Experimental Results

A set of time-varying mass estimating simulation experiments have been performed with the proposed modified dynamic measurement method. All of the experiments are conducted with identical initial conditions of the covariance estimations matrix

$$P(0) = nI, \tag{23}$$

where n is the model estimation order. The noise in measurement impact on the estimates quality has been studied. A set of experiments with noise covariance v of 0, 0.01 and 0.5 have been conducted. The noise variance are chosen according to model sensor random error. Fig. 2-4 illustrate sensor measurements, mass estimation results with the proposed method for model of order *n=3* and mass estimation results with the standard Kalman filter with the three corresponding noise variances. As expected, regardless of noise variance, the Kalman filter rapidly estimates unknown mass only when actual mass is *150kg*, which is the value that is used in the Kalman filter design, based on model (20). With the proposed method, we obtain several times better results than the sensor measurement for experiments without noise and with noise covariance of 0.01 (The exact mass value is obtained in 800*ms* with the sensor and in 170*ms* with the model-free method). For mass value of *150kg* the Kalman filter produces the best result, estimating the unknown quantity for only 20*ms*.

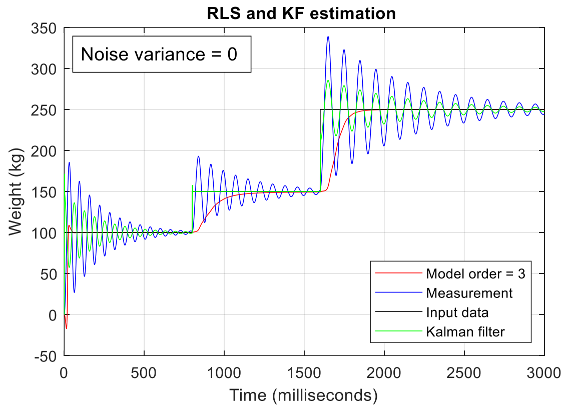

Fig. 2. Results with model of 3rd order and noise covariance $D_v = 0$

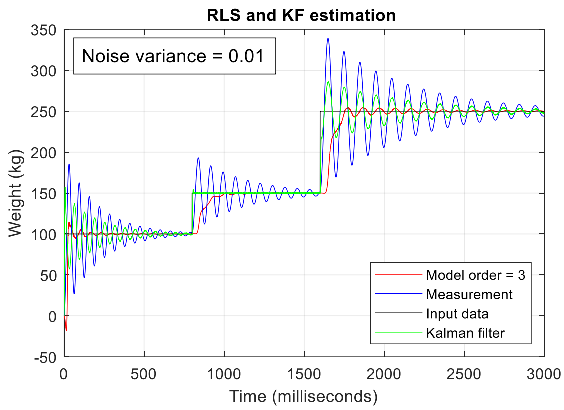

Fig. 3. Results with model of 3rd order and noise covariance $D_v = 0.01$

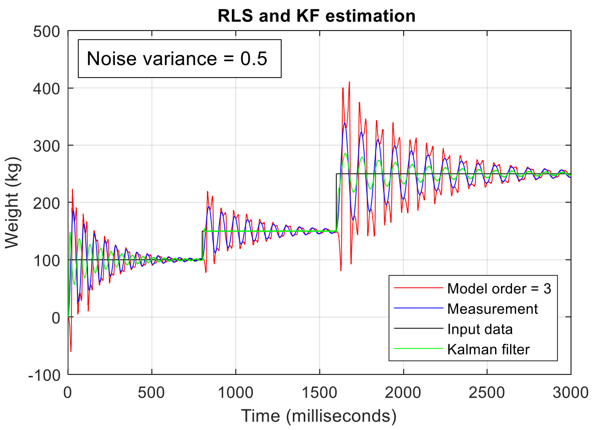

Fig. 4. Results with model of 3rd order and noise covariance $D_v = 0.5$

The reason for the poor results obtained from the model-free method when the noise variance increases is that the model is of insufficient order. This hypothesis is confirmed by the results shown in fig. 5, where we observe measured mass by the sensor and output signals of estimated autoregressive models of 5th and 20th orders.

In fig. 5 we find that the sensor measured signal matches perfectly to the estimated model of 20th order. In order to make an adequate decision on the estimated model order and achieve reasonable computation complexity and quality of the model, we use the Akaike information criterion.

$$J_m(\hat{\theta}) = \left(1 + 2\frac{\dim\theta}{N}\right) J(\hat{\theta}) \,, \qquad (24)$$

where $J(\hat{\theta}) = e^T e$ is the loss function and e is the residuals vector, computed based on the estimated parameters. The criterion (24) is presented on fig. 6 for model orders varying from 5 to 40. After model order 17 the criterion decrease is insignificant and the minimum is at order 25. In order to achieve compromise between complexity and accuracy of the model, following some experiments, we have determined that model of order 20 produces sufficiently accurate and fast estimation.

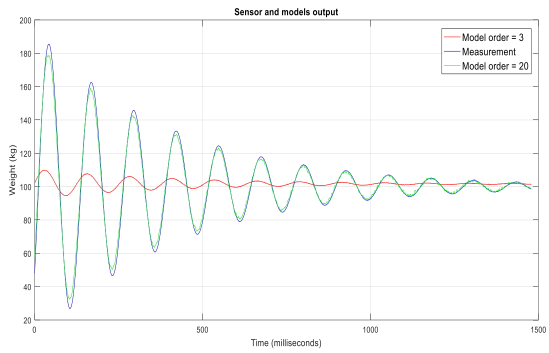

Fig. 5. Mass measured by the sensor and estimated mass with models of 3th and 20th orders

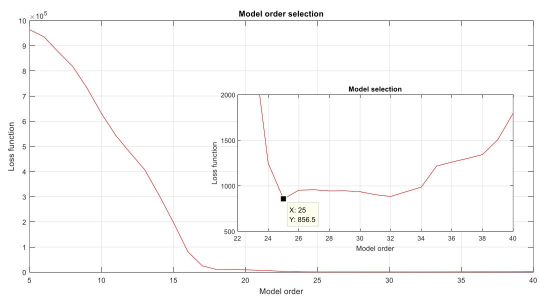

Fig.6. Akaike information criterion for 5th to 40th model order

Fig. 7 shows estimation results for models with orders from 5th and 20th and measurement noise covariance 0.05. When the mass is time-varying with model order 20, the suggested method estimates accurately the unknown mass for *20ms*, meaning that we obtain 40 times better results than the sensor measurement. Furthermore, this result is almost the same as the results with Kalman filter, designed based on the exact model of the measuring process.

TABLE I
SOURCE CODE OF PROPOSED ALGORITHM

```
// lamda
FOR m1 := 0 TO k DO              // Trace of P
    FOR m2 := 0 TO k DO
        IF m1 = m2 THEN
            Ptr := Ptr_p + P_p[m1,m2];
            Ptr_p := Ptr;
        END_IF
    END_FOR
END_FOR
Ptr_trace := Ptr_p;
Ptr_p := 0.0;

first := phi_t;                  // phi'*P
second := P_p;
Matrix_multiplication();

first := third;                  // phi'*P*G_rec
second := G_rec_out;
Matrix_multiplication();

f := 1-1/Ptr_trace*third[0,0];
 // P_matrix
 first := G_rec_out;             //G_rec*phi'
 second := phi_t;
 Matrix_multiplication();

 first := third;                 //G_rec*phi'*P
 second := P_p;
 Matrix_multiplication();

 FOR m1 := 0 TO k DO             //P = P - G_rec*phi'*P
     FOR m2 := 0 TO k DO
         P[m1,m2] := 1/f*(P_p[m1,m2] - third[m1,m2]);
     END_FOR
 END_FOR

 P_p := P;
```

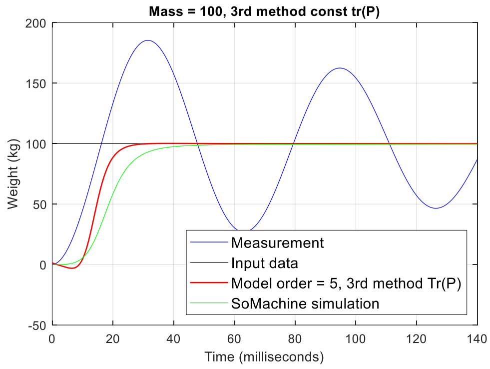

Fig. 8. Results from the modified estimation method maintaining constant trace of the covariance matrix

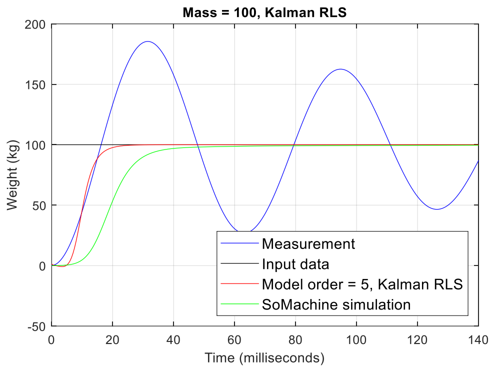

Fig. 9. Results from Kalman filter estimation, designed based on the actual value of the mass

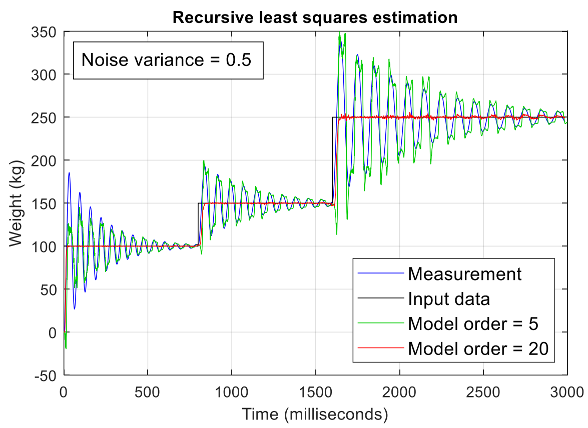

Fig. 7. Results for estimated model of order 5th and 20th

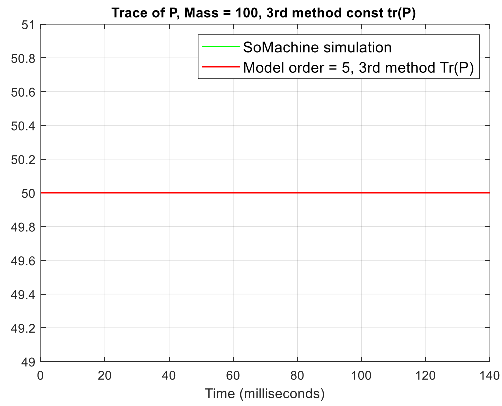

Fig. 10. Trace of the covariance matrix with suggested method

V. SIL SIMULATION OF THE SUGGESTED METHODS

To validate the proposed model-free estimation method in conditions close to the real ones, we have implemented algorithms in a software-in-the-loop simulation using the programmable logic controller software SoMachine. Table 1 presents the realization of the algorithm maintaining constant trace of the covariance matrix with the programming language Structured Text. Simulation results are presented using Matlab in fig. 8 to fig.11.

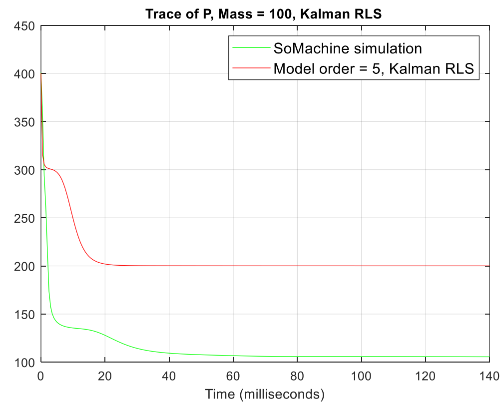

Fig. 11. Trace of covariance matrix with Kalman filter estimation

In fig. 8 and fig. 8 there is a delay in the *SoMachine* simulation, which is caused by the fact that when conducting an experiment in real time in the beginning there is no measurement information and therefore an estimation cannot be computed right away. In order to obtain initial estimations, we have to accumulate as many measurements as the selected estimation model order.

VI. CONCLUSION

This paper proposes a modified method improving measurements in dynamic systems, without using model of the process (model-free method). The method is derived from an existing method and is further developed for time-varying quantity estimation. A mass measurement process in control system is introduced as an example. The proposed method is compared to the standard Kalman filter, designed based on measuring process model with an average mass value. As approach to reduce the impact of noise to the performance of estimated quantity we propose to estimate the regressive model of higher order. The appropriate model order is chosen, based on the Akaike information criterion. The properties of the proposed method have been examined for three different measurement noise variance levels. Obtained results show the advantages of the modified improving measurement method even when using sensors with lower accuracy class. The SIL simulation results approved workability of suggested method and developed software for conventional programmable logic controller.

REFERENCES

[1] S. Eichstadt, C. Elster, T. Esward, and J. Hessling,"Deconvolution filters for the analysis of dynamic measurement processes: a tutorial" Metrologia, vol. 47, no. 5, p. 522–533, 2010, https://doi.org/10.1088/0026-1394/47/5/003

[2] I. Markovsky "An application of system identification in metrology," Control Engineering Practice, vol. 43, p. 85–93,2015, https://doi.org/10.1016/j.conengprac.2015.07.001

[3] W. Shu, "Dynamic weighing under nonzero initial conditions," IEEE Transactions on Instrumentation and Measurement,vol. 42, no. 4, p. 806–811, 1993, https://doi.org/10.1109/19.234489

[4] C. Elster, A. Link. T. Bruns, "Analysis of dynamic measurements and determination of time-dependent measurement uncertainty using a second-order model," Measurements Science and Technology, vol. 18, no. 12, p. 3682–3687, 2007, https://doi.org/10.1088/0957-0233/18/12/002

[5] M. Nied´zwiecki, P. Pietrzak, "High-Precision FIR-Model-Based Dynamic Weighing System," IEEE Transactions on Instrumentation and Measurement, vol. 65, no. 10, pp. 2349–2359, 2016, https://doi.org/10.1109/TIM.2016.2575180

[6] M. Nied´zwiecki and A. Wasilewski. Application of adaptive filtering to dynamic weighing of vehicles. Contr. Eng. Practice, 4, p.635–644, 1996, https://doi.org/10.1016/0967-0661(96)00045-7

[7] I. Markovsky Comparison of adaptive and model-free methods for dynamic measurement". In: IEEE Signal Proc. Letters 22, p. 1094–1097,2015. DOI: 10.1109/LSP.2014.2388369

[8] G. Quintana Carapia, I. Markovsky, R. Pintelon, P. Zoltan Csurcsia, and D. Verbeke. Experimental validation of a data-driven step input estimation method for dynamic measurements. IEEE Transactions on Instrumentation and Measurement, vol. 69, no. 7, p. 4843-4851,2020, doi: 10.1109/TIM.2019.2951865.

[9] Goodwin, G., C., Hill, D., J. Paianiswami, Towards an Adaptive Robust Controller. Proc. IFAC Symp. Identification and System Parameter Estimation, p. 997-1002, 1985, https://doi.org/10.1016/S1474-6670(17)60692-X

[10] Velev K. Adaptive Systems, Arkas Sofia, 1995 (in Bulgarian)

[11] Baraharska M. Slavov T., I Markovski Model-free method for time varying dynamic measurements in control system. Proceedings of TU-Sofia, vol 70 no.4 p51-60,2020 (in Bulgarian), https://doi.org/10.47978/TUS.2020.70.03.020