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H_∞ CONTROL DESIGN OF A MULTITANK SYSTEM

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Abstract: *This paper considers MATLAB® modelling and simulation of H∞ controller and its realization on the Multitank System. The first task is to study the physical plant the laboratory Multitank System and to apply a given mathematical model for optimal controller design. The general objective of the derived regulator is to reach and stabilize the level in the tanks by an adjustment of the pump operation or/and valves settings. Finally, it is necessary to simulate the obtained closed-loop system and to test its workability.*

Keywords: *Multitank system, H_∞ design, modeling, control, simulation*

1. INTRODUCTION

The Multitank System relates to liquid level control problems commonly occurring in industrial storage tanks. For example, steel producing companies around the world have repeatedly confirmed that substantial benefits are gained from accurate mould level control in continuous bloom casting [1]. The goal of the Multitank System (MTS) design is to study and verify in practice linear and nonlinear control methods. Different control strategies PID, LQR, adaptive or fuzzy logic [2], [3] can be used in this case.

The MTS has been designed to operate with an external PC-based digital controller. The control computer communicates with the level sensors, valves and pump by a given I/O board and the power interface. The I/O board is controlled by a real-time software which operates in MATLAB®/SIMULINK® environment.

The H_∞ controller [4] guarantees stability and acceptable performance with respect to time responses, disturbances and noises. The H_∞ methods have advantages to classical controllers in case when multivariable plants have to be regulated. They are capable to eliminate the undesired impact of the different channels of a particular system to control.

The paper is aimed at designing an H_∞ controller which makes the closed loop system reach and stabilize the level in the tanks by an adjustment of the pump operation and valves settings. We also simulate the behavior of the closed loop system with the designed controller in MATLAB® and SIMULINK®.

The remainder of the paper is organized as follows. In Section 2 we describe the MTS model. In Section 3 we present some theoretical facts regarding the H_∞ control problem. Section 4 is connected with consideration of the obtained simulation results before we end up in Section 5 with some concluding remarks.

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2. MULTITANK SYSTEM DESCRIPTION

The Multitank System [5] consists of a number of tanks placed above each other (fig. 1). Some of the tanks have a constant cross section, while others are spherical or prismatic, so having variable cross section. Liquid is pumped in the upper tank from the supply tank by the pump driven by a DC motor. The liquid outflows the tank due to gravity. The output orifices act as flow resistors, but can also be controlled from the computer.

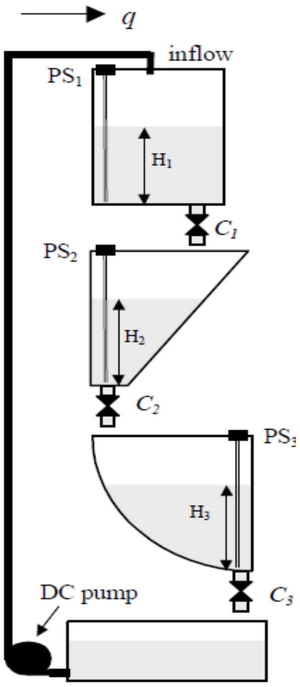

Fig. 1. *Multitank System*

The levels in the tanks are measured with pressure transducers. The frequency signals of the level sensors are connected to the digital inputs of RT-DAC/USB multi-purpose I/O board. There are four control signals send out from the board to the multi-tank system: three valve controls and one pump control signal. The appropriate PWM control signals are transmitted from digital outputs of the I/O board to the power interface, and next to the valves and to the DC motor. The speed of the DC motor is controller by a sequence of PWM pulses. The liquid levels in the tanks are the system states and they also represent the system outputs.

Modern methods of design of advanced controllers require high quality models of the process. The classical procedure of model development consists of the following steps: development of the mathematical model based on physics of the process; simplification of the model, i.e. turn into standard form; development of a simulation model; tuning the model parameters; verification of the model.

Liquid levels H_1, H_2, H_3 in the tanks are the state variables in the system (Fig. 1). For the tank system there are four controlled inputs: liquid inflow q, and valves settings C_1, C_2, C_3. Therefore several models of the tanks can be analysed, classified as pump-controlled system, valve-controlled system and pump/valve controlled system.

Several issues have been recognized as potential problems to perform high accuracy control of level of flow in the tanks: nonlinearities caused by shapes of the tanks; saturation-type nonlinearities introduced by min and max allowed level in the tanks; nonlinearities introduced by valve geometry and flow dynamics; nonlinearities introduced by pump and valves input/output characteristic curve.

The laminar output flow rate of an "ideal fluid" from a tank (fig.2) is governed by the Bernoulli law. This equation is obtained by performing calculation of the potential and kinetic energy of the fluid [6]

$$Q_r = \mu S\sqrt{2gH_0} \tag{1}$$

where

S – output area of the orifice,

μ – the orifice output coefficient.

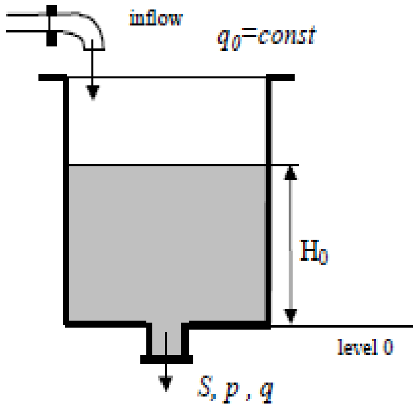

Fig. 2. *Outflow of ideal fluid*

Assuming a laminar outflow of an "ideal fluid" for a cascade of n-tanks, the model describing dynamics of the process can be obtained by means of mass balance

$$\begin{aligned}
&\frac{dV_1}{dH_1}\frac{dH_1}{dt} = q - C_1 H_1^{\alpha_1},\\
&\frac{dV_2}{dH_2}\frac{dH_2}{dt} = C_1 H_1^{\alpha_1} - C_2 H_2^{\alpha_2},\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,\\
&\frac{dV_n}{dH_n}\frac{dH_n}{dt} = C_{n-1} H_{n-1}^{\alpha_{n-1}} - C_n H_n^{\alpha_n}
\end{aligned} \tag{2}$$

where

V_1 , $V_2, \ldots,$ V_n – fluid volumes in the tank,

C_1 , $C_2, \ldots,$ C_n – fluid volumes in the tank,

$\alpha_i = 1/2$.

In this paper we consider an optimal approach to control the tank system. The block diagram of the controlled process is presented on figure (3).

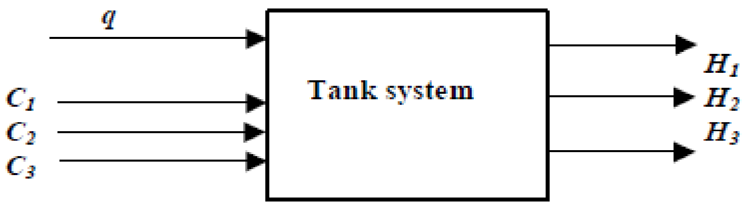

Fig. 3. *Control through the pump input and computer controlled valves*

The system of differential equations describing the cascade interconnected tanks in the equilibrium state are show below:

$$\begin{cases} \frac{dH_1}{dt} = k_{11}\Delta H_1 + 0\Delta H_2 + 0\Delta H_3 + k_{12}\Delta C_1 + 0\Delta C_2 + 0\Delta C_3 + \frac{1}{aw}q \\ \frac{dH_2}{dt} = k_{21}\Delta H_1 + k_{22}\Delta H_2 + 0\Delta H_3 + k_{23}\Delta C_1 + k_{24}\Delta C_2 + 0\Delta C_3 + 0q \\ \frac{dH_3}{dt} = 0\Delta H_1 + k_{32}\Delta H_2 + k_{33}\Delta H_3 + 0\Delta C_1 + k_{34}\Delta C_2 + k_{35}\Delta C_3 + 0q \end{cases} \quad (3)$$

Applying Teylor series expansion the state-space model of the linearized multi-tank system is presented below. The plant has four inputs and three outputs, which coincide with the system states.

$$\begin{pmatrix} \dot{H}_1 \\ \dot{H}_2 \\ \dot{H}_3 \end{pmatrix} = \begin{pmatrix} k_{11} & 0 & 0 \\ k_{21} & k_{22} & 0 \\ 0 & k_{32} & k_{33} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} + \begin{pmatrix} \frac{1}{aw} & k_{12} & 0 & 0 \\ 0 & k_{23} & k_{24} & 0 \\ 0 & 0 & k_{34} & k_{35} \end{pmatrix} \begin{pmatrix} q \\ C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} q \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} \quad (4)$$

3. H_∞ CONTROL PROBLEM

The H_∞ optimal control problem [7] is related to finding all stabilizing controllers K such that

$$\|F_l(P,K)\|_\infty = \max_w \sigma(F_l(P,K)(jw)). \quad (5)$$

Here with F_l we denote the linear fractional transformation P-plant and K-controller for the scheme below:

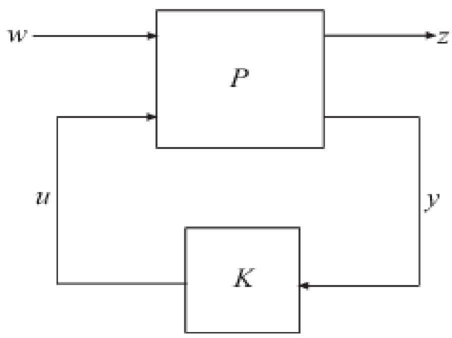

Fig. 4. *Lower linear fractional transformation of P and K*

w – disturbance signals,
z – performance signals,
u – control signals,
y – output signals.

In fact we solve the suboptimal H_∞ control problem where the relation below should be satisfied:

$$\|F_l(P,K)\|_\infty \prec \gamma \quad . \tag{6}$$

To perform the computational procedure we apply the so called γ-iteration where γ should be less than one.

In the H_∞ design we would like to achieve not only stability but also nominal performance, i.e. the inequality below should be satisfied:

$$\left\|W_p(I+PK)^{-1}\right\|_\infty \prec 1 \, , \tag{7}$$

where W_p is a low pass filter and $(I+PK)^{-1}=S$ is the sensitivity function, i.e. the relation between the disturbance and the output of the system.

4. SIMULATION RESULTS

To perform the H_∞ design we apply the γ-iteration and achieve a value of γ 0.95, which is less than one thus the nominal performance test is satisfied.

On (5) we present the singular values of the closed-loop system, in low frequency domain we have big amplification, which means that we can ensure good attenuation of disturbance signals.

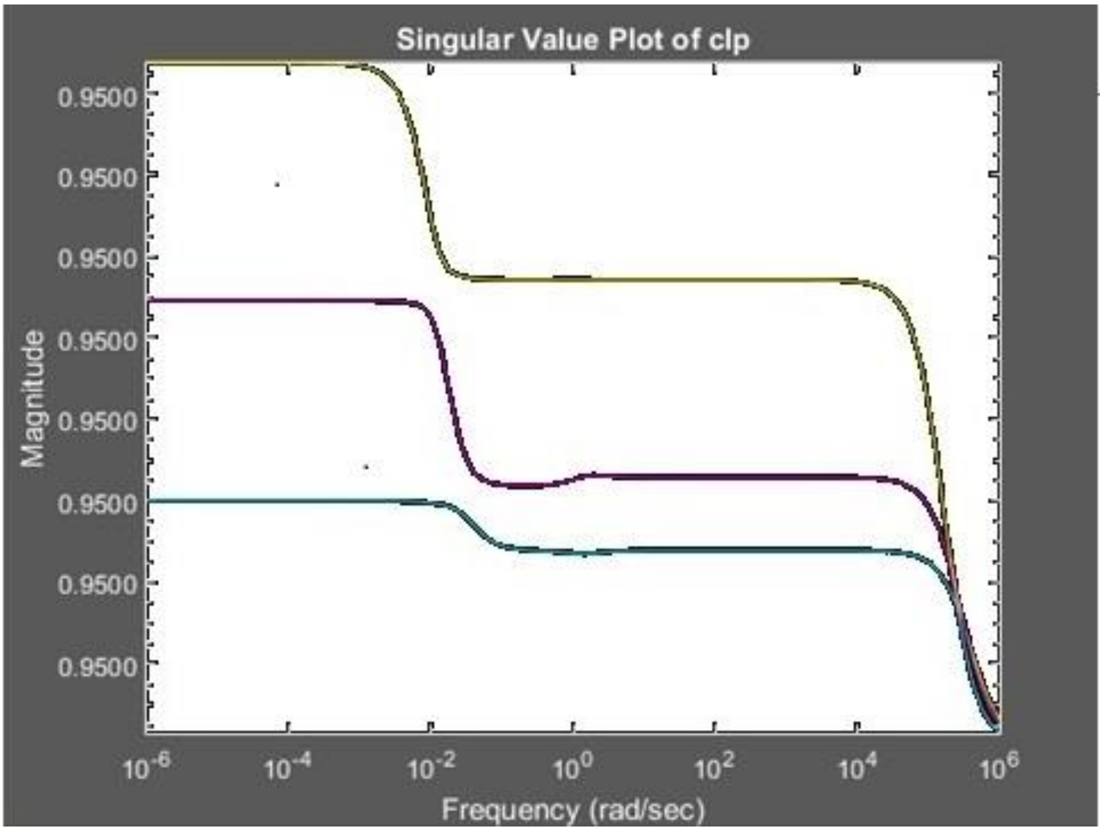

Fig. 5. *Singular values of the closed-loop system*

Further on fig. 6 we show the frequency responses of the sensitivity function and the inverse performance filter. It is obvious that the nominal performance specification (6) is satisfied.

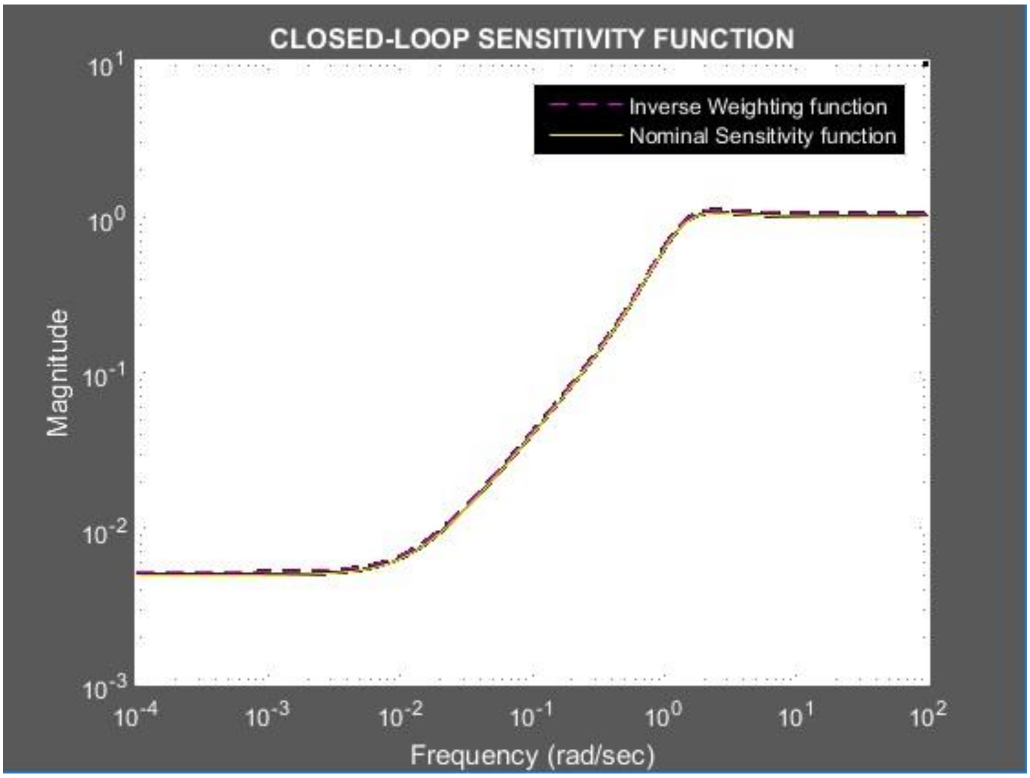

Fig. 6. *Closed-loop sensitivity function*

On figures (7) and (8) we present the time domain responses of the closed-loop system and the control signals.

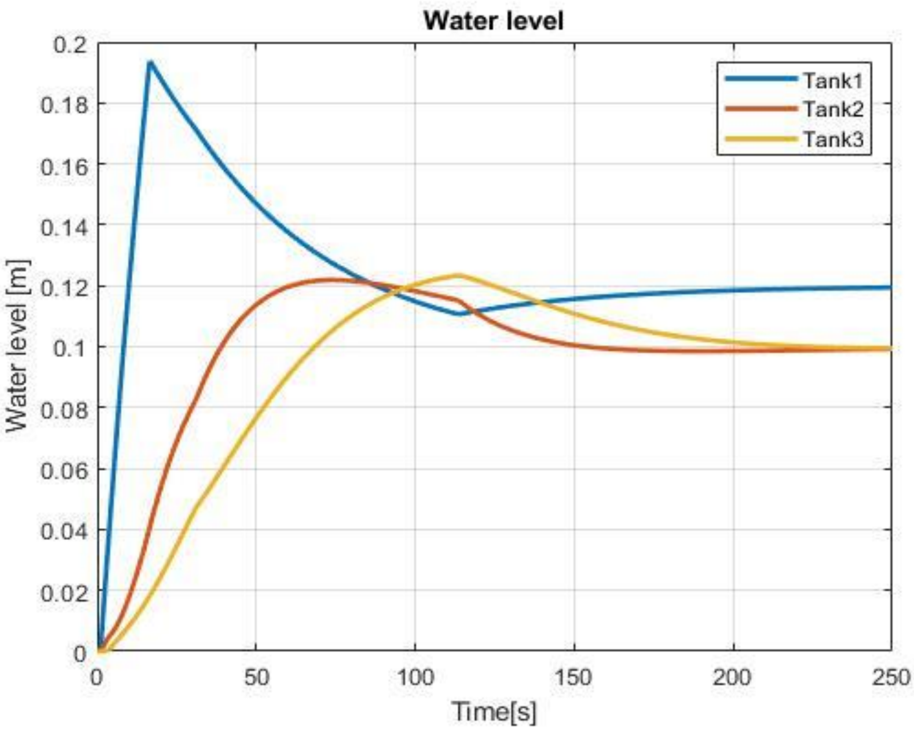

Fig. 7. *Time responses of the closed-loop system*

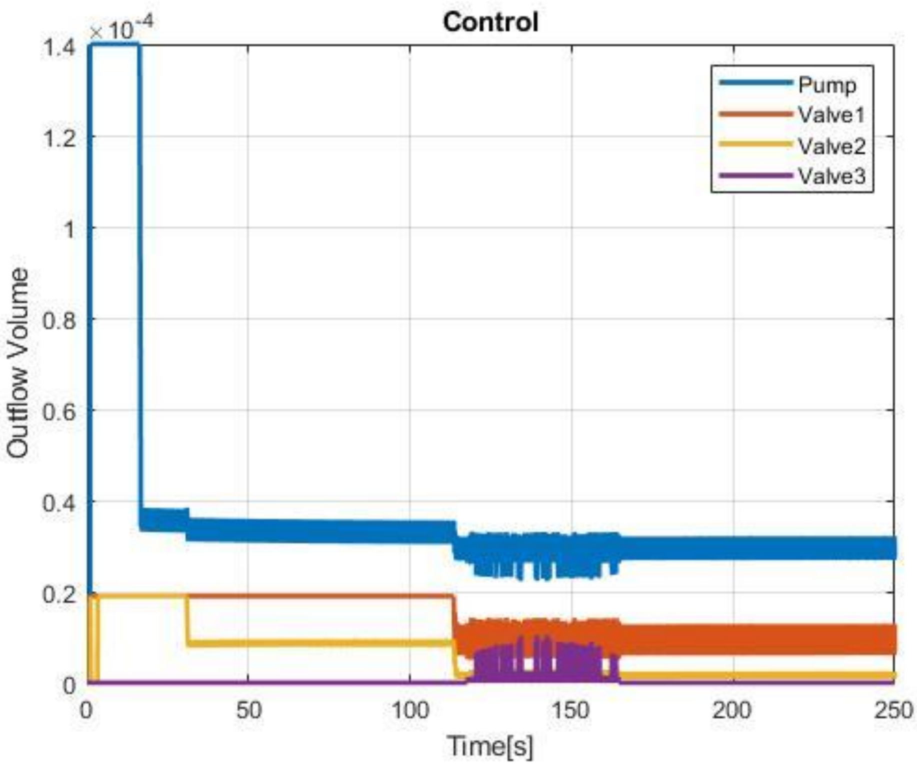

Fig. 8. *Control signals*

Using the designed H_∞ controller we can obtain good satisfaction of the technological requirements of the controlled multitank system.

On the last two figures we show the time domain responses of the closed-loop system with respect to load disturbances see (fig. 9). Here we observe good attenuation of the disturbance signals and also acceptable satisfaction of the technological requirements of the closed-loop system.

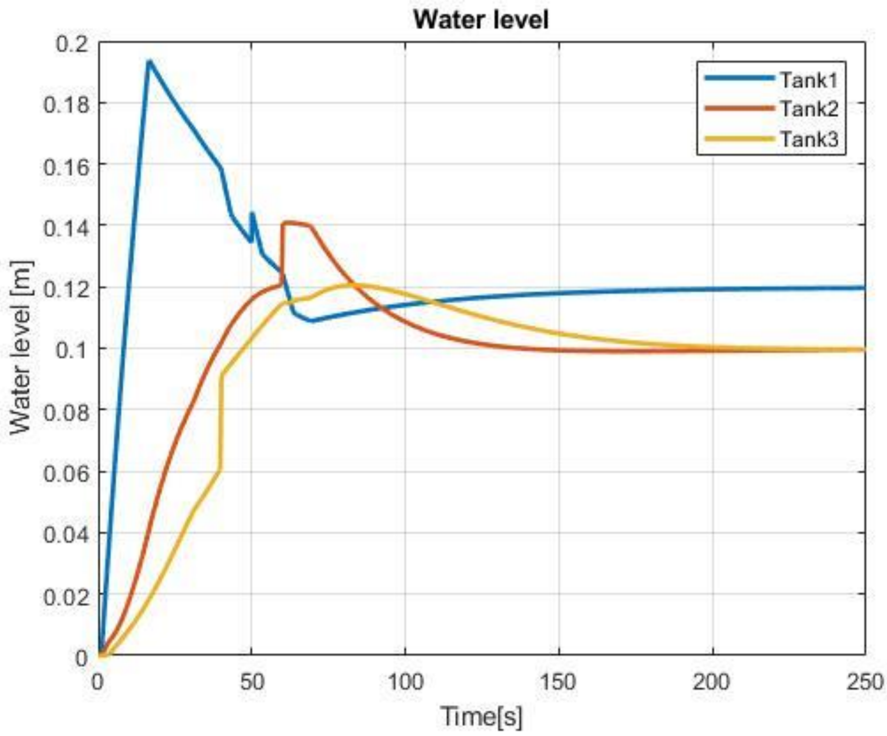

Fig. 9. *Time responses with respect to load disturbances*

5. CONCLUSION

In this paper we investigated the dynamic behavior of the multitank system. The considerations were connected with the application of the H_∞ approach. The performance for the multitank system being controlled using H_∞ technique was studied. The simulations obtained from the H_∞ control synthesis were presented. They clearly reveal the advantages of the considered optimal control technique. The usage such an approach leads to the fact that the time domain responses satisfy the technological requirements of the multitank system model.

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